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(Article begins on next page)

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Memristive diode bridge with LCR filter

F. Corinto and A. Ascoli

The class of nonlinear dynamical systems known as memristive systems was defined by Chua and Kang back in 1976. Since then, a great deal of studies have addressed the search for physically-realizable memristive systems. In this Letter we prove that the class of memristive systems encloses an elementary electronic circuit comprising a full-wave rectifier with second-order RLC filter.

Introduction: The existence of a resistor endowed with memory was conjectured by Chua back in 1971 [1]. Just a few years later, Chua and Kang realized that the memory-resistor (memristor) is just one element from a class of nonlinear dynamical systems, the memristive systems, characterized, in general, by a state-dependent algebraic relation between input and output and by a system of ordinary differential equations governing the time evolution of the state [2]. Since then, scientists have devoted a great deal of efforts in the search for physical realizations of memristive systems. In this field of research the most well-known breakthrough was undoubtedly achieved in 2008 at Hewlett-Packard (HP) Laboratories (Labs), where Williams recognized the first memristor nano-device [3]. Subsequently, a thorough experimental investigation of the switching dynamics of a set of metal-dioxide-metal memristive nano-devices [4] led to a better understanding of the key physical mechanisms underlying the nonlinear behavior under observation. In 2011 we proposed a versatile boundary condition-based model (BCM) for memristive switching nano-structures [5]. The BCM state equation includes a boundary condition-controlled switching window function. The tunability of the window' switching mechanism allows the BCM to capture the nonlinear behavior of a large class of memristive nano-films, including all dynamics attributed to the HP memristor and reported in [3]. Taking inspiration from the BCM switching function, we recently investigated the possibility to realize a memristive electronic system through switching networks, frequently used in power electronics applications. This investigation led us to prove that the the Graëtz bridge with RLC series filter is a memristive system. In particular, the proposed circuit manifests the fingerprint of memristive systems [6], i.e. a pinched hysteretic current-voltage loop for any state initial condition and for any non-zero amplitude and any non-zero and non-infinite frequency of any periodic sign-varying driving source. Peculiarity of our circuit is that it acts as a nonlinear resistor at direct current (dc) and at infinite frequency of any periodic sign-varying driving source with non-zero amplitude. It is common belief that the only memristor realizations employing already-existing components are the complex topologies based upon the mutator-based active circuits and presented in [1]. This circuit represents the first-ever realization of a memristive electronic system using only passive components (4 diodes, an inductor, a capacitor and a resistor).

Theoretical analysis of the proposed circuit: Consider the circuit of Fig. 1. A voltage source v_g is applied across a full-wave rectifier cascaded with a RLC series filter. The relation between the voltage across and the current through diode D_k, named as v_k and i_k respectively ($k=\{1,2,3,4\}$), is modeled as $i_k = I_S\left(\exp\left(\frac{v_k}{nV_T}\right)-1\right)$, where I_S symbolizes the reverse saturation current, n is the emission coefficient, and $V_T = KTq^{-1}$ stands for the thermal voltage, where $K = 1.38\cdot 10^{-23}\,J\,K^{-1}$ is the Boltzmann's constant, T represents the absolute temperature and $q = 1.6\cdot 10^{-19} C$ refers to the elementary electronic charge.

Applying Kirchhoff's Voltage Law to the diode bridge mesh and to the mesh containing v_g, D_1 and D_2, and Kirchhoff's Current Law to the nodes respectively associated to anode and cathode of D_1, some algebraic manipulation reveal that at each time instant the following identities constrain the voltages across each pair of parallel diodes: $v_1 = v_3$ and $v_2 = v_4$, where $v_2 = v_1 - v_g$ and v_1 is found to be given by

$$v_1 = nV_T \ln\left(\frac{i_L + 2I_S}{2I_S\exp\left(-\frac{v_g}{2nV_T}\right)\cosh\left(\frac{v_g}{2nV_T}\right)}\right). \quad (1)$$

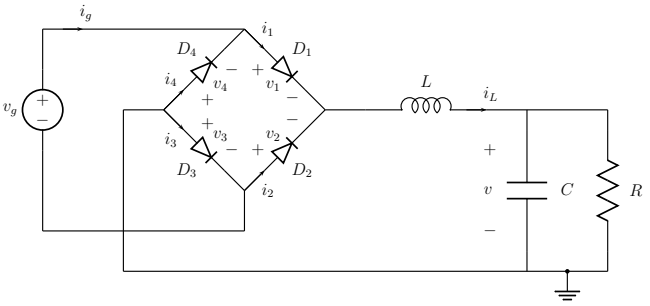


Fig. 1. A novel elementary memristive electronic cell.

Writing down the constitutive equations of the dynamic elements (i.e. C and L), deriving a closed-form expression for i_g, i.e.

$$i_g = (i_L + 2I_S)\tanh\left(\frac{v_g}{2nV_T}\right)$$

and further defining state variables as $x_1 = v\,(V_T)^{-1}$ and $x_2 = i_L\,(I_S)^{-1}$, system input and output as $u = v_g\,(V_T)^{-1}$ and $y = i_g\,(I_S)^{-1}$ respectively, and a new time variable as $\tau = t\,(t_0)^{-1}$, where $t_0 = 2\pi\,(\omega_0)^{-1}$ stands for the time normalization factor and $\omega_0 = [(LC)^{-1} - (RC)]^{\frac{1}{2}}$ denotes the resonant frequency of the second-order low-pass filter loading the Graëtz bridge, after some algebraic manipulation we get:

$$\frac{d\mathbf{x}}{dt} = f(\mathbf{x},u,t) \quad (2)$$

$$y = g(\mathbf{x},u,t)u \quad (3)$$

where $f(\mathbf{x},u,t):\mathbb{R}^2\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}^2$, describing the time evolution of the vector state (i.e. $\mathbf{x} = [x_1,x_2]'\in\mathbb{R}^2$), and $g(\mathbf{x},u,t):\mathbb{R}^2\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$, modeling the input-output behavior, are defined as

$$f(\mathbf{x},u,t) = \begin{bmatrix} \beta(x_2-\alpha x_1) \\ \gamma\left(u - x_1 - 2\ln\left(\frac{x_2+2}{2\exp\left(-\frac{u}{2n}\right)\cosh\left(\frac{u}{2n}\right)}\right)\right)\end{bmatrix} \quad (4)$$

$$g(\mathbf{x},u,t) = (x_2+2)\frac{\sum_{m=0}^{\infty}\frac{(\frac{u}{2n})^{2m}}{(2m+1)!}}{\sum_{m=0}^{\infty}\frac{(\frac{u}{2n})^{2m}}{(2m)!}}, \quad (5)$$

where $\alpha = \frac{V_T}{RI_S}$, $\beta = \frac{I_S t_0}{CV_T}$ and $\gamma = \frac{V_T t_0}{LI_S}$ are dimensionless parameters. Note that (5), equivalent to $\frac{i_g V_T}{v_g I_S}$ and thus standing for the memductance of the system unless a proportionality factor, is derived through the use of Taylor series expansions of hyperbolic functions. Equations (2)-(3) are nothing but the defining equations for the class of memristive systems [2], and, expressing $f(\cdot,\cdot,\cdot)$ and $g(\cdot,\cdot,\cdot)$ by means of equations (4) and (5) respectively, they define the *mathematical model* of the proposed circuit. This proves that the elementary circuit of Fig. 1 is a *second-order memristive system*. Note that the key mechanisms at the origin of its memristive behavior are the voltage constraints involving each pair of parallel diodes, i.e. $v_1 = v_3$ and $v_2 = v_4$.

Theory validation: An extensive number of PSpice simulations of the circuit of Fig. 1 enabled us to confirm the validity of the theory. Let us describe some simulation result. Diode $D1N4148$ is used in the Graëtz bridge. The values for the passive components are taken as $R = 1.5K\Omega$, $C = 4\mu F$ and $L = 2.5\mu H$. The initial conditions of the voltage across the capacitor and of the current through the inductor are respectively chosen as $v(0) = 0.01\,V$ and $i_L(0) = 0.01\,A$. The circuit is driven by a sine-wave voltage source expressed by $v_g = v_{g0}\sin(2\pi ft)$ with amplitude $v_{g0} = 1.25$ and frequency $f = 10\,Hz$. Under such parameter setting, the proposed circuit manifests the typical pinched hysteretic current-voltage loop characterizing memristive systems, as it is shown in Fig. 2(a). Sweeping frequency above $10\,Hz$, the lobes of the i_g-v_g loop get increasingly squeezed (while stretching along the i_g axis), as it is demonstrated in Figs. 2(b)-(c), where f is set to 100 and to $1000\,Hz$ respectively. From these graphs it may be inferred that at infinite frequency the circuit behaves as a nonlinear resistor. Note that even sweeping frequency below $10\,Hz$ yields a gradual flattening of the loop lobes until nonlinearly resistive behavior

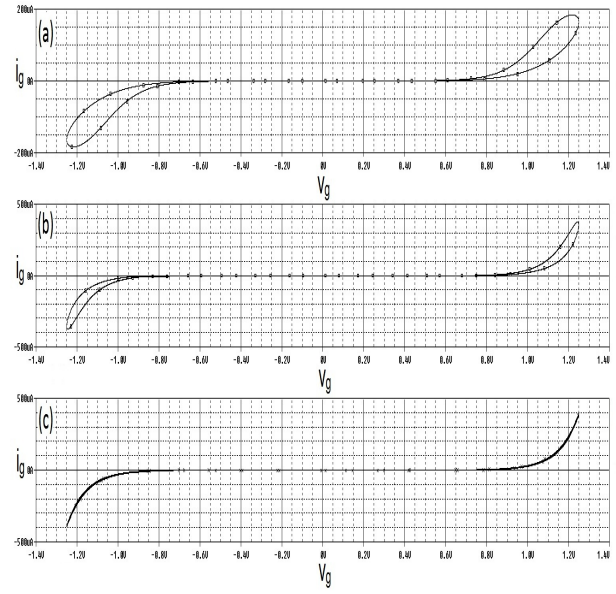


Fig. 2 *Current-voltage characteristics derived from PSpice simulations of the circuit of Fig. 1 under sine-wave input with frequency equal to* 10 *(plot (a)),* 100 *(plot (b)) and* 1000 Hz *(plot (c)).*

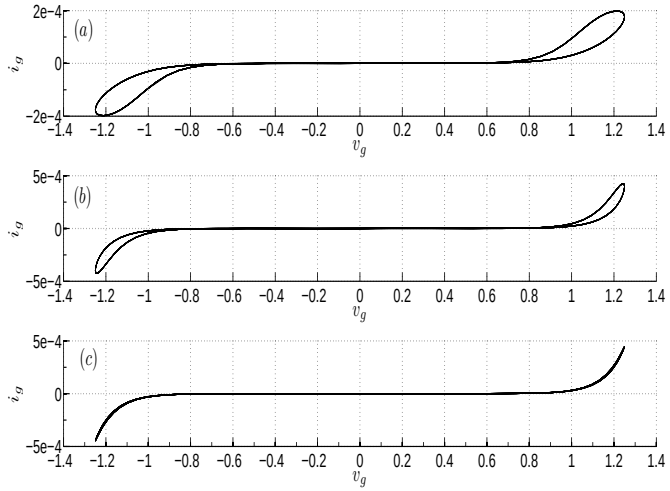


Fig. 3 *Current-voltage characteristics observed in numerical simulations of the mathematical model of the proposed circuit for a sine-wave input with* f *set to* 10 *(plot (a)),* 100 *(plot (b)) and* 1000 Hz *(plot (c)).*

arises at dc. In order to ascertain the robustness of the i_g-v_g pinched hysteresis, besides input frequency f we also swept input amplitude v_{g0}, initial conditions $v(0)$ and $i_L(0)$, values of passive components, and even periodic sign-varying input signals including triangle and square waves.

The theoretical analysis was further validated through the run of a large series of Matlab simulations of dynamical system (2)-(3) with $f(\cdot,\cdot,\cdot)$ and $g(\cdot,\cdot,\cdot)$ respectively defined by equations (4) and (5). Let us present those simulation results enabling a comparison with Fig. 2. With regards to the diode parameters used in the model, the values of reverse saturation current and emission coefficient refer to diode $D1N4148$, i.e. $I_S = 2.682\,nA$ and $n = 1.836$, while $V_T = 25\,mV$ (at $T = 293\,K$). Choosing the same values for R, L and C as in Fig. 2, $t_0 = 1.9910^{-5}s$. State initial condition is chosen as $\mathbf{x}(0) = [x_1(0), x_2(0)] = [v(0)\,(V_T)^{-1}, i_L(0)\,(I_S)^{-1}]$. The input to the system is expressed by $u = u_0 \sin(2\pi k\tau)$ with dimensionless amplitude $u_0 = v_{g0}\,(V_T)^{-1}$ and dimensionless frequency $k = f\,t_0$. Setting $v(0)$, $i_L(0)$ and v_{g0} and sweeping f as in Fig. 2, the resulting i_g-v_g characteristics are illustrated in Fig. 3. The qualitative and quantitative agreement with Fig. 2 is evident.

Conclusion: In this Letter a theoretical proof for the memristive nature of a simple electronic system is presented. The circuit is composed of an elementary diode bridge and of a RLC series circuit, respectively introducing nonlinearity and dynamical behavior into the system. It is the first time a purely passive circuit, employing already-existing components, is found to manifest memristive dynamics. Being inexpensive and easy to fabricate, its analysis may be introduced in the academic curricula for undergraduate students as a demonstration for the existence of memristive behavior in nature. Agreement between PSpice simulations of the circuit and numerical solutions to the system model validate the theoretical proof. This was further confirmed through experimental verification. Due to lack of space, the results of this investigation shall be published elsewhere.

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