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Tunnel Static Behavior Assessed by a Probabilistic Approach to the Back-Analysis

Claudio Oggeri and Pierpaolo Oreste
Department of Environmental,
Land and Infrastructural Engineering (DIATI), Politecnico di Torino, Italy

Abstract: Problem statement: The steps of the validation procedure of the project of a tunnel are briefly illustrated in this study, starting from the geological and structural surveys on the excavation walls and the measurement of the physical and mechanical parameters during the excavation works. Unfortunately, however, the knowledge of the rock mass, which is fundamental to the project, is usually approximate before the study is started. This knowledge improves considerably once construction of the tunnel is started, when it is possible to have direct access to the rock and analyze its behavior in relation to the excavation and support works. **Approach:** The measurement of displacements and stresses in the rock mass and in the support structures represents a different methodology for the evaluation of the Geotechnical characteristics of a rock mass and therefore also of the support work conditions. To correctly interpret the measurements it is necessary to make use of a more complex procedure, called back-analysis, that, starting from an estimation of the unknown parameters of the rock mass obtained through a preliminary characterization, integrated and modified by sampling of the rock mass during the construction stage and by the performed stress and displacement measurements, is able to define the unknown parameters of the rock mass. **Results and Conclusion:** Back-analysis in engineering in the rock field occurs, however, in an uncertainty context, which complicates the problem. The preliminary estimation of the Geotechnical characteristics of the rock mass has in fact a degree of reliability that is a function of the intensity of the preliminary investigations. The performed measurements present a certain precision in relation to the various typologies of error that can occur. The final result of the back-analysis therefore also consists in the definition of the Geotechnical parameters of the rock mass that are considered to be of influence in the problem under examination, with a certain reliability and precision that is obviously greater than that relative to the initial estimation of the same parameters. The purpose of this study is to present a global approach to back-analysis in a probabilistic context that is aimed to obtain a reliable calibration of the parameters of the rock mass that are necessary to study the behavior of the support structures.

Key words: Geotechnical parameters, support structures, measurements present, structural surveys, definitely confirmed, displacement measurements, preliminary estimation

INTRODUCTION

The knowledge of the rock mass, which is fundamental for the tunnel design, is usually approximate before the study is started. This knowledge improves considerably once construction of the tunnel is started, when it is possible to have direct access to the rock and analyze its behavior in relation to the excavation and support works (Oreste, 2005; Oreste, 2009a). The design of a tunnel should therefore be validated during work procedures when all the assumed hypotheses of the preliminary stage can be definitely confirmed. In the case where, during the study procedures, elements emerge that are in contrast to the preliminary assumptions, the project should first be re-verified and then, if necessary, modified or integrated and finally validated. The validation process of a tunnel project should therefore be able to rely on all the information that becomes available during its construction.

The behavior of underground cavities and therefore also of the supports that are necessary to guarantee stability of the voids, depends, to a great extent, on the Geotechnical characteristics of the rock mass (above all strength and deformability) (Oreste, 2003; 2007; 2008; 2009b; 2009c; Osgoui and Oreste, 2007; 2010; Oreste, 2002). The definition of the Geotechnical characteristics is today made in two different ways:

- Through the geomechanical characterization of the natural materials, with attribution of the physical, mechanical and hydraulic parameters at small and

Corresponding Author: Claudio Oggeri, DIATI, Politecnico di Torino, Italy

large scale (that is, at the laboratory sample scale and the problem scale, respectively)

- Through the measurement of sizes described by the stress and strain behavior of the rock during the construction of the study or during the construction of nearby auxiliary works that have to be built

In the first case the evaluation of the Geotechnical parameters occurs before and during the construction of the study and is based on laboratory tests and in situ sampling, with the help of experience gained from other analogous sites, experience that is organized in a rational manner through the different widespread technical classifications. Unfortunately, however, the preliminary knowledge of the rock mass is never certain; in some cases (very deep mountain pass tunnels or tunnels in complex and chaotic formations) it is even rather approximate and basically based on surface geological studies, therefore the use of geomechanical characterization during the first stages of construction the study results to be fundamental (Oreste and Longo, 2010).

The measurement of displacements and stresses in the rock mass and in the support structures represents a different methodology for the evaluation of the Geotechnical characteristics of a rock mass and therefore also of the support work conditions. To correctly interpret the measurements it is necessary to make use of a more complex procedure, called back-analysis, that, starting from an estimation of the unknown parameters of the rock mass obtained through a preliminary characterization, integrated and modified by sampling of the rock mass during the construction stage and by the performed stress and displacement measurements, is able to define the unknown parameters of the rock mass.

Back-analysis in the rock engineering field occurs, however, in an uncertainty context, which complicates the problem. The preliminary estimation of the Geotechnical characteristics of the rock mass has in fact a degree of reliability that is a function of the intensity of the preliminary investigations. The performed measurements present a certain precision in relation to the various typologies of error that can occur. The final result of the back-analysis therefore also consists in the definition of the Geotechnical parameters of the rock mass that are considered to be of influence in the problem under examination, with a certain reliability and precision that is obviously greater than that relative to the initial estimation of the same parameters.

The purpose of this study is to present a global approach to back-analysis in a probabilistic context that is aimed to obtain a reliable calibration of the parameters of the rock mass that are necessary to study the behavior of the support structures and to validate the preliminary tunnel design.

MATERIALS AND METHODS

The significance of the design validation during the tunnel construction: The designing of a tunnel presents particular characteristics that are different from other engineering works: The knowledge of the rock mass, which is at the basis of the design hypothesis, can only be approximate before the excavation works are started. The geomechanical parameters and the stress-strain behavior laws of the rock mass are estimated, starting from the results of the diagnostic investigations, the in situ and the laboratory tests. The entity of the preliminary investigations is however always limited because of economic reasons and sometimes because of the short time available and, in the case of deep tunnels, only the surface area of the rock mass involved in the tunnel excavation is studied. The obvious variety of natural materials furthermore sometimes prevents one from extrapolating the situations that exist between one survey and another in a reliable manner.

Even the numerical simulation of the response of the rock mass to the excavation operations can only offer rough indications. Numerical calculation methods in fact require a simplified behavior law for the rock mass at the scale of the problem which cannot be obtained through the usual laboratory tests.

For these reasons, during the construction of a tunnel, the project hypothesis should be verified through a validation procedure of the initial project. A great number of data can in fact be obtained during the study construction stage: Direct geological and structural surveys on the excavation walls (first of all, rock samples of a suitable size for laboratory tests), carrying out mechanical and geophysical in situ tests, carrying out investigations during advancement, the measurement of the stress-strain state in the rock and in the support structures, surveying the functioning parameters of the perforation and consolidation machines and obtaining the functioning parameters of the TBMs. All these data are added to the information that was available at the beginning during the design stage, allowing an updating of the calculation model to verify the initial project and to define any possible modifications that prove necessary. The initial, possibly modified project is therefore subject to a validation procedure.

The evaluation in the design stage of stresses and strains around a tunnel: The stress and displacement that develop around a tunnel and in the supports during and after its construction are generally estimated in the design stage. Their magnitudes are the subject of the measurements that are carried out during work. The technician in charge of planning, performing and interpreting the measurements should therefore preventively and qualitatively know the trend and the order of magnitude of these values.

The stress and strain state around a tunnel and on the inside of the supports is conditioned by different factors, which summarized, can be thus divided:

- The quality of the rock mass, identified from a quality index
- Dimensions, geometry and depth of the tunnel
- Stress conditions of the rock and, in particular, the nature and intensity of the horizontal stresses in undisturbed conditions
- The entity, number and type of supports installed
- The distance from the excavation face along the tunnel axis

The design of a tunnel is based on different methodologies:

- An analytic approach, usually reserved to simple geometric situations or parts of a general system
- An empirical-observational approach, based on case histories of already constructed works or on the phenomenological knowledge of the rock on the whole
- A numerical approach, in which the rock and the support works are modelled through different calculation codes, to be adopted in function of the structure of the rock and the type of work

The numerical approach is nowadays quite common. The use of simple analytical calculation procedures still play an important role in the preliminary comprehension of the phenomenon and the definition of the physical and mechanical parameters of influence.

There however exists some limitations to a perfect adherence of a numerical modelling to the physical reality of the problems and this is due to the heterogeneity of the geological formations, to the non simple geometry, to the spatial variability of the parameters of the rock, to the presence of water and to the problematic knowledge of the original stress state in the rock mass. Then there are problems that are difficult to transfer to a model, which, however, are of fundamental importance: for example mention can be made of phenomena that are dependent on time, the influence of mechanical and physical anisotropies, the relevance of technological aspects connected to the modality and times of excavation and of the installation of the supports.

For these reasons approximate schemes are adopted, usually formulating simplifying theories on the behavior of the natural materials.

In spite of all this, the use of numerical modelling should not be played down as, from the engineering point of view, even before that of the physical or mathematical point of view, what is necessary to establish is a connection between cause and effect and to know how to understand the evolution of the phenomenon as a tendency.

In order to work in a reliable way, it is necessary to have some connections with reality. This meeting point is represented by in situ measurements carried out during excavation of the tunnel. Experience has shown how it is necessary to have an agreement between numerical modelling and measurements; that means that the choice of measurement parameters, the installation of the instruments and the frequency of the measurements should be as connectable as the model can offer. On the other hand it is opportune that the model be chosen and validated taking the geostructural characteristics of the rock mass and the scale of the study into consideration (equivalent continuous, discontinuous).

The geomechanical characterization of the rock mass, the numerical modelling of the problem under examination, the planning, the performing and the interpreting of the measurements cannot be carried out separately, but should be continuously integrable during the construction of the study in order to improve the numerical simulation and therefore the comprehension of the real phenomenon so as to be able to take the necessary countermeasures and, if necessary, to redefine the support system.

RESULTS

Monitoring measurements during the work: The investigations and measurements that are carried out before, during and even after the excavation of the tunnel, investigate a large series of parameters. In the field of underground construction the most frequently carried out measurements refer to one of the following groups of magnitudes, according to the particular problem: relative displacements, absolute displacements and stresses in the linings and in the rock mass, pressures on the lining and forces in the anchorages in the rock, or water table pressures. Preference is usually given to displacement measurements (such as convergence measurements of the tunnel walls) as they represent, from the mathematical point of view, integral magnitudes that are not subject to typical local effects. Stresses and strains are instead differential magnitudes that can present values that are very different from point to point and should therefore be calculated on sufficiently extended areas so as to be able to furnish appreciable indications.

With measurements carried out during the study, one usually searches for the following responses:

- A check on the safety
- The determination of the properties of the materials and, if possible, of their undisturbed stress state (obtained through back-analysis techniques); the determination of the physical and mechanical characteristics of the rock mass results to be particularly interesting at this stage as they refer to the problem in real magnitude, this being different from the preliminary tests which instead involve only a limited portion of the rock mass
- Verification of the validity of the instrument choice, with reference to the excavation method
- Comparison of the prediction theories with the real structural behavior

The displacement measurements that are usually carried out can be divided into the following types:

- Measurement of the variations of the dimensions of the tunnel (measurement of the relative convergence)
- Measurements of the displacements of the surrounding rock with reference to an area that is considered stable (measurement of distension, which can be performed working on the walls of the cavity or, when possible, from the external surfaces)
- Displacement measurements between the two borders of a relevant structural discontinuity
- Strain measurements in the body by the lining
- Strain measurements of the excavation face, especially when working in large sections, in soft or loose rock and when there are important reinforcing interventions of the nucleus (also called extrusion measurements)

The loading measurements on the supports concern, in particular, the bolts, the tendons, the steel sets and the tie-bolt heads.

The strain measurements concern, in particular, the aspects of interaction between the natural formation and the support works, whether they are of a preliminary type, such as cast concrete shells, or of a permanent type.

The measurement stations should be installed so as to be able to identify the expected variations without these being masked by other secondary phenomena. Furthermore, it is a good idea that the positions of the measurement stations can be clearly identified in the numerical model used for the simulation of the tunnel. In this way one reduces the possibility of loosing information simply because of geometric type reasons (for example, because of a different orientation of a bi-dimensional model from the section to be measured) or for geological reasons (for example, because of a lithological variation between the installed measurement section and the section studied in a bi-dimensional model).

It is also advisable to not put one's trust in a single measurement position, in order to prevent local malfunctioning or anomalies from conditioning first the good results of the measurements and then the comparison with the calculation.

The volume that is involved by the instrumentation is of interest in that it is from this that one derives the either local or extended value of the measurements that have been carried out. Some parameters of the rock (for example, the strain) or the stress conditions induced around the excavation are greatly influenced by the reference volume. The rock structure makes the measured value vary with the direction and with the entering of the rock, above all in fractured or anisotropic rock masses. For example, the stress measurements along the border of an excavation, carried out with flat jacks, are greatly conditioned by the redistribution of the stresses, with steep gradients close to the angular areas.

As the volume of interest grows, the measurement data tend to become homogeneous while local measurements show a greater dispersion.

The precision required of the measurement instrumentation varies according to the order of magnitude of the expected phenomena. However, for practical reasons, it is common that the potentiality of the instruments to be used in tunnels is adaptable to the various requirements. It should be remembered, however, how the cost of a measurement device is connected to the precision according to a power function: the choice should therefore respond to a cost-benefit analysis.

For example, the measurement of displacements is repeatable within 0.01 mm, the load measurements within 100 N, the pressure measurements within 10 kPa, the extensimetric deformation measurements within 5 $\mu\varepsilon$ and the inclinometric measurements within 1 mm/30 min.

The responses of the various instruments are however different in terms of overall capacity to understand the physical trend of the phenomenon without feeling local effects: the convergence and the speed of convergence, the measurements with a flat jack, the load cells on the bolts or on the steel arches are, for example, measurements that are more easily interpreted than those based on extensometers mounted at the extradox of the supports. This is probably due to the damage that the instruments can undergo during the different procedures in the construction phase.

Back-analysis in uncertain environments: In order to improve and make the first estimation of the geomechanical parameters of the rock obtained by the geomechanical characterization more reliable, one should proceed with the treatment of the results of the measurements through adequate back-analysis techniques (Oreste, 2005). By back-analysis in the excavation and rock engineering field one means that particular procedure, developed above all with numerical analysis methods of the stress-strain state of the rock mass and the supports which, starting from the displacement and strain measurements obtained in situ during the construction of the study, permits the calibration of the calculation model of the initial estimations:

- Of the geomechanical parameters of the natural material
- Of the initial strain state (undisturbed) in the rock mass

Modern back-analysis represents one of the most delicate stages of the whole planning program as one has only limited times available to be able to supply, during the construction, the necessary guidelines for any possible improvements of the original project and for the design of the unforeseen interventions which are however necessary to guarantee the stability of the study and an economic efficiency (Fig. 1).

These results to be even more important in the construction of underground tunnels and voids, when a certain variability of the geomechanical characteristics of the rock mass is encountered along the section which was not possible to ascertain in detail during the preliminary analysis.

Back-analysis therefore usually consists in the search for unknown parameters, of which one only has a preliminary estimation, that minimize the difference between the results of the calculation with the numerical model and the results of the performed measurements.

To perform a correct back-analysis it is usually necessary to choose:

- A suitable calculation model that is able to determine the stress and strain state in the rock mass, with the evolution of the excavation stages
- The function error, which measures the distance between the forecasts and the available measurements, with a variety of the unknown parameters

From what has emerged from the previous sections of this study, both the preliminary estimation of the geomechanical parameters of the rock mass and the results of the measurements, present a certain level of uncertainty in relation to the intrinsic variability of the rock mass quality, to the reduced availability of data and to the precision of the measurement instruments. The back-analysis should therefore be able to develop in an uncertain environment, furnishing a new estimation of the geomechanical parameters of the rock mass that are able to guarantee a greater reliability than the initial estimation (a smaller variance).

The uncertainty of the monitoring system can be described through the covariance matrix $C_{\Delta\bar{\eta}}$ of the errors $\Delta\bar{\eta}$ of the single m measurement. It is a square matrix of m x m dimensions expressed as (Eq. 1):

$$C_{\Delta\bar{\eta}} = E\left|\Delta\bar{\eta}\Delta\bar{\eta}^T\right| \quad (1)$$

where, the operator E|---| is the expected mean.

For statistically independent measurements, for which the operator conditions and instruments of one does not have repercussions on the other, $C_{\Delta\bar{\eta}}$ is a diagonal matrix with all positive values, which represent the variability of each single measurement. As a first approximation, the variance of a measurement can be intended as the square of the precision of a measurement instrument.

The initial estimation of the unknown vector $x^0 = \left(x_1^0, x_2^0 \ldots, x_n^0\right)$ can also be considered as a probabilistic variable, for which one assumes (Eq. 2 and 3):

$$x^{(0)} = E|x| \quad (2)$$

$$C_{x^{(0)}} = E\left|\left[x - x^{(0)}\right]\left[x - x^{(0)}\right]^T\right| \quad (3)$$

where, x is the unknown vector column, of n terms;

$C_x^{(0)}$ is the square matrix of the covariance of the initial estimation, of n x n dimensions.

If the initial estimation of the unknown parameters is not correlated in probabilistic terms (unfortunately they often are as the initial estimation refers to the same quality index for the majority of the unknown parameters), the matrix $C_x^{(0)}$ is diagonal. With an increase of the uncertainty of the initial estimations, there is an increase in the values of the components of the matrix $C_x^{(0)}$.

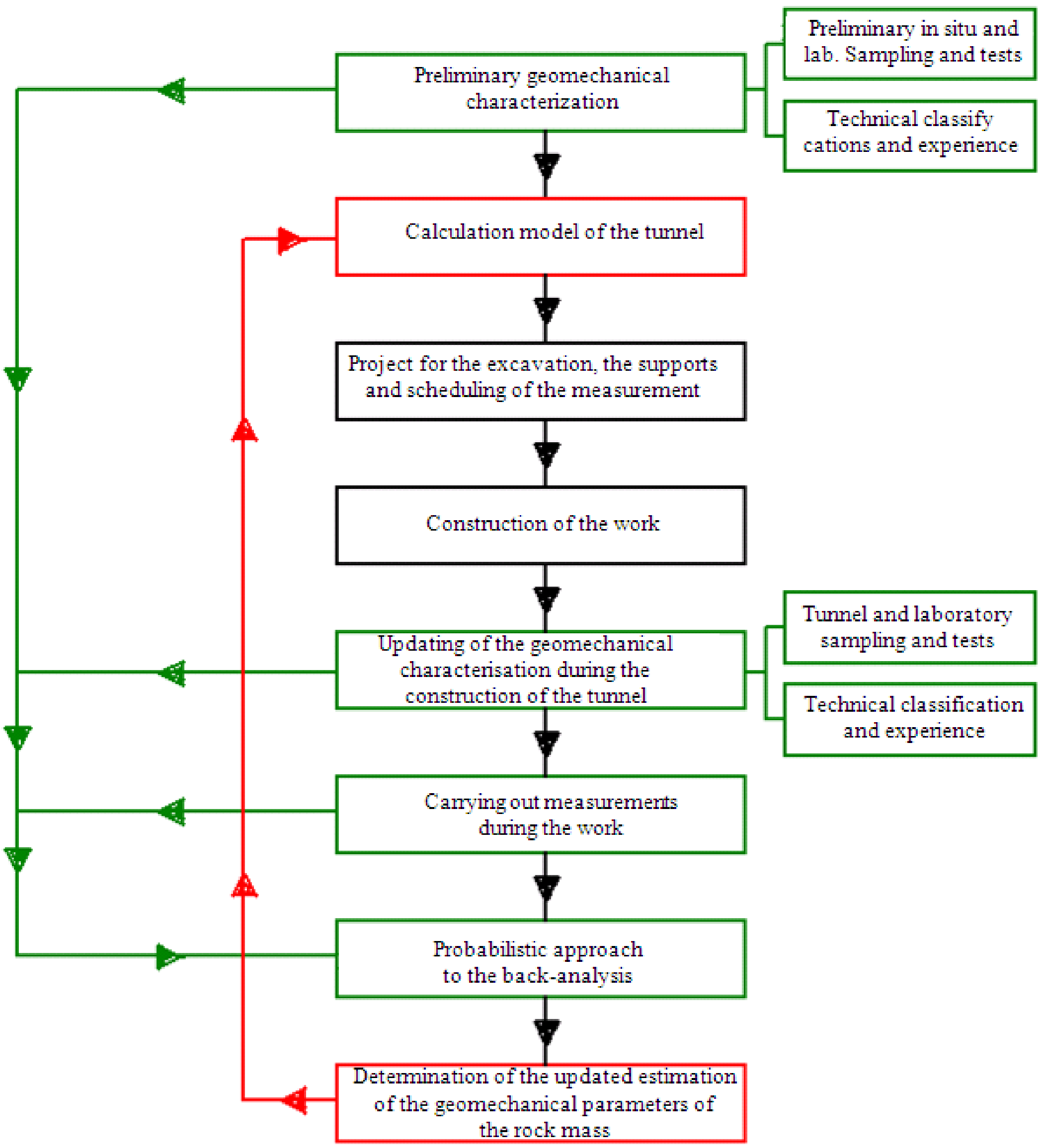

Fig. 1: Proposed procedure for the refining of the project on the support structure of a tunnel, thanks to the use of back-analysis, according to the probabilistic approach, which requires a first initial estimation of the rock parameters (and its degree of reliability) and the results of the measurements (with its precision) performed in a tunnel during construction

If one wishes to search for a new estimation of the unknown parameters so that the distance between the unknown vector and the preliminary estimation (expressed as the difference between the unknown vector of the geomechanical parameters and the initial estimation, related to the variance of the initial estimation). Added to the distance between the *in situ* measurements and the calculation results (expressed as the difference between the results of the calculation and the carried out measurements, related to the variance of the measurements) is minimal, the function error ε takes on the following form:

$$\varepsilon = [\bar{\eta} - \eta]^T \left[C_{\Delta\bar{\eta}}\right]^{-1} [\bar{\eta} - \eta] + \left[x^{(0)} - x\right]^T \left[C_{x^{(0)}}\right]^{-1} \left[x^{(0)} - x\right] \tag{4}$$

where, $\bar{\eta}$ is the column vector of the mean of the *in situ* measurement, of m terms;

η is the vector column of the results of the numerical modelling, corresponding to the measurements carried out *in situ*.

In the simplest case in which the results of the numerical model η are linear functions of the unknown parameters x:

$$\eta = f(x) = \eta' + L\{x - x'\} \quad (5)$$

Where:

- η' = The column vector of m known terms, independent of x
- L = the matrix of dimensions m x n (m lines and n columns)
- x' = the column vector of n known terms, equal to the value of the unknown vector x for which η = η/
- η', = L and x' are obtained from the numerical model by varying one of the unknown parameters at a time and obtaining the results of the calculation

Deriving the function error ε Eq. 4 with respect to x, one obtains the following simple linear equation system (Eq. 6):

$$\left[L^T C_{\Delta\bar{\eta}}^{-1} L + \left(C_{x^{(0)}}\right)^{-1}\right] x = L^T C_{\Delta\bar{\eta}}^{-1} [\bar{\eta} - \eta' + Lx'] + \left[C_{x^{(0)}}\right]^{-1} x^{(0)} \quad (6)$$

which, once solved, furnishes the solution of the back-analysis problem:

$$x^* = [I - M_0 L] x^{(0)} + M_0 \bar{\eta} - M_0 [\eta' - Lx'] \quad (7)$$

where, I is the matrix identity:

$$M_0 = \left[L^T C_{\Delta\bar{\eta}}^{-1} L + \left(C_{x^{(0)}}\right)^{-1}\right]^{-1} L^T C_{\Delta\bar{\eta}}^{-1}$$

And the matrix of the x covariance, which permits one to describe the newly obtained estimation in probabilistic terms:

$$C_{x^*} = [I - M_0 L] C_{x^{(0)}} [I - M_0 L]^T + M_0 C_{\Delta\bar{\eta}} M_0^T \quad (8)$$

In the more general case, the function f, given in Eq. 5, is not linear but can be linearized through a Taylor series truncated at the first order terms. In this case, Eq. 7 no longer directly produces the solution x^*, but only its approximation; one therefore proceeds iteratively, through the following calculation steps, starting from the initial estimation $x^{(0)}$:

The vector x' is placed equal to $x^{(i)}$ in Eq. 5;

The term η' and the matrix L in Eq. 5 are obtained through a parametric analysis with the numerical model (the matrix L is determined by approximation through the secant method in point $x^{(i)}$):

- $x^{(i+1)}$ is obtained from Eq. 7;
- the iteration proceeds until the difference in norm between the two following approximations is lower than a pre-established tolerance; once the convergence is reached, the covariance matrix of the solution is calculated on the basis of Eq. 8.

The availability of matrix C_{x^*} allows one to obtain a quantitative evaluation of the reliability of the results of the back-analysis and to choose the number and quality of measurements to carry out *in situ* to obtain the unknown parameters with the desired precision.

DISCUSSION

A tunnel design requires taking into account a number of aspects, some of which are only known with a certain approximation before the study is begun. The mechanical characteristics of the rock mass which influence the dimensioning of the support structure and the excavation method are, in particular, initially evaluated starting from geognostic investigations and in situ and laboratory tests.

It is then during the construction of the tunnel, however, that the initial design should be verified, even eventually integrated or modified, in other words validated.

The further investigations in the tunnel and the results of monitoring measurements of the static behavior of the cavity following excavation operations, leads to the improvement of the estimation of the mechanical characteristics of the rock mass and reduces the level of uncertainty. It is necessary to perform a back-analysis to correctly interpret monitoring measurements, that is, a type of procedure that permits the back definition of rock parameters that, considered in the calculation, produce results that are close to the performed measurements.

The monitoring measurements should be planned, carried out and interpreted in close connection to the tunnel calculation model, which nowadays is usually of a numerical type. At least those measurements needed in the back-analysis procedures should in fact be foreseen. The type, precision and number of measurements that should be carried out should be defined in relation to the available numerical model and to the required precision of the final estimation of the geomechanical parameters of the rock mass that one wishes to obtain.

As both the preliminary estimation of the rock characteristics and the measurements made during excavation of the tunnel present a level of uncertainty that can be described with a probability distribution, the back-analysis can be developed according to the probabilistic type approach.

CONCLUSION

From the analysis of the probabilistic approach to the theory of the back-analysis, shown in this study, it is possible to conclude how it is necessary, to obtain a satisfactory calibration of the geomechanical parameters of the rock mass, to be able to refer to a preliminary characterization based on geognostic investigations that are adequate for the problem under examination. A large uncertainty in the initial estimation would in fact reflect on the final results even in the presence of precise measurements and suitable calculation models.

The evaluation of the stability conditions of the supports, once the back-analysis is terminated, can be obtained by analyzing the stress state induced inside the foreseen structure, carrying out parametric analysis which allows one to vary the geomechanical parameters of the rock in their variability interval, expected with a certain confidence.

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