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1 Introduction

In this paper we address the Two Echelon Capacitated Vehicle Routing Problem (*2E-VRP*), an extension of a classical Capacitated Vehicle Routing Problem (CVRP), in which a two level routing network is addressed. At the first-level, goods are transferred by urban trucks from the depot to intermediary depots, called *satellites*, where they are consolidated into smaller environment friendly vehicles performing the delivery to the customers, through the second-level routing [1].

This approach allows to efficiently keep large trucks far from city centers, while the last leg of the distribution activities is provided by small and environmental-friendly vehicles. This family of problems differs from the classical multi-echelon distribution systems addressed in literature, mainly focused on the utilization of facilities and the flows between levels. On the contrary, in *2E-VRP* the key element concerns the management of the fleet and the global routing of vehicles. The goal, as in traditional routing problems, is to ensure an efficient and low-cost operation of the system, where the demand is satisfied and the total cost of the traffic on the overall transportation network is minimized.

Literature on this problem is somehow limited. For what concern meta-heuristic methods, a multi-start heuristic able to reach good solutions in short computational time has been presented in [2], while two math-heuristics have been proposed in [1]. An Adaptive Large Neighborhood Search heuristics (ALNS) has been recently introduced. Since it is still not published, for more explanation about the method, we refer the reader to [4].

In this work we present a metaheuristic based on a reactive GRASP combined with a diversification heuristic and a feasibility search strategy. This algorithm produces an elite set of high quality solutions which are post-optimized by means of a Path Relinking.

2 Proposed method

The customer-to-satellite assignment problem plays a crucial role when addressing the *2E-VRP*. In fact, assuming one knows the optimal customer-to-satellite assignments, the *2E-VRP* can be partitioned into at most n_s+1 CVRP instances, where n_s is the number of satellites, one for the 1st level and one for each satellite with at least one customer assigned to it. Thus, as in the matheuristic presented in [1], and in the metaheuristic in [2], we focus on the customer-to-satellite assignment by searching for the optimal assignment, delegating state-of-the-art CVRP methods to solve the corresponding subproblems. To address these satellite-to-customer assignments, we propose a method that hybridizes two well-known metaheuristics, GRASP [5] and Path Relinking [3]. GRASP is used to build an elite set of solutions of size N, which then serve as starting point for an intensification-diversification phase performed by the Path Relinking. GRASP appears particularly well suited for the structure of problem considered, for which a global configuration may be obtained assigning one customer at a time, choosing among a limited subset of possibilities.

2.1 GRASP

At each iteration of the construction phase, a customer is assigned to a satellite following a greedy randomized rule, until all the customers have been assigned to a satellite. Assignment probabilities are computed following a probability function which is inversely proportional to the customer-to-satellite distance. A complete solution can be computed solving the resulting VRPs. This procedure does not imply the feasibility of the obtained solution, in fact, some assignment configurations could require for the routing phase, a number of vehicles higher than the number of available vehicles. If this happens, we try to rebuild a feasible solution by means of a Feasibility-Search strategy, (FS). This procedure consists into removing customers from the satellite to which the less filled vehicle belongs and reassign them to another satellite, in order to free that vehicle, until the global fleet-size constraint is satisfied. At each step of the procedure, we remove the customer for which the difference between the distance between itself and the nearest *available* satellite, and between itself and the satellite to which it is currently assigned, is minimum. A satellite is considered available if there is at least one vehicle with enough free room to take in charge the customer demand without violating the vehicle capacity constraints. Proceeding in this way, we certainly obtain a feasible solution. Once a feasible solution is reached, we apply the local search phase of the algorithm, which explores a neighborhood containing all the solutions differing from the current solution by one assignment only. The order according to which this neighborhood is explored is given by a distance-based rule (for more details we refer to [2]). The local search is performed following a first-improvement approach. At each GRASP iteration, a strong diversification move, called Depot Closure (DC), is applied with probability $q\%$. This move consists into closing a randomly-selected satellite s, meaning that all customer probabilities to be assigned to s are fixed to zero. These changes, which are maintained for the current iteration only, allow to analyze solutions very far in the solutions space, from the previously analyzed ones. An Elite Set (ES) containing the best K feasible solutions found is kept in memory. Every N GRASP iterations, the composition of solutions belonging to ES is analyzed. If a customer has been assigned to the same satellite in more than $p\%$ of these, it is deterministically assigned to that satellite for the following iterations, otherwise, in order to insert diversification with respect to the initial phase, assignment probabilities are modified following a uniform distribution among all the satellites. The algorithm stops after M iterations where M is an integer multiple of N.

2.2 Path Relinking

The Path Relinking phase consists in randomly choosing two solutions Z' and Z'' belonging to the ES, and linking them. At each step of the linking phase, an element of Z'' is introduced into Z', More precisely, a customer assigned to satellite s_1 in Z' and to satellite s_2 in Z'', is assigned to satellite s_2 in Z', without changing the other assignments. The order according to which we process the customers to be reassigned is given by a less-distance based rule, such that, at each iteration, we choose the customer, among those still assigned to different satellites in Z' and Z'', for which the difference among the distance between itself and the satellite to which it is assigned in Z'' and the distance between itself and the satellite to which it is assigned in Z',is minimum.

3 Computational results

Computational tests were undertaken on benchmark instances, with 50 customers and 3 satellites, introduced in [6]. Table 1 displays the average values of the objective functions obtained by our method and other heuristics in the literature. Columns 2 and 3 display the results obtained by the Reactive GRASP, with and without Path Relinking, respectively. Columns 4 and 5 show the values of the Greedy (FC) and the Multi-start (MS) heuristics presented in [2], while results obtained by the matheuristics proposed in [1] are given in Columns 6 and 7. Finally, Column 8 reports the values obtained by the ALNS metaheuristic [4]. From the results we can notice the proposed metaheuristic is very competitive,

METHOD	GRASP+DC+PR	GRASP+DC	FC	MS	DIVING	SEMI	ALNS
AVG VALUES	1396.58	1409.02	1523.55	1437.88	1521.44	1476.32	1382.34

Table 1: Summary of computational results

obtaining better results than most state-of-the-art methods. It is also very close, within 1% in average to ALNS. Moreover, one notices examining the detailed results that, the method we propose performs better than ALNS on instances with customer distributions (see [6] for more details) similar to what may typically be observed in urban settings. A detailed result analysis will be presented at the conference.

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