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Efficient Computation of Transient Responses of Frequency-Dependent Nonlinearly Loaded Transmission Lines

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Abstract- **In this paper, we address the combined time and frequency domain analysis of nonlinearly loaded low-loss transmission lines. We show that a variety of interconnects are characterized by transfer functions, whose impulse responses have a fast initial-time structure and a slow long-time part. A piecewise linear approximation of the transient functions with nonuniform sampling is proposed as an effective method to obtain high accuracy with low computational costs.**

I. Introduction

The problem of signal propagation along transmission lines loaded by nonlinear elements has a central role in the modern technology of signal processing and transmission. Fast digital and analog circuits, at any level of integration, offer a wide choice of examples of single and multiconductor transmission lines connected with nonlinear devices. The decreasing rise time of signal waveforms emphasizes the importance of the propagation effects, and the signal corruption caused by parasitic phenomena as losses and skin effect is now a relevant issue in many applications.

The analysis of nonlinearly terminated lossy transmission lines is addressed in this paper with a modified version of a method belonging to the class of *mixed* techniques [1], which characterize the line via scattering parameters. The time-domain Green's functions of the interconnects are derived via inverse Fourier transformation of the frequency-domain scattering parameters. These impulse responses are then convolved with the impinging voltage waves, and a numerical solver is applied to account for the nonlinear terminations.

For ideal (lossless and dispersionless) lines, such Green's functions can be obtained in closed form, and are expressed in terms of Dirac δ functions located at the multiples of the line delay. On the contrary, if losses and dispersion cannot be neglected, the inverse transformation of the spectral scattering parameters can be a problem. In this case the impulse responses are usually derived via numerical inversion techniques, [1]-[3]. However, there exist several circumstances in which spectral peaks appear in the magnitude plot of the line transfer functions, which translate into multiple-echoes of the corresponding impulse responses. The presence of multiple short echoes separated by many times their duration make it extremely difficult to obtain the pulse behavior of the line via an inverse FFT algorithm. Moreover, a very large amount of samples is required for an adequate representation of the Green's functions, and this lengthens the convolution computation of the transient response. In this paper, we show that the time-domain characterization of the interconnects can be often obtained via analytical inversion of the spectral functions. The resulting impulse responses represent approximated expressions which are advantageously employed in the subsequent computations. In particular, we concentrate on the transient analysis of low-loss interconnects defined as those for which $R_{DC}\mathcal{L} < 2Z_{LC}$, where R_{DC} is the line ohmic resistence per unit length, $\mathcal{L}$ is the line length, and Z_{LC} is the characteristic impedance of an associated ideal line without losses. We will show that the behavior of the corresponding impulse responses present a fast short-time structure, together with the usual long-time shape of the curve, due to ohmic losses.

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In order to provide a precise representation of the line impulse response over long time intervals with the minimal amount of data, we propose a nonuniformly-sampled piecewise linear approximation of the functions involved in the transient.

II. Pulse Behavior of Low-Loss Lines

For a two-conductor interconnect the mode propagation factor is

$$\begin{aligned} K(\omega) &= \beta(\omega) - j\alpha(\omega) \\ &= \sqrt{-[R + j\omega L][G + j\omega C]}, \end{aligned} \qquad (1)$$

where α and β represent the attenuation and phase functions, respectively. In a scattering parameters formulation [1], [4], the line Green's function is related to (1) by the following relationship

$$h(t) = \mathcal{F}^{-1}\left\{e^{-jK(\omega)\mathcal{L}}\right\}, \qquad (2)$$

$\mathcal{L}$ being the interconnect length. The features of the impulse response $h(t)$ are determined by the characteristics of the functions $R(\omega)$, $C(\omega)$, $L(\omega)$, and $G(\omega)$, that represent the frequency-dependent per-unit-length parameters of the real interconnection lines. In practice, the hypothesis of a small dispersion allows us to employ a frequency independent approximation for the per-unit-length capacitance. On the contrary, the values of the per-unit-length resistance and inductance may be greatly influenced by the skin effect, *i.e.*, by the magnetic flux penetration inside the conductors [5]. The quantity G parameterizes the effect of dielectric losses.

The effects of the parameters of $H(\omega)$ on the features of impulse response $h(t)$ can be explained through the attenuation curve $\alpha(\omega)$, which determines the amplitude of the transfer function.

In [4] it is demonstrated that the impulse response of a low-loss line with low values of $R_{DC}v/Z_{LC}$ contains both a fast time structure, due to the high frequency part of the transfer function, and slow time components, due to the low frequency ohmic part of the transfer function. In order to gain further insight into the structure of $h(t)$, it is useful to study the exact impulse responses of two canonical forms of $H(\omega)$. The first form corresponds to a line with pure ohmic losses, and is defined for $R = R_{DC}$, $G = 0$, for all frequencies. The second form corresponds to a line with pure skin losses and is characterized by $R = (1+j)R_o\sqrt{\omega}$ and $G = 0$, for all frequencies.

The exact impulse response corresponding to the first canonical form of $H(\omega)$ is [6]

$$\begin{aligned} h_r(t) &= e^{-\xi/2}\sigma\Big[\delta(\sigma(t-\tau)) + \\ &\quad + \frac{\xi}{2}\frac{e^{-\sigma(t-\tau)}}{\sqrt{\sigma(t-\tau)(\sigma(t-\tau)+\xi)}} \cdot \\ &\quad \cdot I_1\Big(\sqrt{\sigma(t-\tau)(\sigma(t-\tau)+\xi)}\Big)\Big], \end{aligned} \qquad (3)$$

where the following normalization parameters have been introduced: $\tau = \mathcal{L}/v$, that represents the line delay, $\sigma = R_{DC}v/2Z_{LC}$ and $\xi = R_{DC}\mathcal{L}/Z_{LC}$, that are loss parameters; $I_1(\cdot)$ is the modified Bessel function of order 1. We indicate with h_{rns} the nonsingular part of h_r in (4)

$$h_{rns}(x,\xi) = \sigma\frac{e^{-x}}{\sqrt{x(x+\xi)}}I_1\Big(\sqrt{x(x+\xi)}\Big), \qquad (4)$$

where $x = \sigma(t-\tau)$ represents a normalized time. For low-loss lines (*i.e.*, $\xi < 2$), h_{rns} is weakly dependent on ξ, and can be approximated by $h_{rns}(x, \xi = 0)$.

The impulse response corresponding to the second canonical form is [5]

$$h_s(t) = \frac{1}{\eta}\frac{1}{\sqrt{\pi\left[\frac{t-\tau}{\eta}\right]^3}}e^{-\eta/(t-\tau)} \qquad (5)$$

where $\eta = \xi K\mathcal{L}/8Z_{LC}$. Both functions have a fast initial part and then decrease slowly for increasing value of the arguments. The durations w of the fast part of h_{rns} and h_s are estimated to be $w_r = 6/\sigma$ and $w_s = 10\eta$, respectively. The long tails of the pulses yield a significant contribution when the functions h_r and h_s are convolved with the input wave voltages [4].

The impulse response of a real low loss line includes the features of both canonical functions h_r and h_s. Its long-time part is due to the low-frequency ohmic part of H and evolves as h_r with duration independent of $\mathcal{L}$, while its short-time part is caused by the high-frequency skin effect and evolves as h_s. In order to have an accurate short and long time

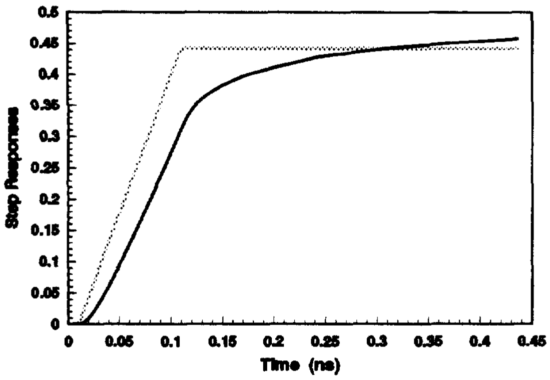

Figure 1: Time-domain step responses of the two-conductor interconnect described in the text; the rise time of the input signal is $T = 100$ ps.

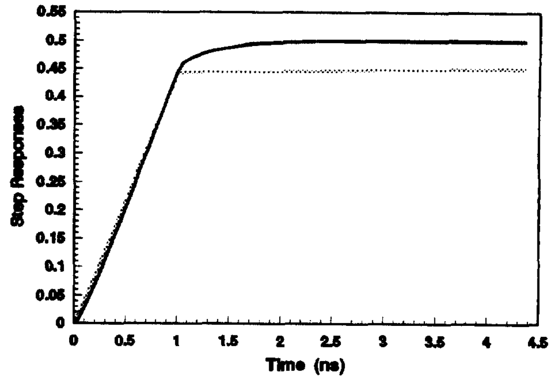

Figure 2: Time-domain step responses of the two-conductor interconnect described in the text; the rise time of the input signal is $T = 1$ ns.

representation of $h(t)$, the usual uniformly-sampled staircase representation is not suitable. In fact, a sampling pitch adequate to the initial fast part of the function implies that the number of samples needed to describe the function for the whole time interval of interest is too large, thus lenghtening the computation of the convolutions required by the solution of the transient equations. In this paper, a nonuniformly spaced piecewise linear representation of h and of all time functions involved in the transient problem is exploited in order to obtain an accurate representation with high numerical efficiency.

III. Numerical Example

The structure under analysis is a single low-loss line loaded at both ends by matched terminations. The line parameters are chosen to be representative of the ceramic packages interconnection technology [7]: the asymptotic characteristic impedance is $Z_{LC} = 50\ \Omega$, the ohmic series resistance is $R_{DC} = 50\ \Omega/\mathrm{m}$, the propagation velocity is $v = 2\times 10^8$ m/s, and the line length is $\mathcal{L} = 0.25$ m. Based on these data, the loss parameter ξ equals 1/4, meaning that both the ohmic and the skin components of the line impulse response are expected to be relevant, and the time duration of the fast parts of h_r and of h_s are $w_r = 60$ ns and $w_s = 40$ ps, respectively. The line is excited by a step-like voltage source, whose expression is

$$v_s(t) = \frac{1}{T}\left[tu(t) - (t-T)u(t-T)\right], \qquad (6)$$

where $u(t)$ is the unit step function and T is the rise time. The numerical algorithm used for the transient analysis of this network exploits the piecewise linear representation of the functions involved.

The results are summarized in Fig.s 1, 2, and 3, where the load voltages are plotted for different choices of the line impulse response and of the rise time of the input signal. The solid curves denote the load voltage obtained by assuming h_s as the impulse response of the line; the dotted curves represent the same variable, but h_r is chosen to model the line response. Fig. 1 refers to the case $T = 100$ ps. In this situation, the best approximation of the solution is the one obtained through h_s, since the dotted curve completely fails to predict the shape of the rising ramp.

The results corresponding to a rise time of 1 ns are reproduced in Fig. 2. This simulation is representative of a situation where neither of the canonical models are adequate to represent the complete line impulse response. In fact, if the interconnection is modelled considering only the skin effect (*i.e.,* using h_s alone), the output voltage reaches the full amplitude value with excessive fastness. The reason is because the steady state level is strictly connected with the low frequency characteristics of the line transfer function, and in such region the skin effect plays a minor role. On the other hand, a simulation based on a constant series resistance model fails to predict the correct shape of the upper knee of the curve. This is essentially due to the inadequate description of the high frequency components of the line transfer function.

Finally, the case of a signal rise time of 10 ns was considered. Since any increase of the rise time T produces a narrower frequency spectrum of the travelling step, we expect less pronounced dispersion effects on the output waveform. In fact, the effect of h_s merely amounts to a slight smoothing of the step knee as shown by the solid curve (see Fig. 3). The dotted curve, corresponding to the h_r model, is the best approximation, thereby showing that the low frequency effects are dominant in this case.

IV. Conclusions

This paper deals with a mixed (*i.e.*, in the frequency and time domain) method for the transient analysis of low-loss lines.

A significant result of our analysis is the identification of a fast and slow structure into the line impulse response, and the recognition that their presence is due to skin effect and ohmic losses, respectively. We have shown also that the two phenomena can be separately illustrated by means of exact analytical solutions. The influence of the fast and slow parts of the impulse response on the transmission of step signals has been evidenced by an example where the line is excited by generators with different rise times.

Also, we propose a nonuniform piecewise linear approximation of the line impulse response and of all functions involved in the transient analysis, in order to guarantee both an adequate representation of all time scales and to deal with a minimum amount of data.

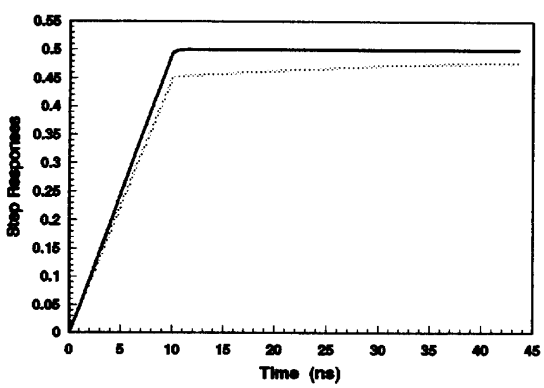

Figure 3: Time-domain step responses of the two-conductor interconnect described in the text; the rise time of the input signal is $T = 10$ ns.

Acknowledgments

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