

Inventory Management, Spare Parts and Reliability Centred Maintenance for Production Lines

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# Inventory Management, Spare Parts and Reliability Centred Maintenance for Production Lines

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## 1. Introduction

1<sup>st</sup> Premise: Ever since he was a young student, at the secondary school, Fausto Galetto was fond of understanding the matters he was studying: ***understanding for learning*** was his credo (φιλομαθής σπουη); for all his life he was keeping this attitude, studying more than one ton of pages: as manager and as consultant he studied several methods invented by professors, but never he used the (many) wrong ones; on the contrary, he has been devising many original methods needed for solving the problems of the Companies he worked for, and presenting them at international conferences [where he met many bad divulgers, also *professors "ASQC certified quality auditors"*]; after 25 years of applications and experience, he became professor, with a dream "improve future managers (students) quality": the incompetents he met since then grew dramatically (also with documents. F.Galetto got from students ERASMUS, (Fijiu Antony et al., 2001, Sarin S. 1997). 2<sup>nd</sup> Premise: "*The wealth of nations depends increasingly on the quality of managers.*" (A. Jay) and "Universities grow future managers." (F. Galetto) Entailment: due to that, the author with this paper will try, again, to provide the important consequent message: *let's, all of us, be scientific in all Universities, that is, let's all use our rationality. "What I want to teach is: to pass from a hidden non-sense to a non-sense clear."* (L. Wittgenstein). END [see the Galetto references]

"In my university studies .... in most of the cases, it seemed that students were asked simply to regurgitate at the exams what they had swallowed during the courses." (M. Gell-Mann "The Quark and the Jaguar..." [1994]). Some of those students later could have become researchers and then professors, writing "scientific" papers and books ... For these last, another statement of the Nobel Prize M. Gell-Mann is relevant: "Once that such a misunderstanding has taken place in the publication, it tends to become perpetual, because the various authors simply copy one each other."....., similar to "Imitatores, servum pecus" [Horatius, 18 B.C.!!] and "Gravior et validior est decem virorum bonorum sententia quam totius multitudinis imperitiae" [Cicero]. When they teach, "***The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.*** [Deming (1986)]", because those professors are unable to practice maieutics [μαιευτική τεχνη], the way used by Socrates for teaching [the same was for Galileo Galilei in the *Dialogue on the Two Chief World Systems*]. Paraphrasing P. B. Crosby,

in his book *Quality is free*, we could say “Professors may or may not realize what has to be done to achieve quality. Or worse, they may feel, mistakenly, that they do understand what has to be done. Those types can cause the most harm.”

What do have in common Crosby, Deming and Gell-Mann statements? The fact that professors and students betray an important characteristic of human beings: rationality [the “Adult state” of E. Berne (see fig. 1)]. Human beings are driven by curiosity that demands that we ask questions (“why?. ..., why?”) and we try to put things in order (“this is connected with that”): curiosity is one of the best ways to learn, but “learning does not mean understanding”; only twenty-six centuries ago, in Greece, people began to have the idea that the “world” could be “understood rationally”, overcoming the religious myths: they were sceptic [σκεπτομαι=to observe, to investigate] and critic [κρινω=to judge]: then and there a new kind of knowledge arose, the “rational knowledge”.

Till today, after so long time, we still do not use appropriately our brain! A peculiar, stupid and terrific non-sense! During his deep and long experience of Managing and Teaching (more than 40 years), F. Galetto always had the opportunity of verifying the truth of Crosby, Deming and Gell-Mann statements.

Before proceeding we need to define the word “scientific”.

A document (paper or book) is “scientific” if it “scientifically (i.e. with “scientific method”) deals with matters concerning science (or science principles, or science rules)”. Therefore to be “scientific” a paper must both concern “science matters” and be in accordance with the “scientific method”.

The word “science” is derived from the Latin word “scire” (to know for certain) {derived from the Greek words μαθησις, επιστημη, meaning learning and knowledge, which, at that time, were very superior to “opinion” [δοξα], while today opinion of many is considered better than the knowledge of very few!}; knowledge is strongly related to “logic reasoning” [λογικος νους], as it was, for ages, for Euclid, whose Geometry was considered the best model of “scientificness”. Common (good) sense is not science! Common sense does not look for “understanding”, while science looks for “understanding”! “Understanding” is related to “intelligence” (from the Latin verb “intelligere” ([intus+legere: read into]: “intellige ut credas” i.e. understand to believe. Unfortunately “none so deaf as those that won't hear”).

Let's give an example, the Pythagoras Theorem: *In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.* Is this statement scientific? It could be scientific because it concerns the science of Geometry and it can be proven true by mathematical arguments. It is not-scientific because we did not specify that we were dealing with the “Euclidean Geometry” (based, among others, on the “parallel axiom”: from this only, one can derive that the sum of the interior angles of a triangle is always  $\pi$ ): we did not deal “scientifically” with the axioms; we assumed them implicitly.

So we see that “scientificness” is present only if the set of statements (concerning a given “system”) are non-contradictory and deductible from stated principles (as the rules of Logic and the Axioms).

Let's give another example, the 2<sup>nd</sup> law of Mechanics: *The force and the acceleration of a body are proportional vectors:  $F=ma$ , ( $m$  is the mass of the body).* Is this statement scientific? It could be scientific because it concerns the science of Mechanics and it can be proven “true” by well designed experiments. It is not-scientific because we did not specify that we were dealing with “frames of reference moving relatively one to another with constant velocity” [inertial

frames (with the so called “Galilean Relativity”: the laws of Physics look the same for inertial systems)] and that the speed involved was not comparable with the “speed of light in the vacuum [that is the same for all observers]” (as proved by the Michelson-Morley experiment: in the Special Relativity Theory,  $\mathbf{F}=\frac{d(m\mathbf{v})}{dt}$  is true, not  $\mathbf{F}=\mathbf{ma}$ !) and not involving atomic or subatomic particles. We did not deal “scientifically” with the hypotheses; we assumed them implicitly. From the laws of Special Relativity we can derive *logically* the conservation laws of momentum and of energy, as could Newton for the “Galilean Relativity”. For atomic or subatomic particles “quantum Mechanics” is needed (with Schrödinger equation as fundamental law).

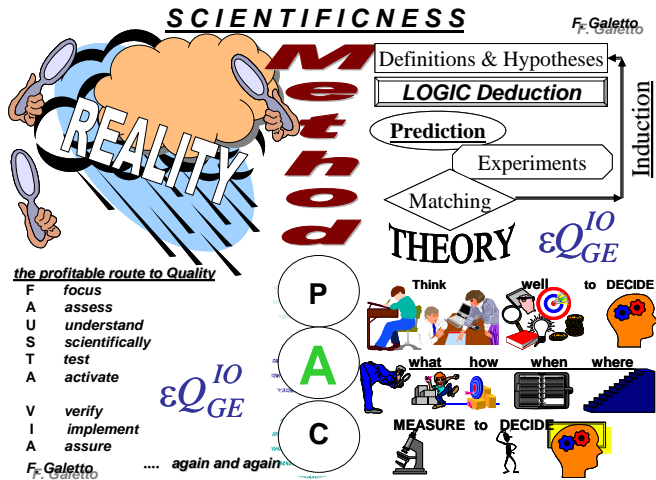


Fig. 1. Scientificness

So we see that “scientificness” is present **only** if the set of statements (concerning a given “system”) do not contradict the observed data, collected through well designed experiments [“scientific” experiments]: only in the XVII century, due to Galilei, Descartes, Newton, ... we learned that. Since that time only, science could really grow.

When we start trying to learn something, generally, we are in the “clouds”; reality (and truth) is hidden by the clouds of our ignorance, the clouds of the data, the clouds of our misconceptions, the clouds of our prejudices; to understand the phenomena we need to find out the reality from the clouds: we make hypotheses, then we deduct logically some consequences, predicting the results of experiments: if predictions and experimental data do match then we “confirm” our idea and if many other are able to check our findings we get a theory. To generate a theory we need Methods. Eric Berne, the psychologist father of “Transactional Analysis”, stated that everybody interacts with other people through three states P, A, C [Parent, Adult, Child, (not connected with our age, fig. 1)]: the Adult state is the one that looks for reality, makes questions, considers the data, analyses objectively the data, draws conclusions and takes logic decisions, coherent with the data, methodically. Theory [θεωρία] comes from the Adult state! Methods [μεθοδος from μετα+οδος = the way through (which one finds out...)] used to generate a Theory come from the Adult state!

People who take for granted that the truth depends on “Ipse dixit” [αυτος εφα, “he said that”], behave with the Parent state. People who get upset if one finds their errors and they

do not consider them ["we are many and so we are right", they say!!] behave with the Child state. [see the books of the Palo Alto group]

To find scientifically the truth (out of the clouds) you must **Focus** on the problem, **Assess** where you are (with previous data and knowledge), **Understand Scientifically** the message in the data and find consequences that confirm (or disprove) your predictions, Scientifically design **Test** for confirmation (or disproof) and then **Activate** to make the Tests. If you and others **Verify** your prediction, anybody can **Implement** actions and **Assure** that the results are SCIENTIFIC (FAUSTA VIA): all of us then have a THEORY. SCIENTIFICNESS is there (fig. 1).

From these two examples it is important to realise that when two people want to verbally communicate, they must have some common concepts, they agree upon, in order to transfer information and ideas between each other; this is a prerequisite, if they want to understand each other: what is true for them, what is their "conventional" meaning of the words they use, which are the rules to deduce statements (Theses) from other statements (Hypotheses and "previous" Theses): **rigour is needed for science**, not opinions!!!

Many people must apply *Metanoia* [μετανοια=change their mind (to understand)] to find the truth.

Here we accept the rules of LOGIC, the *deductive* logic, where the premises of a valid argument contain the conclusion, and the truth of the conclusion follows from the truth of the premises with certainty: any well-formed sentence is either true or false. We define as *Theorem* "a statement that is proven true by reasoning, according to the rules of Logic"; we must therefore define the term *True*: "something" (statement, concept, idea, sentence, proposition) is true when there is *correspondence* between the "something" and the facts, situations or state of affairs that verify it; the *truth* is a relation of *coherence* between a thesis and the hypotheses. Logical validity is a relationship between the premises and the conclusion such that if the premises are true then the conclusion is true. The validity of an argument should be distinguished from the truth of the conclusion (based on the premises). This kind of truth is found in mathematics.

Human beings evolved because they were able to develop their knowledge from inside (the *deductive* logic, with analytic statements) and from outside, the external world, (the *inductive* logic, with synthetic statements), in any case using their intelligence; the *inductive* logic is such that the premises are evidence for the conclusion, but the truth of the conclusion follows from the truth of the evidence only with a certain probability, provided the way of reasoning is correct.

The scientific knowledge is such that any valid knowledge claim must be verifiable in experience and built up both through the *inductive* logic (with its synthetic statements) and the *deductive* logic (with its analytic statements); in any case a clear distinction must be maintained between analytic and synthetic statements.

This was the attitude of Galileo Galilei in his studies of falling bodies. At first time he formulated the tentative hypothesis that "the speed attained by a falling body is directly proportional to the distance traversed"; then he deduced from his hypothesis the conclusion that objects falling equal distances require the same amount of elapsed time. After "Gedanken Experimenten", Designed Experiments made clear that this was a false conclusion: hence, logically, the first hypothesis had to be false. Therefore Galileo framed a new hypothesis: "the speed attained is directly proportional to the time elapsed". From this he was able to deduce that the distance traversed by a falling object was proportional to the

square of the time elapsed; through Designed Experiments, by rolling balls down an inclined plane, he was able to verify experimentally his thesis (it was the first formulation of the 2<sup>nd</sup> law of Mechanics). [fig. 1]

Such agreement of a conclusion with an actual observation does not itself prove the correctness of the hypothesis from which the conclusion is derived. It simply renders that premise much more plausible.

For rational people (like were the ancient Greeks) the criticism [κρίνω = to judge] is hoped for, because it permits improvement: asking questions, debating and looking for answers improves our understanding: we do not know the truth, but we can look for it and be able to find it, with our brain; to judge we need criteria [κριτήριον]. In this search Mathematics [note μάθησις] and Logic can help us a lot: Mathematics and Logic are the languages that Rational Managers must know! Proposing the criterion of testability, or falsifiability, for scientific validity, Popper emphasized the *hypothetico-deductive character of science*. Scientific theories are hypotheses from which can be deduced statements testable by observation; if the appropriate experimental observations falsify these statements, the hypothesis is refused. If a hypothesis survives efforts to falsify it, it may be tentatively accepted. No scientific theory, however, can be conclusively established. A “theory” that is falsified, is NOT scientific.

“Good theories” are such that they **complete** previous “good” theories, in accordance with the collected new data. [fig. 1]

A good example of that is **Bell's Inequality**. In physics, this inequality was used to show that a class of theories that were intended to “complete” quantum mechanics, namely local hidden variable theories, are in fact inconsistent with quantum mechanics; quantum mechanics typically predicts probabilities, not certainties, for the outcomes of measurements. Albert Einstein [one of the greatest scientists] stated that quantum mechanics was incomplete, and that there must exist “hidden” variables that would make possible definite predictions. In 1964, J. S. Bell proved that all local hidden variable theories are inconsistent with quantum mechanics, first through a “Gedanken Experiment” and Logic, and later through Designed Experiments. Also the great scientist, A. Einstein, was wrong in this case: his idea was falsified. We see then that the ultimate test of the validity of a scientific hypothesis is its *consistency* with the totality of other aspects of the scientific framework. This inner consistency constitutes the basis for the concept of causality in science, according to which every effect is assumed to be linked with a cause. [fig. 1]

The scientific community as a whole must judge [κρίνω] the work of its members by the **objectivity and rigour** with which that work has been conducted; in this way the scientific method should prevail.

In any case the scientific community must remember: **Any statement (or method) that is falsified, is NOT scientific.**

Here we assume that the subject of a paper is concerning a science (like Mathematics, Statistics, Probability, Quality Methods); therefore to judge [κρίνω] if a paper is scientific we have to look at the “scientific method”: if the “scientific method” is present, i.e. the conclusions (statements) in the paper follow logically from the hypotheses, we shall consider the paper scientific; **on the contrary**, if there are conclusions (statements) in the paper that do not follow logically from the hypotheses, we shall NOT consider the paper scientific: a wrong conclusion (statement) is NOT scientific. [fig. 1 vs Franceschini 1999]

*“To understand that an answer is wrong you don't need exceptional intelligence, but to understand that is wrong a question one needs a creative mind.” (A. Jay). “Intellige ut credas”.*

*Right questions, with right methods, have to be asked to "nature" (fig. 1). "Intellige ut credas".*

It is easy to show that a paper, a book, a method, is not scientific: it is sufficient to find an example that proves the wrongness of the conclusion. When there are formulas in a paper, it is not necessary to find the right formula to prove that a formula is wrong: an example is enough; to prove that a formula is wrong, one needs only intelligence; on the contrary, to find the right formula, that substitutes the wrong one, you need both intelligence and ingenuity. I will use only intelligence and I will not give any proof of my ingenuity: this paper is for intelligence ... For example, it's well known (from Algebra, Newton identities) that the coefficients and the roots of any algebraic equation are related: it's easy to prove that  $\pm\sqrt{-c/a}$  is not the solution (even if you do not know the right solution) of the parabolic equation  $ax^2 + bx + c = 0$ , because the system  $x_1 + x_2 = -b/a$ ,  $x_1x_2 = c/a$  is not satisfied ( $x_1$  and  $x_2$  are the roots). [Montgomery 1996 and ...]

The literature on "Quality" matters is rapidly expanding. Unfortunately, nobody, but me, as far as I know, [I thank any person that will send me names of people who take care ...], takes care of the **Quality of Quality Methods used for making Quality** (of product, processes and services). "Intellige ut credas". [O' Connor 1997, Brandimarte 2004]

I am eager to meet one of them, fond of Quality as I am. [fig. 1, and Galetto references]

If this kind of person existed, he would have agreed that **"facts and figures are useless, if not dangerous, without a sound theory"** (F. Galetto), **"Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality"** (Deming) because he had seen, like Deming, Gell-Mann and myself **"The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications."** [Deming (1986)] (Montgomery 1996 and ..., Franceschini 1999)

During 2006 F. Galetto experienced the incompetence of several people who were thinking that only the "Peer Review Process" is able to assure the scientificness of papers, and that only papers published in some magazines are scientific: one is a scientist and gets funds if he publishes on those magazines!!! Using the scientific method one can prove that the referee analysis does not assure quality of publications in the magazines of fig. 2.

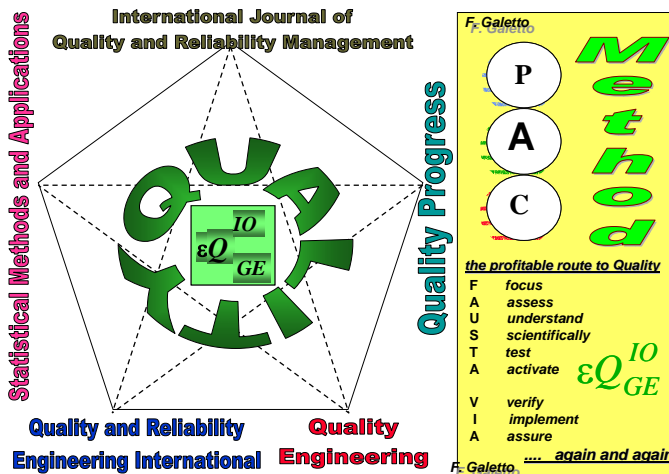


Fig. 2. The "pentology"

The symbol  $\epsilon Q_{GE}^{IO}$  [which stands for the “*epsilon Quality*”] was devised by F. Galetto to show that *Quality* depends, at any instant, in any place, at any rate of improvement, on the *Intellectual honesty* of people who always use experiments and think well on the experiments before actually making them (*Gedanken Experimenten*) to find the truth” [*Gedanken Experimenten* was a statement used by Einstein; but, if you look at Galileo life, you can see that also the Italian scientist was used to “mental experiments”, the most important tool for Science; Epsilon ( $\epsilon$ ) is a greek letter used in Mathematics and Engineering to indicate a very small quantity (actually going to zero); *epsilon Quality* conveys the idea that *Quality* is made of many and many prevention and improvement actions].

The level of knowledge F. Galetto could verify (in 40 years experience and a lot of meetings) is given in table 1.

|             | Logic    | Management | Mathematics | Physics  | Quality Management | Probability | Stochastic Processes | Statistics | Applied Statistical Methods | Parameter Estimation | Test of Hypotheses | Decision Making | System Reliability Theory | Maintainability | Design Of Experiments | Control Charts | Sampling Plans | Quality Engineering | Quality Practice |
|-------------|----------|------------|-------------|----------|--------------------|-------------|----------------------|------------|-----------------------------|----------------------|--------------------|-----------------|---------------------------|-----------------|-----------------------|----------------|----------------|---------------------|------------------|
| Professors  | 1<br>H   | VL<br>95   | VL<br>98    | VL<br>98 | VL<br>99           | VL<br>99    | VL<br>99             | VL<br>99   | VL<br>99                    | VL<br>99             | VL<br>99           | VL<br>95        | VL<br>99                  | VL<br>99        | VL<br>99              | VL<br>99       | VL<br>99       | VL<br>99            | VL<br>99         |
| Managers    | VL<br>95 | VL<br>95   | VL<br>98    | VL<br>98 | VL<br>99           | VL<br>99    | VL<br>99             | VL<br>99   | VL<br>99                    | VL<br>99             | VL<br>99           | VL<br>95        | VL<br>99                  | VL<br>99        | VL<br>99              | VL<br>99       | VL<br>99       | 1<br>VH             | 1<br>VH          |
| Consultants | VL<br>99 | VL<br>95   | VL<br>98    | VL<br>98 | VL<br>99           | VL<br>99    | VL<br>99             | VL<br>99   | VL<br>99                    | VL<br>99             | VL<br>99           | VL<br>95        | VL<br>99                  | VL<br>99        | VL<br>99              | VL<br>99       | VL<br>99       | 1<br>VH             | 1<br>VH          |

Legenda : VL90=probability 90% that knowledge is lower than Very Low;  
 5VH=probability 5% that knowledge is higher than Very High  
 Scale: None, Very Low, Low, Medium, High, Very High, Perfect

Table 1. Level of Knowledge (based on 40 year of experience, in companies and universities)

Many times F. Galetto spoiled his time and enthusiasm at conferences, in University and in Company courses, trying to provide good ideas on *Quality* and showing many cases of wrong applications of stupid methods [see references]. He will try to do it again ... by showing, step by step, very few cases (out of the hundreds he could document).... in order people understand that *QUALITY* is a serious matter. The Nobel price R. Feynman (1965) said that *for the progress of Science are necessary experimental capability, honesty in providing the results and the intelligence of interpreting them... We need to take into account of the experiments even though the results are different from our expectations.* It is apparent that Deming and Feynman and Gell-Mann are in agreement with  $\epsilon Q_{GE}^{IO}$  ideas of F. Galetto. Once upon a time, A. Einstein said “*Surely there are two things infinite in the world: the Universe and the Stupidity of people. But I have some doubt that Universe is infinite*”. Let's hope that Einstein was wrong, this time. Anyway, before him, Galileo Galilei had said [in the *Saggiatore*] something similar “*Infinite is the mob of fools*”. [see references]

All the methods, devised by F. Galetto, were invented and have been used for preventing and solving real problems in the Companies he was working for, as Quality Manager and as Quality Consultant: several million € have been saved.[see Galetto references]

Companies will not be able to survive the global market if they cannot provide integrally their customer the Quality they have paid for [fig. 5, Management Tetrahedron]. So it is of paramount importance to define correctly what Quality means. Quality is a serious and difficult business; it has to become an integral part of management.

The first step is to define *logically* what Quality is.

Let's start with some ideas of Soren Bisgard (2005) given in the paper Innovation, ENBIS and the Importance of Practice in the Development of Statistics, Quality and Reliability Engineering International. He says: "*Since the early 1930s industrial statistics has been almost synonymous with quality control and quality improvement. Some of the most important innovations in statistical theory and methods have been associated with quality.*" .... "*Quality Management also provides an intriguing example. Its scientific underpinning is greatly inspired by statistics, a point forcefully set forth by Shewhart. Quality is typically interpreted narrowly by statisticians as variance and defect reduction. However these efforts should be viewed more broadly as what economist call innovation. When we engage in statistical quality control studies ... we are engaged in process innovation and ... in product innovation.*" Two paragraphs of his paper are entitled: 2. Quality as innovation, and 3. Quality as systematic innovation.

One must say that the paper does not provide the definition of the term Quality, such as "Quality is ..."; however he realises that statisticians have a narrow view of Quality, as "Quality is variance and defect reduction."

As a matter of fact, D. C. Montgomery defines: "*Quality is the inverse of variability*" (saying "*We prefer a modern definition of quality*") and adds "*Quality Improvement is the reduction of variability in processes and products*". To understand the following subject the reader has to know Reliability and Statistics. Let's consider two computers (products A and B): after 3 years, A experiences  $g_A(3)=9$  failures, while B experiences  $g_B(3)=5$  failures; which product has better quality? B, because it experienced less failures! BUT, which product has lower variability? A, because it experienced more failures!

Generally, statisticians (and professors) do not understand this point: they are Gauss-drugged with the "normal distribution"! Let's assume, for the sake of simplicity, that two items have constant increasing failure rate (IFR, they do wear and do not have infant mortality); from the data we can estimate the MTF (Mean Time To Failure), the Reliability  $R(t)$  and the probability density of failures  $f(t)$ : B is more reliable and has more variability [the upper curve]! Therefore according to D. C. Montgomery definition B (that is more reliable) has lower "quality"! Fig 3 provides a hint for understanding ....

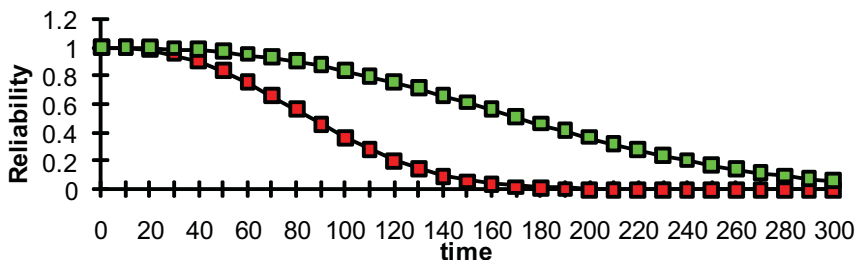


Fig. 3. An item with more variability B, has better Quality

If one thinks to the Formula One Championship and applies Montgomery's definition he finds that if the two Red Bull arrive 1<sup>st</sup> and 2<sup>nd</sup> they have lower quality than the two Ferrari that arrive 7<sup>th</sup> and 8<sup>th</sup>!!!!!! Can you believe it?

There are a lot of "quality definitions; let's see some of the latest definitions of the word "Quality" that can be found in the literature (some of them existed before the date here given; the date shown refers to the latest document I read):

"conformance to requirements" (Crosby, 1979), "fitness for use" (Juran ,1988), "customer satisfaction" (Juran, 1993), "the total composite product and service characteristics of marketing, engineering, manufacture and maintenance through which product or service in use will meet the expectations by the customer" (Feigenbaum, 1983 and 1991), "totality of characteristics of an entity that bears on its ability to satisfy stated and implied needs" (ISO 8402:1994), "a predictable degree of uniformity and dependability at low cost and suited to the market" (*falsely attributed to Deming, 1986; I read again and again Deming documents and I could not find that*), "Quality is inversely proportional to variability" (Montgomery, 1996), "degree to which a set of inherent characteristics (3.5.1) fulfils requirements (3.1.2)" [**ISO 9000:2000, Quality management systems - Fundamentals and vocabulary**, (definition 3.1.1)]

The ISO definition is very stupid; it is like confounding two very different concepts: energy and temperature; "temperature" provides the *degree* of "energy" [=Quality]; therefore *Quality* must be "the set of characteristics".

Quality definition must have Quality in it!

In order to provide a practical and managerial definition, since 1985 F. Galetto was proposing the following one:

**Quality is the set of characteristics of a system that makes it able to satisfy the needs of the Customer, of the User and of the Society.** It is clear that none of the previous eight definitions highlights the importance of the *needs* of the three actors: **the Customer, the User and the Society**. They are still not considered in the latest document, the ISO 9000:2008, Quality management systems - Fundamentals and vocabulary.

Some important characteristics are stated in the Quality Tetrahedron of ten characteristics Safety, Conformity, Reliability, Durability, Maintainability, Performance, Service, Aesthetics, Economy, Ecology; each characteristic has an "operational definition" that permits to state goals and verify achievement [according to FAUSTA VIA]. "Customer/User/Society needs satisfaction" must be converted from a slogan to real practice, if companies want to be competitive. Today, many managers show their commitment with customer satisfaction, but they are not prepared to invest time and money in NEEDS Satisfaction; they do not put the theory into practice; they do not speak with the facts, but only with words.

Unfortunately *management commitment to Quality is not enough*; managers must understand and learn Quality ideas: too many companies are well behind the desired level of Quality management practices. [fig. 5]

The Quality Tetrahedron shows that Management must learn that *solving problems is essential but it is not enough: they must prevent future problems and take preventive actions*: Safety, Reliability, Durability, Maintainability, Ecology, Economy can be tackled rightly only through preventive actions; the PDCA cycle is useless for prevention; it very useful for improvement. Several of the Quality characteristics [in the Quality Tetrahedron] need prevention; reliability is one of the most important: very rarely failures can be attributed to blue collar workers. Failures arise from lack of prevention, and prevention is a fundamental aspect and responsibility of Management. The same happens for safety, durability, maintainability, ecology, economy, ...

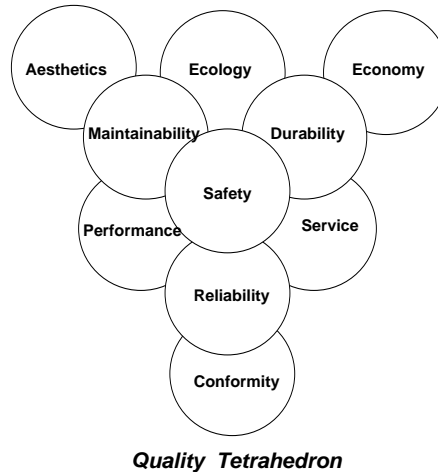


Fig. 4. The Quality Tetrahedron for the Quality definition

So we see that Quality entails much more than “innovation”: you can innovate without Quality! Few decades ago electronic gadgets entered into cars; electronics was an innovation in cars, but the cars failed a lot: innovation did not take into consideration **prevention!** The essence of Quality is prevention. Innovation is a means for competition, rarely for Quality. (see the “Business Management System” in figure 5).

We are in a new economic age: long-term thinking, prevention, Quality built at the design stage, understanding variation, waste elimination, knowledge and scientific approach are concepts absolutely needed by Management, if they want to be good Managers. In this paper, Manager is the person who *achieves the Company goals, economically, through other people, recognise existing problems, prevents future problems, states priorities, dealing with their conflicts, makes decisions thinking to their consequences, with rational and scientific method, using thinking capability and knowledge of people.* These kind of Managers behave according to the “Business Management System”.

The customer is the most important driving force of any company. Companies will not be able to survive the global market if they cannot provide integrally their customer the Quality they have paid for. It is important to stress that “Customer Needs Satisfaction” is absolutely different from “Customer Satisfaction”. Moreover, the previous analysis shows that for a good definition of Quality there are other people involved: the user and the Society.

**Prevention...** We said that the essence of Quality is prevention. Innovation is a means for competition, rarely for Quality. (see the “Business Management System”). Quality is essential for any product (services are defined as products in the ISO 9000:2000 terminology). The measurement of Quality (of product and services) is important if we want to improve and better if we want to prevent problems [F. Galetto from 1973]. Quality depends on effective management of problems prevention and correction (improvement). Effective management needs effective measurement of performance and results, the absolute condition to achieve Business Excellence. A Company that wants to become “excellent” has to find the needs of Customers/Users/Society and to measure how much they are achieved. Moreover that Company has to be “sure” that all its processes are “in

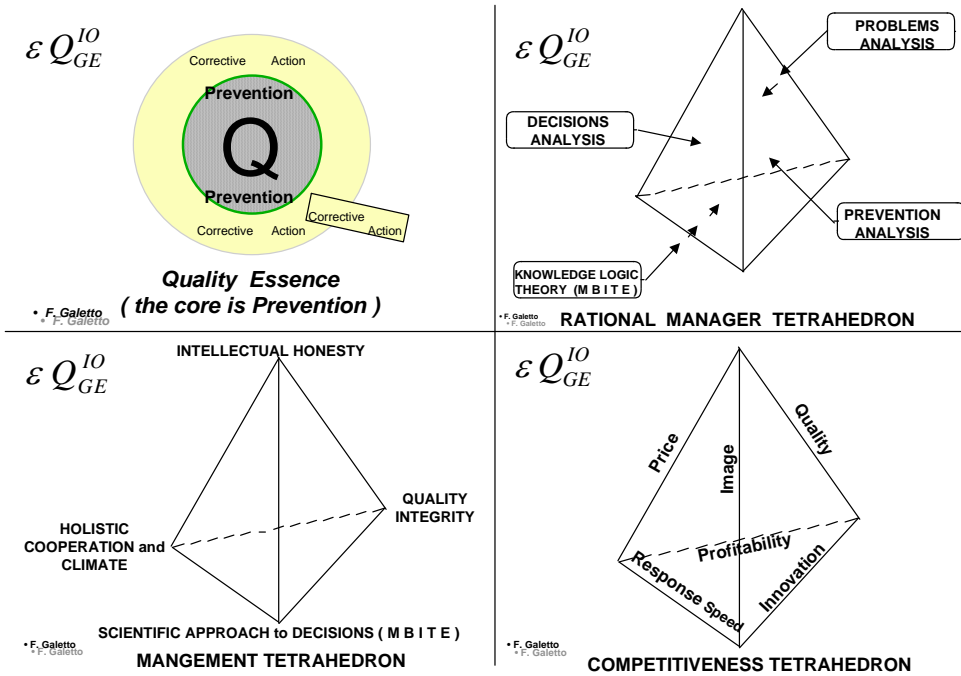


Fig. 5. The Business Management System

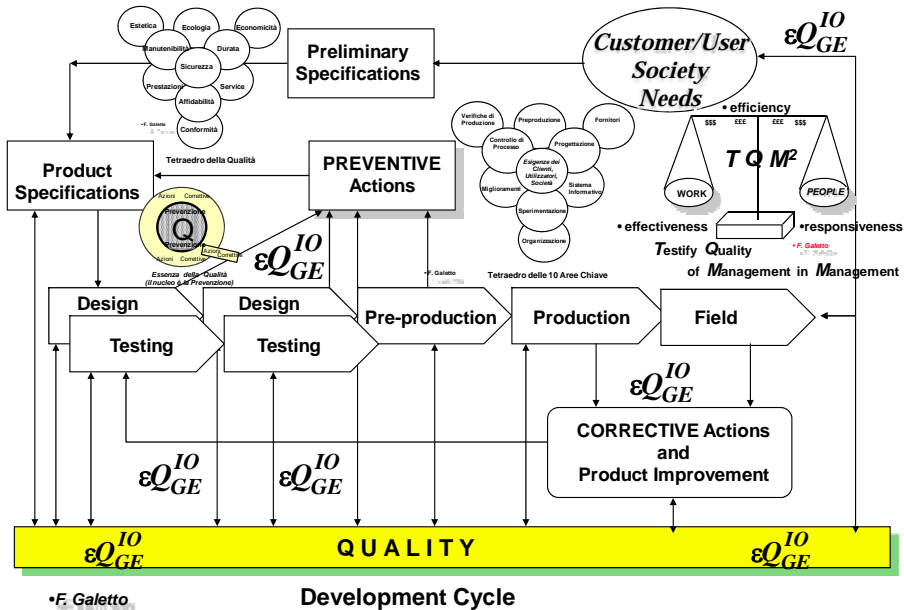


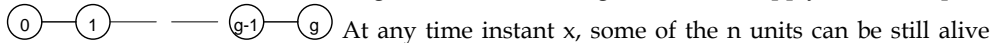
Fig. 6. The Development Cycle

control". If data are available (through properly designed method of collection), statistical methods are the foundation stone for good data analysis and "management decisions" [F. Galetto from 1973].

Prevention is very important and must be considered since from the first stages of product development as shown in figure 6; corrective actions come later.

Reliability is important in all the stages of product development. Reliability tests are essential during product development; collected data have to be analysed by scientific methods that involve Engineering, Statistics and Probability Methods. Reliability is important for preventive maintenance and for the so called RCM (Reliability Centred Maintenance). Let's for example consider the methods for data analyses and maintenance planning, as given in the papers "Total time on test plotting for failure data analyses", (1978), SAAB-SCANIA, and "Some graphical methods for maintenance planning", (1977), *Reliability & Maintainability Symposium*. They are connected with similar ideas of Barlow.

Let's consider a sample made of n "identical" items (n=sample size), that are neither repaired nor replaced after failure. We can view the tested sample as a system ("in parallel") that can be represented as in the graph, where state "i" indicates that "i" items are failed; g indicates the last failure observed during the test. When g=n we can apply the TTT-plot.



At any time instant x, some of the n units can be still alive (survived up to time x), while the other are failed, before x; the sum of all the "survival times" of the n items put on test is denoted TTT(x) [and named the Total Time on Test, up to the time instant x]; the duration of the test depends on the failure of the last item (out of the n) that will fail. If the items fail at times  $t_1, t_2, \dots, t_n$ , then TTT( $t_1$ ) is the Total Time on Test until the 1<sup>st</sup> failure, TTT( $t_i$ ) is the Total Time on Test until the i<sup>th</sup> failure and TTT( $t_n$ ) is the Total Time on Test until the n<sup>th</sup> failure. If  $T_i$  is the *random variable* "Time to the i<sup>th</sup> failure" then TTT( $T_i$ ) is the *random variable* "Total Time on Test until the i<sup>th</sup> failure": the distribution of the *random variable* TTT( $T_i$ ) depends on the distribution of the *random variable*  $T_i$ , which depends on the distribution of the *random variable* T "Time to Failure" of any of the n "identical" items put on test. The n-1 *random variables*  $U_i = TTT(T_i) / TTT(T_n)$ , the *scaled TTT*, have a distribution that depends on the *random variable* T "Time to Failure" of any item; since the distribution  $F_T(t)$  of the r.v. T depends on the failure rate, one can plot a curve  $F_U(t)$  [named TTT-transform] versus  $F_T(t)$ : the curve is contained in the square unit of the Cartesian plane and its shape depends on the type of the failure rate [constant, IFR (increasing), or DFR (decreasing)]. Therefore the TTT-plotting allows understanding the type of failure rate.

The evolution of the system depends from the functions  $b_{i,i+1}(s/r)ds$ , probability of the transition  $i \Rightarrow i+1$  (i.e. the (i+1)<sup>th</sup> failure) in the interval  $s \rightarrow s+ds$ , given that it happened into the state "i" at the time instant r; the functions  $b_{i,i+1}(s/r)$  [named "kernel of the stochastic process" of failures] (Galetto, ...) allow to get the probability  $\overline{W}_i(t|r)$  that the system remains in the state i for the period  $r-t$  and the probability  $R_i(t/r)$  [reliability relative to state i] that the system does not be in the state g at time t (given that it entered in i at time instant r).  $R_0(t/0)$  is then the probability that the system, in the interval  $0-t$ , does not reach the state g, i.e. it experienced less than g failures:  $G(t) < g$ . We have then the fundamental system of Integral Theory of Estimates [valid for any distribution of time to failure of tested units, i.e. for any "kernel"]

$$R_i(t|r) = \overline{W}_i(t|r) + \int_r^t b_{i,i+1}(s|r) R_{i+1}(t|s) ds, \quad i=0, \dots, g-1 \quad e \quad b_{g-1,g}(s|r) = 0$$

For  $g=n$  we get the probability of  $n$  failures  $1 - R_0(t | 0)$  and the Total Time on Test.

If the items on test have constant failure rate  $\lambda$ , then  $b_{i,i+1}(s | r) = n\lambda \exp[-n\lambda(s-r)]$ , when failed items are replaced or repaired, while  $b_{i,i+1}(s | r) = (n-i)\lambda \exp[-(n-i)\lambda(s-r)]$ , when failed items are not replaced or repaired.

After the test one has the data. Let's suppose  $n=7$  and the time to failure (in the sample) are 60, 105, 180, 300, 400, 605, 890. One can estimate  $F_U(t_i)$  and  $F_T(t_i)$ , and plot the 7 points  $[F_U(t_i), F_T(t_i)]$ , obtaining the "empirical" curve.

Now Statistics Theory enters the stage. When the reliability of the items is exponential (constant failure rate), the TTT-transform  $F_U(t)$  versus  $F_T(t)$  is the diagonal of the unit square (the bisector of the coordinate axes). Plotting the "empirical" curve.  $[F_U(t_i), F_T(t_i)]$  one finds a line "near" the bisector ( $\beta=1$ ), and concludes that a constant failure rate is adequate. When the reliability of the items is IFR, the TTT-transform  $F_U(t)$  versus  $F_T(t)$  is a convex curve above the diagonal of the unit square. Figure 7 provides some curves depending on the "shape parameter  $\beta$ ".

To practitioners this can be fantastic. They collect data, elaborate them and then they compare the "empirical" curve with the "Theoretical Curve" given in figure 7; then they "know" the failure distribution and take decisions. Perhaps the situation is more complex ....

- And ... what you do if you do not have the figure 7? Are you able to generate it?
- Now let's suppose that we, as managers, decide to save time for decision and we replace the items failed and we continue the test. Can we use the same figure 7?
- Now let's suppose that we, as managers, decide to save time for decision and we repair the items failed and we continue the test. Can we use the same figure 7?
- Now let's suppose that we, as managers, decide to save time for decision and we test more items (e.g. 28) and we stop the test at the 7<sup>th</sup> failure. Can we use the same figure 7?
- Now let's suppose that we, as managers, decide to save time for decision and we test more items (e.g. 50) and we stop the test at the time instant  $t=200$ . Can we use the same figure 7?

If one is a sensible Manager he will answer: "I do not know. I have to study a lot; I have also to be careful if I go to some consultant". If one is a NOTsensible Manager he will answer: "Yes, absolutely".

Let's see now how a problem is dealt in the paper "Total time on test plotting for failure data analyses"; in Section 5 "SYSTEM FAILURE DATA" we read [verbatim] "*It is also possible to use the TTT-plotting technique for analysing failure data from a repairable system. In this case  $TTT(T_i)$  shall be defined as the time generated by the system until the  $i^{th}$  failure. If  $n-1$  failures have been obtained until time  $T^*$ , the time during which the system was observed, then we substitute  $T^*$  to  $T_n$  and perform the plotting as before. Also the interpretation of the plot remains unchanged. The TTT-transform has, however, no counterpart. The statistical tests described in Section 4 are still applicable.*" [end of Section]. In Section 4 "STATISTICAL TESTS" we read [verbatim] "*Based on the ideas behind the TTT-plotting some statistical test may be obtained. These tests also provides us with some insight in the stochastic properties of the TTT-plot. ....*"

Now let's use Logic, as we said before. IF The TTT-transform does not exist how can one consider "*The statistical tests described in Section 4 are still applicable.*"? They are "*Based on the ideas behind the TTT-plotting ...*" which "has, however, no counterpart." !!!! From the TTT-plot one can only have some hints of the non-applicability of "constant failure rate"!!! Nothing more! Moreover, IF one does not know TTT-transform for repairable systems, he should say "I do not know how to find the TTT-transform" and not "The TTT-transform has, however, no counterpart."

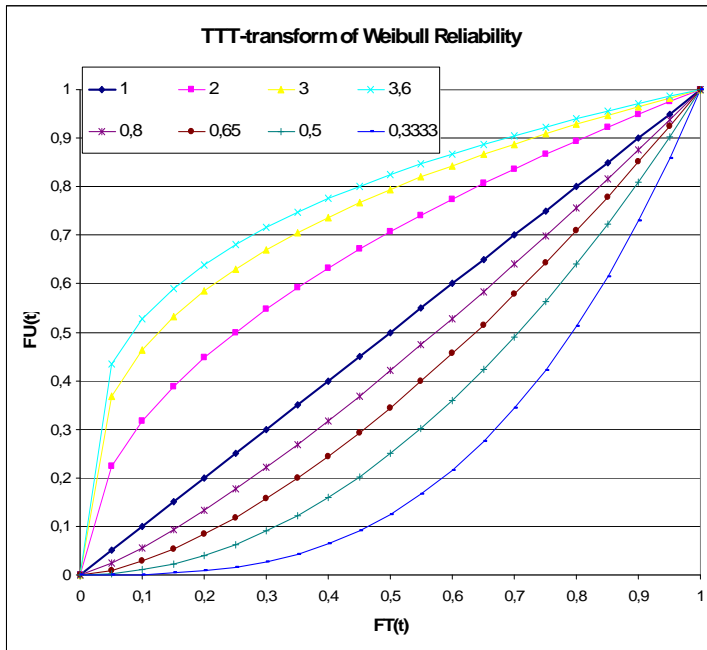


Fig. 7. Weibull TTT-transform [with “shape parameter  $\beta$ ”]

As a matter of fact, in 1977, at the *Reliability & Maintainability Symposium*, Philadelphia, F. Galetto provided the Reliability Integral Theory that solves the problem, with his paper “SARA (System Availability and Reliability Analysis)”. The theory did exist, not the single formulae: any scholar could have found them. The same theory Reliability Integral Theory, is applicable to maintenance problems, as those presented in “Some graphical methods for maintenance planning”; we will deal with this point in a successive paragraph related to preventive maintenance.

## 2. Logistics and inventory

Inventories are stockpiles of raw material, supplies, components, work in progress and finished goods that appear at numerous points throughout a firm's production and logistic channel. Having these inventories on hand cost at least 20% of their value per year, therefore, carefully managing inventory levels makes good economic sense, because in recent years the holding of inventories has been roundly criticised as unnecessary end wasteful. Actually good management of inventories improve customer service and reduce costs. Inventory plays a key role in the logistical behaviour of all manufacturing systems. The classical inventory results are central to more modern techniques of manufacturing management, such as material requirement planning (MRP), just-in-time (JIT) and time based competition (TBC).

**1<sup>st</sup> step: the case of “constant (fixed)” demand** Let's consider the oldest, and simplest, model - the Economic Order Quantity - in order to work our way to the more sophisticated ReOrder Level (ROL) model. One of the earliest applications of mathematics to factory

management was the work of F. W. Harris (1913) on the problem of setting manufacturing lot sizes. He made the following assumptions about the manufacturing system: 1) production is instantaneous, 2) delivery is immediate, 3) a production run incurs a fixed setup cost, 4) there is no interaction between different products, 5) demand is deterministic, 6) demand is constant over time.

Let's consider the problem of establishing the order quantity  $Q$  [lot size] for an inventory system, dealt in “**Logistics courses**” and related books. In this field the assumptions are very similar: a single item is subject to “constant (fixed)” demand “ $\lambda$ ” [demand rate, in units per year], there is a fixed cost  $A$  [ordering cost, in euro] of placing an order and a carrying charge “ $h$ ” [holding cost, in euro per unit per unit time allotted (often year) to each item in inventory]. If no stockouts are permitted and lead time is zero (i.e. orders arrive immediately) there is a quantity  $Q$  (named EOQ: Economic Order Quantity), given by the famous Wilson lot-size formula  $Q = \sqrt{2A\lambda / h}$  that minimise the “total cost per year”. The inventory can be depicted as a system that starts with  $Q$  units (the level,  $I$ , of the inventory): we are certain that  $\lambda t$  units are sold (delivered) in any interval of duration  $t$ ; when the level inventory is zero,  $I=0$ ,  $Q$  products are ordered and arrive immediately... and the system starts again from scratch.

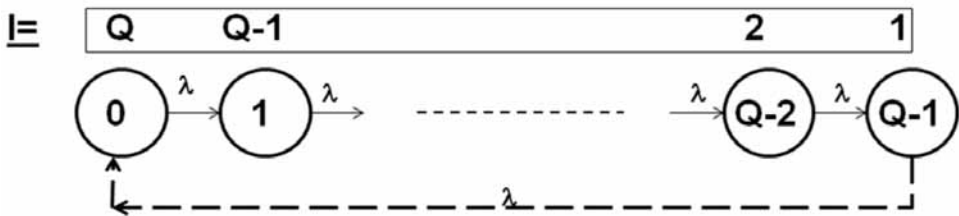


Fig. 8. System inventory states, with fixed and constant demand; state  $i$  means  $i$  products dispatched

The function depicting the curve of the inventory level  $I(t)$  is a saw-tooth line, with constant distance between peaks.

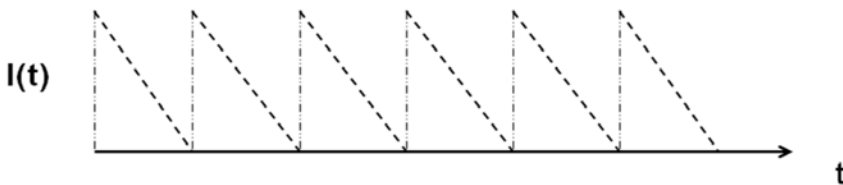


Fig. 9. Level of inventory versus time  $t$

The production cost does not influence the solution and therefore is not considered in the “total cost per year”  $Y(Q) = hQ/2 + A\lambda/Q$ . Taking the derivative of  $Y(Q)$ , and using elementary concepts of calculus, one gets easily the Wilson formula  $Q = \sqrt{2A\lambda / h}$ . In this particular case, I repeat, in this particular case, the number of lots ordered per year is  $N = \lambda/Q$  and the optimal time between orders is  $T = Q/\lambda$ , i.e.  $T = 1/N$ .

Let's now see what happened in a MASTER (after 5 years of Engineering courses) on Maintenance and Reliability, in the lessons for RCM [Reliability Centred Maintenance]:

Wilson formula  $Q = \sqrt{2A\lambda / h}$ , which holds only in the hypotheses we said just before, was provided to student for buying the spare parts, which obviously depend on the number of failures, which obviously depend on the unreliability, which obviously depend on the time failure, which obviously is a **random variable!!!** *A serious teacher should have proved that the formula holds true, before teaching it to students !!!!*

**2<sup>nd</sup> step: the case of random demand with “constant” demand rate and steady state of the stochastic process** We are going now to consider the demand as a random variable, so introducing the need of the use of probability theory. If we maintain all the previous hypotheses, but the number 5 and 6: 1) production is instantaneous, 2) delivery is immediate, 3) a production run incurs a fixed setup cost, 4) there is no interaction between different products, 5) demand is random, 6) demand rate is constant over time. We can depict the system as before [and in fig. 10]

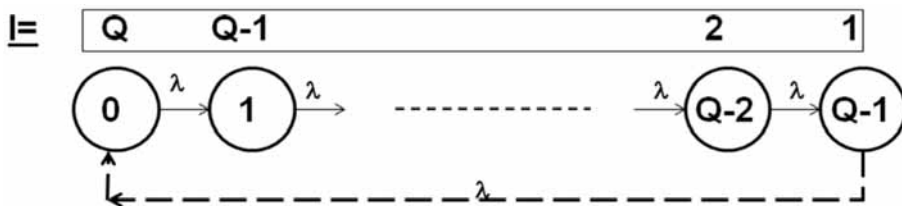


Fig. 10. System inventory states, with random demand and constant demand rate; state  $i$  means  $i$  products dispatched

where now the “time to sell a new unit (time between demands)” is a random variable exponentially distributed.

The function depicting the curve of the inventory level  $I(t)$  is a saw-tooth line, with variable [randomly] time distance between peaks.

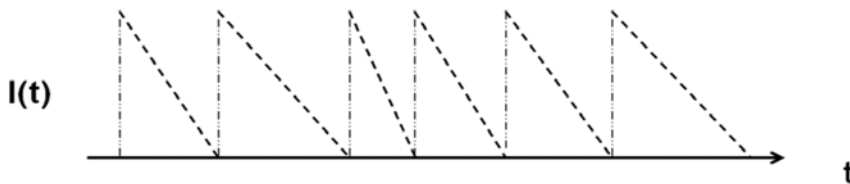


Fig. 11. Level of inventory versus time  $t$

Therefore the probabilistic structure of the inventory system is a Markov process, periodic with period  $Q$ . The mean time (holding time) in any state is  $m=1/\lambda$ , the steady-state transition probability from one state  $i$ , to the next state  $i-1$  is constant  $\varphi_i = \varphi_{i-1} = 1/Q$  [use Markov chains theory]. The “reward structure” is such that the order cost  $A$  is associated with the transition from state  $Q-1$  to  $0$ , while the holding cost, per unit time, for state  $i$  is  $y_i = (Q-i)*h$ ,  $i=2$  to  $Q$ , and  $y_1 = h+\lambda A$ ; the average cost per unit time,  $g$  (cost rate), for operating the system in the steady state is  $g = [\lambda A + hQ(Q+1) / 2] / Q$

The value  $Q$  that optimise the cost rate, in the steady state of the stochastic process, i.e. when the time is tending to infinity, is found as the solution of the previous equation. If  $Q$  is large, we can ignore the discrete nature of  $Q$  [ $Q$  is an integral number], assuming it can be

considered as a continuous variable: so we can differentiate and set the derivative equal to zero; the solution is (the famous Wilson lot-size formula)  $Q = \sqrt{2A\lambda/h}$ . If  $Q$  is small, we cannot ignore the discrete nature of  $Q$  [ $Q$  is an integral number], and the solution has to be found numerically.

**3<sup>rd</sup> : the case of random demand with “variable” demand rate and steady state of the stochastic process** We are going to consider again the demand as a random variable, (need probability theory), maintaining all the previous hypotheses [as in the 2<sup>nd</sup> case], but the number  $Q$ : 1) production is instantaneous, 2) delivery is immediate, 3) a production run incurs a fixed setup cost, 4) there is no interaction between different products, 5) demand is random, 6) demand rate is NOT constant over time, but it varies with time, identically after any transition from a state to the following one.

We can depict, again, the system as before [and in fig. 12]

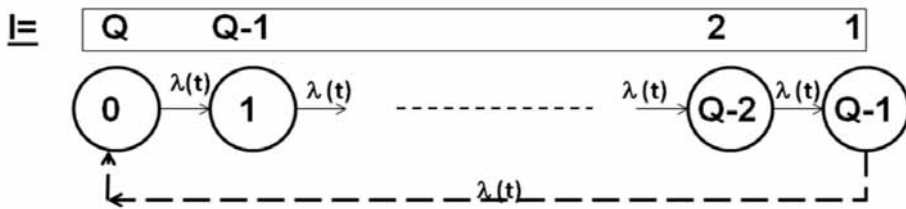


Fig. 12. System inventory states, with random demand and variable demand rate; state  $i$  means  $i$  products dispatched

where now the “time to sell a new unit (time between demands)” is a random variable “identically” [but not exponentially] distributed; let indicate the probability density of the time between transitions as  $f(t)$  [related to the “rate”  $\lambda(t)$ , with cumulative distribution  $F(t)$ ]; its mean is  $m$ .

The mean number of state transitions in the interval  $0 \rightarrow t$ ,  $M(t)$  is the solution of the integral equation

$$M(t) = F(t) + \int_0^t f(r)M(t-r)dr \tag{1}$$

The related intensity of state transitions, at time  $t$ , is  $m(t) = dM(t)/dt$ , the solution of the integral equation

$$m(t) = f(t) + \int_0^t f(r)m(t-r)dr \tag{2}$$

In the process steady state we have  $M(t) \cong t/m$  and  $m(t) \cong 1/m$ , for  $t \rightarrow \infty$ . The function depicting the curve of the inventory level  $I(t)$  is a saw-tooth line, with variable [randomly] time distance between peaks, too. [fig. 13]

Therefore the probabilistic structure of the inventory system is a semi-Markov process, periodic with period  $Q$ . The mean time (holding time) in any state is  $m$  [the mean of the distribution] identical for all the states; then the steady-state transition probability from one state  $i$ , to the next state  $i-1$  is constant  $\varphi_i = \varphi_{i-1} = 1/Q$  [use semi-Markov processes theory]. The “reward structure” is such that the order cost  $A$  is associated with the transition from

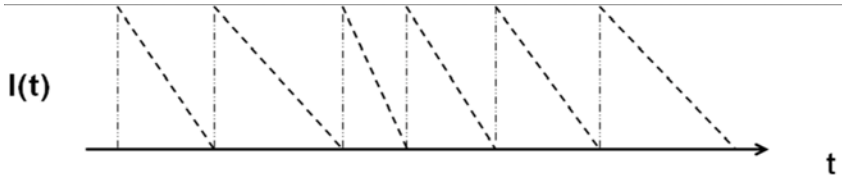


Fig. 13. Level of inventory versus time t

state  $Q-1$  to  $0$ , while the holding cost, per unit time, for state  $i$  is  $y_i = (Q-i)h$ ,  $i=2$  to  $Q$ , and  $y_1 = h+A/m$ ; the average cost per unit time,  $g$  (cost rate), for operating the system in the steady state is the same, as before,  $g = [A / m + hQ(Q + 1) / 2] / Q$ .

The value  $Q$  that optimise the cost rate, in the steady state of the stochastic process, i.e. when the time is tending to infinity, is found as the solution of the previous equation. If  $Q$  is large, we can ignore the discrete nature of  $Q$  [ $Q$  is an integral number], assuming it can be considered as a continuous variable: so we can differentiate and set the derivative equal to zero; the solution is  $Q = \sqrt{2A / (mh)}$  (similar to the famous Wilson lot-size formula); if different types of distributions are used, but with the same mean, one gets the same optimum  $g$ . If  $Q$  is small, we cannot ignore the discrete nature of  $Q$  [ $Q$  is an integral number], and the solution has to be found numerically.

Notice that we can manipulate the formula, obtaining the following  $g = \frac{A}{mQ} + \frac{hmQ(Q+1)/2}{mQ}$  that shows very clearly a fundamental fact of renewal processes:

the gain rate, **in the steady state of a process**, is the ratio of the cost during a renewal cycle and the length of the cycle [ $mQ$ , that is the mean sum of  $Q$  random variables, identically distributed]; we will find the same idea in the formulae of preventive maintenance.

Notice that nobody says that the formulas in the various books and papers are to be considered only for the **steady state**.

It is very interesting noting that, after a long time  $t^*$ , at which the stochastic process reaches "almost surely" its steady state, the cost for the interval  $t^* \text{---} t^*+t$  is

$$gt = \frac{1}{Q} [At / m + Q(Q+1)ht / 2] \tag{3}$$

which shows that  $t/m$  is the mean number of orders for the interval  $t^* \text{---} t^*+t$  (in the steady state) and  $Q(Q+1)ht/2$  is the mean number of products, for holding which we pay, for the interval  $t^* \text{---} t^*+t$  (in the steady state).

**4<sup>th</sup> step: the case of random demand with "constant" demand rate and steady state of the stochastic process** We are going to consider the demand as a random variable, so introducing the need of the use of probability theory, but we consider a lead time different from  $0$ , we maintain some of the previous hypotheses, but the number 2, 5 and 6: 1) production is instantaneous, 2) **delivery takes a constant time  $L$ , named Lead Time, after the order**, 3) a production run incurs a fixed setup cost, 4) there is no interaction between different products, 5) **demand is random**, 6) **demand rate is constant over time**. We can no longer depict the system as before; we need to distinguish between the net inventory  $I(t)$  and the inventory position  $IP(t)$ . The net inventory  $I(t)$  is the actual number of products we have on hand that we can send to our customers, after a time  $L$ , form their order. The inventory

position  $IP(t)$  is the sum of  $I(t)$ , the actual number of products we have on hand, the outstanding orders not yet arrived at time  $t$ , minus the products backlogged.; the order of  $Q$  products is placed, at any time  $t_0$ , when  $IP(t_0)$  equals the ROL (the Re-Order Level); unfortunately, in the meantime [duration  $L$ ] a stockout might occur: while we wait for the lot arrival (replenishment of the inventory), at time  $t_0 + L$ , the net inventory  $I(t)$  and the inventory position  $IP(t)$  decrease because of selling (and dispatching) products. If it happens that  $I(t_{STO})=0$ , at a time  $t_{STO}$ , we face an inventory **STockOut**, that generates a cost: customers are unsatisfied...; we lose to sell products, a case named "Lost Sales". The cost involved in this case are: the order cost  $A$ , the cost of holding the inventory (that varies with time), and the "penalty cost" due to stockout. The "time to sell a new unit (time between demands)" is a random variable exponentially distributed. The function depicting the curve of the Inventory Position level  $IP(t)$  is a saw-tooth line, with variable [randomly] time distance between peaks, exponentially distributed. [fig. 14]

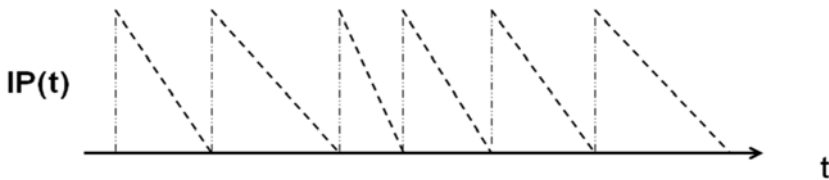


Fig. 14. Level of the inventory position versus time  $t$

Therefore the probabilistic structure of the inventory system is a Markov process, periodic with period  $Q$ . The mean time (holding time) in any state is  $m=1/\lambda$ , the steady-state transition probability from one state  $IP=i$ , to the next state  $IP=i-1$ , **in the process steady state**, is still constant  $\varphi_i = \varphi_{i-1} = 1/Q$  [use Markov chains theory and fig. 15].

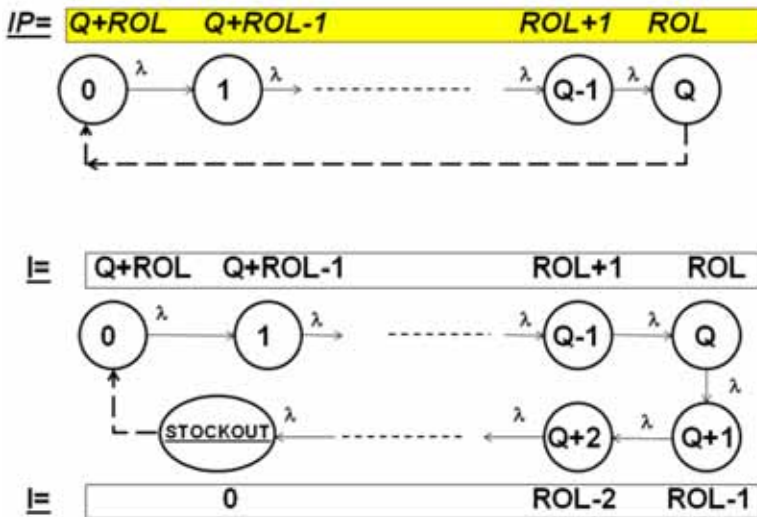


Fig. 15. System inventory states, with random demand and constant demand rate; state  $i$  means  $i$  products dispatched

The “reward structure” is such that the order cost  $A$  is associated with the transition from state  $Q+ROL$  and  $ROL$ ; the carrying inventory cost is associated with the mean number of products on hand times the time they are in the inventory, while the stockout cost is related to the probability that happens the event  $I(t)=0$ , in spite that we have  $ROL$  product when we order the lot of  $Q$  products. [we will use, for short,  $R$  for the  $ROL$ , ReOrder Level]

Let  $t_0$  be the time instant when  $IP(t_0)=R$ ; the net inventory  $I(t_0+L) = R - \text{demanded quantity } X_L$ , during the lead time  $L$ , is a random variable with the same type of distribution as Inventory Position  $IP(t_0+L)$ ; for any interval  $t_0 \text{---} t_0+\Delta t$  the holding cost is a random variable as well

$$h \int_{t_0}^{t_0+\Delta t} I(u)du = hTTI(t_0 \text{---} t_0 + \Delta t) \tag{4}$$

where we name “total time of inventory”,  $TTI(t_0 \text{---} t_0+\Delta t)$ , for any interval  $t_0 \text{---} t_0+\Delta t$ , the time for which we have to pay for the products we have on hands [net inventory ] and for the time they are on hands.

The mean of this random variable is

$$h \int_{t_0}^{t_0+\Delta t} E[I(u)]du = hE[TTI(t_0 \text{---} t_0 + \Delta t)] \tag{5}$$

Being  $I(t)=IP(t)-Q$ , for any time  $t$ , the “total time of inventory”,  $TTI(t_0 \text{---} t_0+\Delta t)$  depends on the transitions between the states  $0, 1, 2, \dots, Q$  and the related probabilities. Therefore the mean of  $TTI$  is

$$E[TTI(t_0 \text{---} t_0 + \Delta t)] = \frac{\Delta t}{\lambda} \{ (R+Q) + (R+Q-1) + (R+Q-1) + \dots + (R+1) \} - Q\Delta t \tag{6}$$

Using simple concepts of Algebra, we get

$$E[TTI(t_0 \text{---} t_0 + \Delta t)] = \frac{\Delta t}{2\lambda} \{ (R+Q)(R+Q+1) - R(R+1) \} - Q\lambda\Delta t / \lambda \tag{7}$$

Letting  $T_Q$  be the random time for selling  $Q$  products, and so reorder a new lot of products, we have, for any planning cycle  $t_0 \text{---} t_0 + T_Q$

$$E[TTI(t_0 \text{---} t_0 + T_Q)] = \frac{Q}{\lambda} \{ R + (Q+1) / 2 - \lambda L \} \tag{8}$$

Therefore the expected cost of inventory is

$$h \frac{Q}{\lambda} \{ R + (Q+1) / 2 - \lambda L \} \tag{9}$$

We proved this formula using probability theory; in all the books I read **never** there was the proof! Why?

The quantity  $ss=R - \lambda L$  is the safety stock that we hold in order to prevent stockouts.

If  $t_0$  is the time instant when  $IP(t_0)=R$ ; the stockout happens when the net inventory  $I(t_0+L) = R - \text{demanded quantity } X_L$ , during the lead time  $L$ , which is a random variable, falls below zero:  $P_{STO}=P[I(t_0+L) \leq 0]$ ; it means that, at some instant,  $t_{STO} \leq t_0+L$ ,  $I(t_0+ t_{STO})=0$ . Letting  $T_R$  be the random time for selling  $R$  product, we have  $P_{STO}=P[T_R \leq L]$ .

If  $T_R > L$  the system is able to provide products, we have on hand (net inventory), to all customers asking for them, filling their demands; that’s why the probability  $S(R,L) = P[T_R > L] = 1-P_{STO}$ , is named Service Level (type 1), or Fill Rate.

Noting that  $P[T_R > L]$  is the “reliability of a stand-by system of  $R$  products” failing with failure rate equal to  $\lambda$ , one can take advantage of the use of all the ideas of Reliability Theory for the field of Inventory Management.

Here we are doing that.

Let  $T_{STO}$  be the random variable “Time To Stock Out” of the inventory system and  $N_{STO}(t)$  be the random variable “Number of Stock Outs” of the system, in the interval  $0 \dots t$ ; at time  $t$  the system has a “residual life”  $\rho(t)$  until the next Stock Out,  $\rho(t) = T_{N_{STO}(t)} - t$ ; since the transitions depend on the exponential distribution  $\rho(t)$  is independent from the Number of the experienced Stockouts. Let  $S(R, t+x | t) = P[\rho(t) > x]$  be the type 1 Service Level, related to the interval  $t \dots t+x$ ; F. Galetto proved [chapter 6 of *Affidabilità, Volume 1: Teoria e Metodi di Calcolo*, (1995) CLEUP, Padova. Italy] that the type 1 Service Level  $S(R,t+x | t) = P[\rho(t) > x]$  is the solution of the integral equation

$$S(R, t + x | t) = S(R, t + x | 0) + \int_0^t f_{STO}(s)S(R, t + x | s)ds \tag{10}$$

where  $f_{STO}(t)$  is the probability density function of the 1<sup>st</sup>  $T_{STO}$ , with mean denoted as  $MTTSTO$  and named Mean Time To StockOut.

If  $t \rightarrow \infty$  the type 1 Service Level  $S(R,t+x | t)$  depends only on  $x$ ; F. Galetto proved [chapter 6 of *Affidabilità, Volume 1: Teoria e Metodi di Calcolo*, (1995) CLEUP, Padova. Italy] that  $S(R, x) = P[\rho(\infty) > x]$  is related to the density of stock outs  $f_{STO}(x | \infty) = S(R, x | 0)/MTTSTO$ .

Therefore, after a long time  $t$  that the inventory system is running, the steady state type 1 Service Level  $S(R, L)$  is

$$S(R, L) = \int_L^\infty S(R, s)ds \tag{11}$$

### 3. What one can find in documents

The following excerpts are copied directly from books; it is not important to report the names of the authors! None of the authors say that their formulae hold only in the **steady state** of the process. Notice that a lot of attention is needed in order to find the correspondence between the different notations.

From a book one can find, where  $d$  is the random demand,  $LT$  is the lead time,  $F_{dLT}(R)$  is the cumulative probability of sales during  $LT$ ,  $p$  is the cost (penalty) of stock out. Notice that there is no proof of this formula in the book.

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Under these assumptions the total cost is

$$C_{tot} = C_{or} + C_{in} + C_{so}$$

$$= A \cdot \frac{E(d)}{Q} + h \cdot (R + Q/2 - E(d) \cdot LT) \quad (5.15)$$

$$+ p \cdot \frac{E(d)}{Q} \cdot (1 - F_{dLT}(R)). \quad (5.16)$$

From another book one can find, where CSL is the Cycle Service Level [i.e. the fraction of replenishment cycles that end with all customer demand being met (a replenishment cycle is the interval between two successive replenishment deliveries). The CSL is equal to the probability of not having a stockout in a replenishment cycle, H is the cost of holding one item for one unit of time,  $C_u$  is the cost of one item, D is the average demand for one unit of time.

In the notations of the previous book  $D_L=E(d)$ ,  $H=h$ ,  $ROP=R$ , replenishment cycle is equal  $LT$ , CSL is then  $F_{dLT}(R)$ ,  $ss=R - \lambda L$  (safety stock).

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Desired cycle service level =  $CSL$ ,

Mean demand during lead time =  $D_L$ ,

Standard deviation of demand during lead time =  $\sigma_L$ .

From Equation 11.3, recall that  $ROP = D_L + ss$ . The store manager needs to identify safety inventory  $ss$  such that the following is true:

$$\text{Probability}(\text{demand during lead time} \leq D_L + ss) = CSL.$$

**Demand During Stockout Is Lost**

For the case where unfilled demand during the stockout period is lost, the optimal cycle service level  $CSL^*$  is given as follows:

$$CSL^* = 1 - \frac{HQ}{HQ + DC_u} \quad (12.7)$$

In this case we have assumed that  $C_u$  is the cost of losing one unit of demand during the stockout period. In Example 12.4, we evaluate the optimal cycle service level if demand is lost during the stockout period.

From another book, again, one can find, where  $G(r)=S(Q, r)$  is the Service Level (type 1),  $I(Q, r)$  is the average net inventory, D is the expected demand per year (in units), k is the cost per stockout.

In the notations of the previous book  $D=D_L=E(d)$ ,  $h=H=h$ ,  $r=ROP=R$ ,  $S(Q, r)=CSL=F_{dLT}(R)$ ,  $k=p$ .

Notice that that the three books provide to the students (or the managers) **three different formulae for the same concept**, the type 1 Service Level!!!

| Book | Formula for Service Level    | Equivalence <b>only IF</b>          | Equivalence <b>only IF</b>          |
|------|------------------------------|-------------------------------------|-------------------------------------|
| 1    | $F_{dLT}(R)$                 | $F_{dLT}(R) = 1 - HQ / (HQ + DC_u)$ | $F_{dLT}(R) = kD / (kD + hQ)$       |
| 2    | $CSL = 1 - HQ / (HQ + DC_u)$ | $DC_u = kD$                         | $1 - HQ / (HQ + DC_u) = F_{dLT}(R)$ |
| 3    | $G(r) = kD / (kD + hQ)$      | $kD / (kD + hQ) = F_{dLT}(R)$       | $kD = DC_u$                         |

It is very clear that it is very improbable that the cost per stockout is equal to the cost per unit.

**Stockout Cost Approach.** As an alternative to the backorder cost approach, we can make verbal formulation (2.32) into a mathematical model by writing the sum of the annual setup or purchase order cost, stockout cost, and inventory carrying cost as

$$Y(Q, r) = \frac{D}{Q}A + kD[1 - S(Q, r)] + hI(Q, r) \tag{2.50}$$

Going through the usual optimization procedure (taking the derivative with respect to  $r$ , setting the result equal to zero, and solving for  $r$ ) yields the following expression for the optimal reorder point:

$$G(r^*) = \frac{kD}{kD + hQ} \tag{2.52}$$

**A case from Fctory Physics**

“Jack, the maintenance manager, has collected historical data that indicate one of the replacement parts he stocks has annual demand ( $D$ ) of 14 units per year. The unit cost  $c$  of the part is \$150, and since the firm uses an interest rate of 20 percent, the annual holding cost  $h$  has been set at  $0,2 \cdot (\$150) = \$30$  per year. It takes 45 days to receive a replenishment order, so average demand during a replenishment lead time is  $\theta = \frac{14}{365} \cdot 45 = 1,726$ ” [units/lead time] “The part is purchased from an outside supplier, and Jack estimates that the cost of time and materials required to place a purchase order  $A$  is about \$15. The one remaining cost required by our model is the cost of either a backorder or stockout. Although he is very uncomfortable trying to estimate these, when pressed, Jack made a guess that the annualized cost of a backorder is about \$100 per year, and the cost per stockout event can be approximated by  $k = \$40$ . Finally, Jack has chosen to model demands using the Poisson distribution.

[...] The order quantity is computed by using ” [the Economic Order Quantity formula].

“which yields  $Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \cdot (15) \cdot (14)}{30}} = 3,7 \approx 4$  units.”

Let's provide clearly the relevant data: annual demand  $D=14$ . NOTICE "estimated from historical data", without any confidence interval!, Lead time  $L = 45$  days cost of order  $A = 15$  \$, holding cost  $h = 30$  \$ per unit per year stockout cost  $k= 40$  \$, demand distribution: Poisson. Since the demand distribution is Poisson, the time between demand is exponentially distributed, and the system can be modelled with a Markov chain in the steady state of the process.

On the contrary, the Factory Physics authors "approximate the Poisson by the normal, with mean 1.726 and standard deviation  $\sigma=1.314$ "; then they compute  $Q=3.7 (\cong 4)$  and  $r=2.946 (\cong 3)$  [with the formula  $G(r) = kD / (kD + hQ)$ ].

Using "reliability theory", we draw the transition diagram, with transition (selling) rate  $\lambda$  [solid lines] and replenishment [dotted lines]; in the steady state we can write the steady transition probability matrix  $P$  that provides us with the MTTSTO, the Cost per Unit Time, the Service Level. We compared our findings with the ones of the Factory Physics authors who "approximate the Poisson by the normal...": while the Factory Physics authors found a type 1 Service Level = 0.824, we found 0.903 a better value. We considered also other couples of values for  $Q$  and  $R$  and we found again better results; we provide the readers all the transition diagrams (fig. 16, 17, 18, 19).

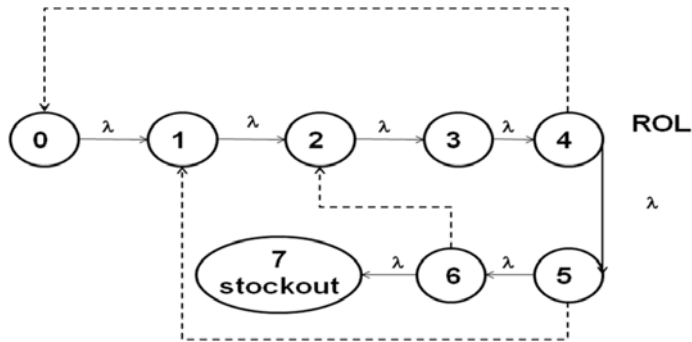


Fig. 16. System inventory states (random demand at constant rate); case  $Q=4, R=3$  [Factory Physics]

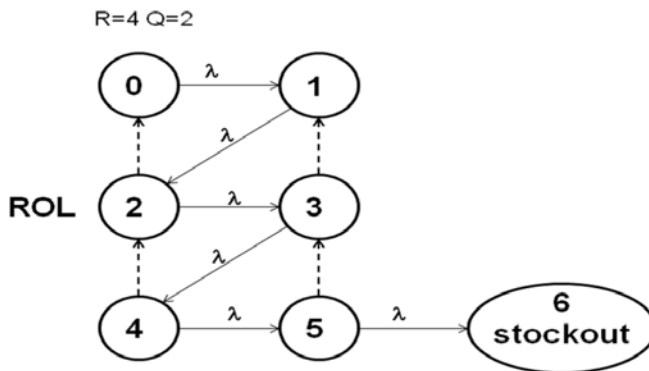


Fig. 17. System inventory states (random demand at constant rate); case  $Q=2, R=4$  [Factory Physics]

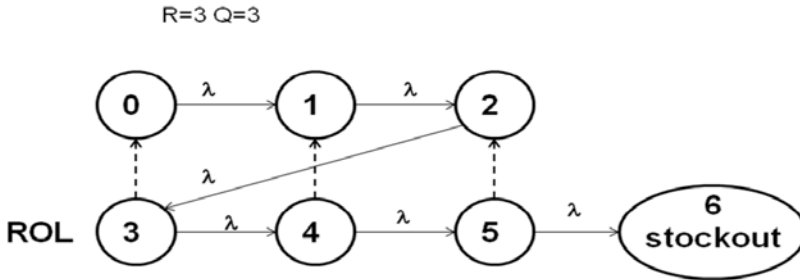


Fig. 18. System inventory states (random demand at constant rate); case Q=3, R=3 [Factory Physics]

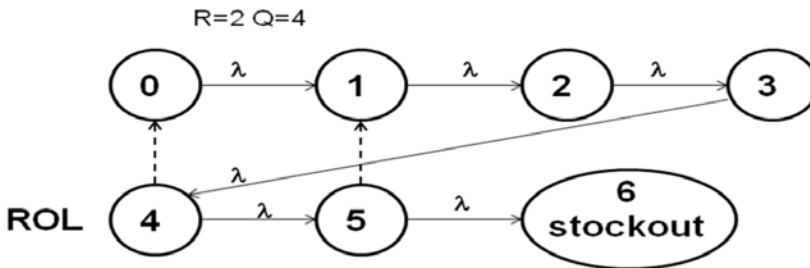


Fig. 19. System inventory states (random demand at constant rate); case Q=4, R=2 [Factory Physics]

The results are, in the steady state,

|                     |               |               |               |               |
|---------------------|---------------|---------------|---------------|---------------|
| R=                  | 4             | 3             | 2             | 3             |
| Q=                  | 2             | 3             | 4             | 4             |
| Cost rate (\$/year) | <b>230.58</b> | <b>203.24</b> | <b>178.30</b> | <b>167.32</b> |
| Service Level       | <b>0.969</b>  | <b>0.903</b>  | <b>0.750</b>  | <b>0.903</b>  |

It is easily seen that Q=2 and R=4 provide better Service Level (97% vs 82% found by FP) at a higher cost per year, in the steady state. In case of failures in a production line the cost of unavailability is much higher than 40\$ ... !

It is interesting to notice that the Factory Physics authors did not find that Q=2 and R=4 is the best solution, provided that 97% of Service Level is considered adequate. In any case it is really better than the solution given to students by the Factory Physics authors.

We used the exponential distribution because we accepted that the “arrival of failures” was according a Poisson distribution: this implies that the reliability of each item is exponential with failure rate  $\lambda/N$ , where N is the number of items in use; the “Mean Number of Failures in the interval 0-----t”, M(t), is equal to  $\lambda t$  and the variance is  $\lambda t$ , as well.

The distribution of the time to failure of the items was assumed exponential; many times it is not so.

Therefore we are going to develop a method adequate for any distribution.

In order to do that we will use the following distribution of the “time to sell one item”; we do so because it is related to the normal distribution of the items sold during the time;

therefore the inventory  $I(t)$  is a normal stochastic process, during the lead time  $L$ : this is the standard assumption in all the books I read.

The probability density of the time to sell one product is ( $\mu_1$  is the mean time to sell one product)

$$f(t|1, \mu_1) = e^{-\frac{(t-\mu_1)^2}{2t\mu_1^2}} / \sqrt{2\pi t^3} \tag{12}$$

If we consider the time to sell  $Q$  products,  $T_Q$ , this has a distribution of the same type, as a result of the convolution of  $Q$   $f(t|1, \mu)$ ; we indicate it as  $f_Q(t|\eta, \mu)$  (where we have  $E[T_Q]=\mu$  and  $Var[T_Q]=\mu^3/\eta$ . Besides  $\eta=Q^2$ .)

$$f_Q(t|\eta, \mu) = \sqrt{\frac{\eta}{2\pi t^3}} e^{-\eta \frac{(t-\mu)^2}{2t\mu^2}} \tag{13}$$

The mean holding time in a state  $m_i$ , and steady state transition probabilities,  $p_{ij}$  are found in the same manner as in the case of the Poisson process [not with the same formulae].

Using this distribution for the previous cases, as obvious, we get quite different **steady state results**

|                     |               |               |
|---------------------|---------------|---------------|
| R=                  | 4             | 2             |
| Q=                  | 2             | 4             |
| Cost rate (\$/year) | <b>182.27</b> | <b>148.10</b> |
| Service Level       | <b>0.959</b>  | <b>0.584</b>  |

The best solution for the **steady state** cost rate is  $Q=4$  and  $R=2$ , while for the service level is  $Q=2$  and  $R=4$  (as before).

#### 4. 5<sup>th</sup> step: the case of random demand with “constant” demand rate BUT NOT steady state of the stochastic process

We maintain some of the previous hypotheses, but the number 2, 5 and 6: 1) production is instantaneous, 2) **delivery takes a constant Lead Time, after the order**, 3) a production run incurs a fixed setup cost, 4) there is no interaction between different products, 5) demand is random, 6) demand rate is constant over time.

Before we considered the process in its steady state, i.e. after a long time  $[t \rightarrow \infty]$ ; now we consider the case of finite time  $t$ .

As in the previous paragraph, we distinguish between the net inventory  $I(t)$  and the inventory position  $IP(t)$ . If, at some instant of time  $t_{STO}$ ,  $I(t_{STO})=0$ , we face an inventory **STockOut**, that generates a cost: customers are unsatisfied...; we lose to sell products, a case named “Lost Sales”: as before, the costs involved are the order cost  $A$ , the cost of holding the inventory (that varies with time, and the “penalty cost” due to stockout.

We assume again that the “time to sell a new unit (time between demands)” is a random variable exponentially distributed, with rate  $\lambda$ ; so the Inventory Position level  $IP(t)$  is a saw-tooth line, with variable [randomly] time distance between peaks, exponentially distributed [Poisson Process]. Therefore the probabilistic structure of the inventory system is a Markov

process, periodic with period  $Q$ . Letting  $P_i(t)=P[\text{process in the state } i, \text{ at time } t]$ , be the probability that process in the state  $i$ , at time  $t$ , we can write a system of differential equation; in order to make the system simple we consider the case  $R=2$  and  $Q=4$ . [fig. 21]

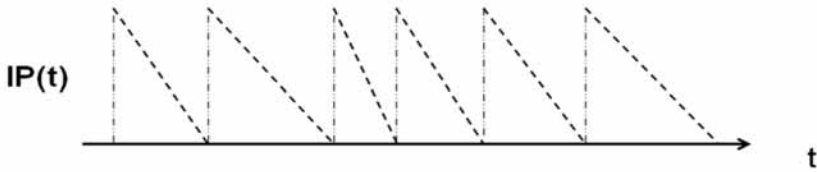


Fig. 20. Level of the inventory position versus time  $t$

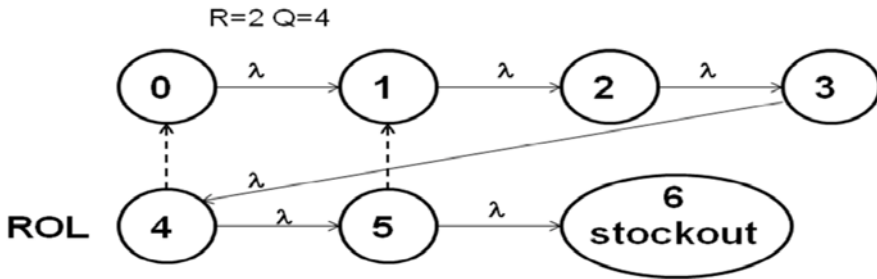


Fig. 21. System inventory states (random demand at constant rate); case  $Q=4, R=2$  [Factory Physics]

The equations

$$\begin{aligned}
 P_0'(t) &= -\lambda P_0(t) + \delta(t-L)P_4(t) & P_1'(t) &= \lambda P_0(t) - \lambda P_1(t) + \delta(t-L)P_5(t) \\
 P_2'(t) &= \lambda P_1(t) - \lambda P_2(t) & P_3'(t) &= \lambda P_2(t) - \lambda P_3(t) & P_4'(t) &= \lambda P_3(t) - \lambda P_4(t) \\
 P_5'(t) &= -\lambda P_5(t)
 \end{aligned} \tag{14}$$

can be written in matrix form as

$$\dot{P}(t) = AP(t) \tag{15}$$

where we have the vector  $P(t)$  [its derivative  $P'(t)$ ] and the matrix  $A$  of the transition rates,  $\lambda$  and  $\delta(t-L)$  ["Dirac impulse"].

The Service Level  $S(t, L)$ , i.e. no stockout for the interval  $0 \rightarrow t$ , and Lead Time  $L$  (for replenishment) is  $S(t, L) = P(t)u$ , dot product of the vector  $P(t)$  and the vector  $u = [1, 1, 1, 1, 1, 1]$ .

The instantaneous Inventory Level  $I(s, L)$ , for any time  $s$  of the interval  $0 \rightarrow t$ , and Lead Time  $L$  (for replenishment) is

$$I(s, L) = \sum_0^5 (R + Q - i) P_i(s) \tag{16}$$

The cost of Inventory, for the interval  $0 \rightarrow t$  is  $I(t, L) = h \int_0^t I(s, L) ds$

The Number of Orders is the number of times [random variable] the system enters the state 4; its expected value  $M(t)$  is the mean number of orders. The probability of stockout is the probability of entrance in to the state 6.

Putting all together we have the relationship that provides the cost  $C(t, L)$  of the system; the cost rate, usually used in books, is here  $C(t, L)/t$ ; it is in the following figure 22.

As we can see the cost rate increases and, after 6 months decreases. The cost rate is lower in the case  $Q=4$  and  $R=2$  with lower service level, as we found in the steady state analysis; both curves decrease very slowly to their steady state rate.

|                                 |               |               |
|---------------------------------|---------------|---------------|
| R=                              | 4             | 2             |
| Q=                              | 2             | 4             |
| Cost rate (\$/year) at 6 months | <b>643.69</b> | <b>378.11</b> |
| Service Level at 6 months       | <b>0.974</b>  | <b>0.861</b>  |

If the service level 0.86, due to the consequences of downtime, is not adequate, we have to choose ...

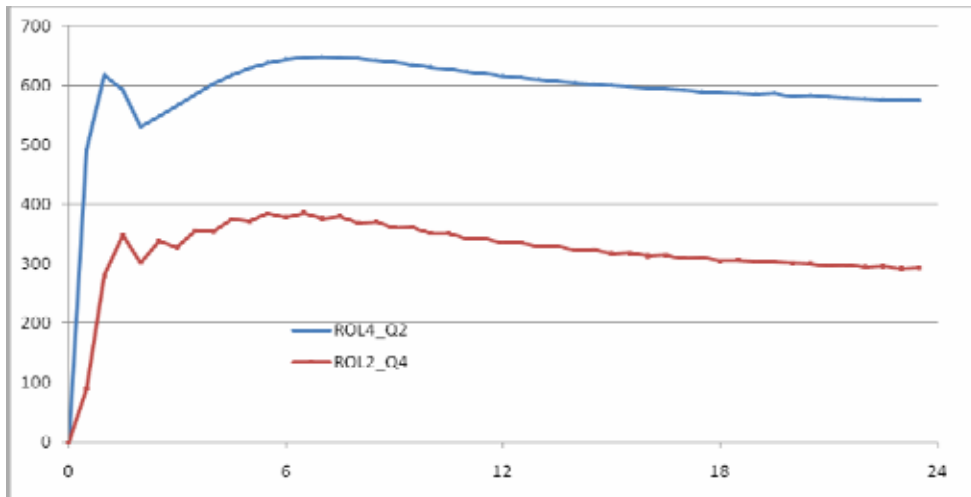


Fig. 22. Cost rate of two inventory systems (random demand at constant rate):  $Q=4, R=2$  vs  $Q=2, R=4$  [Factory Physics]

It is therefore very important to consider the interval in which we want to optimise. This subject has been pointed out many times by F. Galetto, in relation with the Preventive Maintenance. We are going to show it because it has an amazing similarity with inventory management.

When the values  $Q$  and  $R$  increase the dimension of the vectors and matrices  $\dot{P}(t) = AP(t)$  increase accordingly.

### 5. Preventive maintenance and RCM, an analogy with inventory management

We said before that it is very important the interval in which we want to optimise and that this subject has been pointed out many times by F. Galetto, related to the Preventive

Maintenance. We are now going to show it because it has an amazing similarity with inventory management, and shows that there is no need of Poisson processes. In 1977, at the *Reliability & Maintainability Symposium*, Philadelphia, F. Galetto, with his paper "*SARA (System Availability and Reliability Analysis)*", provided the Reliability Integral Theory that solves various reliability problems that cannot be dealt through Markov processes. The theory did exist, not the single formulae: any scholar could have found them. The same theory Reliability Integral Theory, is applicable to maintenance problems, as those presented in "Some graphical methods for maintenance planning"; in that paper, mentioned before, following ideas of Barlow, the optimum replacement interval is found, by B. Bergman, minimising the formula, cost ratio (given in Barlow;  $c_1$  is the cost of preventive replacement,  $c_2$  is the cost due to failures, while  $c$  [Bergman] is the cost of preventive replacement, assuming 1 the cost due to failures:  $c = c_1 / c_2$ )

$$C(t_p) = \frac{cF(t_p) + R(t_p)}{\int_0^{t_p} R(x)dx}, \quad \text{very similar to} \quad C(t_p) = \frac{c_1F(t_p) + c_2R(t_p)}{\int_0^{t_p} R(x)dx},$$

Barlow and Bergman say that those formulas are valid "in the long run": that means after an infinite time, that is after an infinite number of failures! But for a finite time span  $0-t$ ? The cost ratio is computed as the ratio of the mean cost for one cycle divided by the mean duration of one cycle, as it can be done for renewal processes, in the steady state! The same is found in logistics books, ... but they do not say ... that it is valid ONLY in the steady state!!!! [see the cases ...]

One can understand this problem through the F. Galetto papers presented in 1977, SARA at the *Reliability & Maintainability Symposium*, Philadelphia, and CLAUDIA, 21<sup>st</sup> EOQC Conference, Varna (Bulgaria).

Some hints are given here, for preventive maintenance:

Let  $N_0(t, t_p)$  be the number of replacements of unfailed items and  $N_1(t, t_p)$  be the number of failed items over the interval  $0-t$ . They are random variables; their expectations are indicated as  $M_0(t, t_p)$  and  $M_1(t, t_p)$ .

Let  $C(t, t_p)$  be the total expected cost over the interval  $0-t$ , when the unfailed items are "renewed" [at cost  $c_0$ ] at their life  $t_p$ , and failed items are replaced [at cost  $c_1$ ;  $c_1$  includes the cost of the consequences of the failure] with new items. It is

$$C(t, t_p) = c_0 M_0(t, t_p) + c_1 M_1(t, t_p) \tag{17}$$

Let  $R(t, t_p)$  be the reliability for the interval  $0-t$ , when unfailed items are "renewed" at their life  $t_p$ , and  $b(t, t_p)$  the related probability density function. To optimise the cost over the interval  $0-t$ , we need either to compute  $M_0(t, t_p)$  and  $M_1(t, t_p)$  or to compute  $C(t, t_p)$  from the following integral equation

$$C(t, t_p) = c_0 \sum_{i=1}^n R^i(t_p) + \int_0^t [c_1 + C(t-s, t_p)] b(s, t_p) ds \tag{18}$$

where  $n$  is such that  $nt_p \leq t \leq (n+1)t_p$  and  $R(t_p)$  is the reliability of the item for the period of duration  $t_p$ . [formula derived using the "Integral Theory of Reliability" devised by F. Galetto in 1971]

In the formula the 1<sup>st</sup> term gives account of unfailed items during the interval 0–t: it excludes that any failure happens. The failures are considered in the integral from 0 to t; let the 1<sup>st</sup> failure occur in a interval s–s+ds, with probability b(s,t<sub>p</sub>)ds, and cost c<sub>1</sub>; the other failures occur in the remaining interval s–t, and their cost is C(t-s, t<sub>p</sub>): being s any instant we integrate over the interval 0–t.

It is clear that the optimum interval for the preventive replacement depends on t, the duration of the interval 0–t, over which we want to optimise, and the “cost ratio” c<sub>0</sub>/c<sub>1</sub>. The optimum value t\*<sub>p</sub>, as found in the literature on preventive maintenance, is the solution of

$$h(t^*_p)MTTF(t^*_p) = c_1/(c_1-c_0) + (c_0/(c_1-c_0)) R(t^*_p)/F(t^*_p) \tag{19}$$

which optimise the ratio C(t,t<sub>p</sub>)/t, that, for t → ∞, does not depend on t. [steady state] This is not satisfactory, because it does not take into account properly of the probability theory, and it is not said over which period 0–t we want to optimise. The foundations for finding the solution of the integral equation for a finite time span t is found using “Integral Theory of Reliability”.

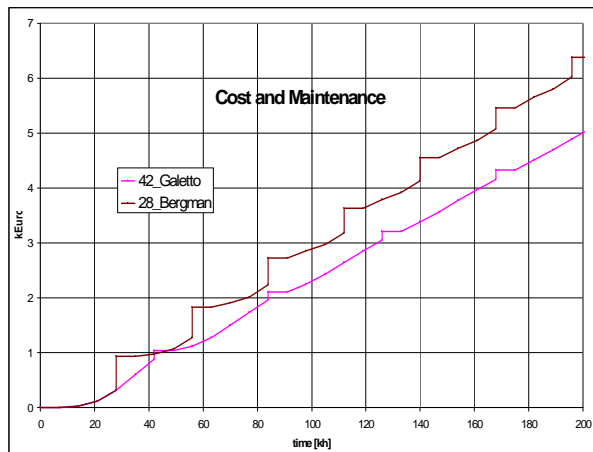


Fig. 23. Comparison of “optimum” replacement intervals

Let's suppose we have the data on a sample of n=7 items; and the time to failure (in the sample) are 60, 105, 180, 300, 400, 605, 890. The items have IFR (as one can easily find). To find the optimum replacement interval, both Barlow and Bergman use a graphical procedure on the TTT-plot: they find the “tangent” to the empirical TTT-plot and passing through the point (-c, 0) for Bergman while for Barlow is (-c<sub>2</sub>/(c<sub>1</sub>-c<sub>2</sub>), 0). The procedure is the same, but the points are different!!! Why? Assuming a cost c=0.9 for preventive replacement and 1 for failure, Bergman finds t\*<sub>p</sub> = 27.9

On the contrary, optimising the integral equation (with η and β estimated from the data), F. Galetto finds, over an interval 0–200, as optimum interval t\*<sub>p</sub> = 42. The graph in fig. 23 shows easily that t\*<sub>p</sub> = 42 is better than t\*<sub>p</sub> = 28 (approximating 27.9); the optimum interval depends on the interval 0–t: rarely t= ∞ is a good choice! [fig.1, 5, 32, 33, 34, 35]

So we see that for more than 40 years we had the theory to solve various reliability problems. The same ideas can be used to solve a problem given in example 14.1, pag. 345, of the book *Practical Reliability Engineering*, Wiley & Sons (1997). There it is written:

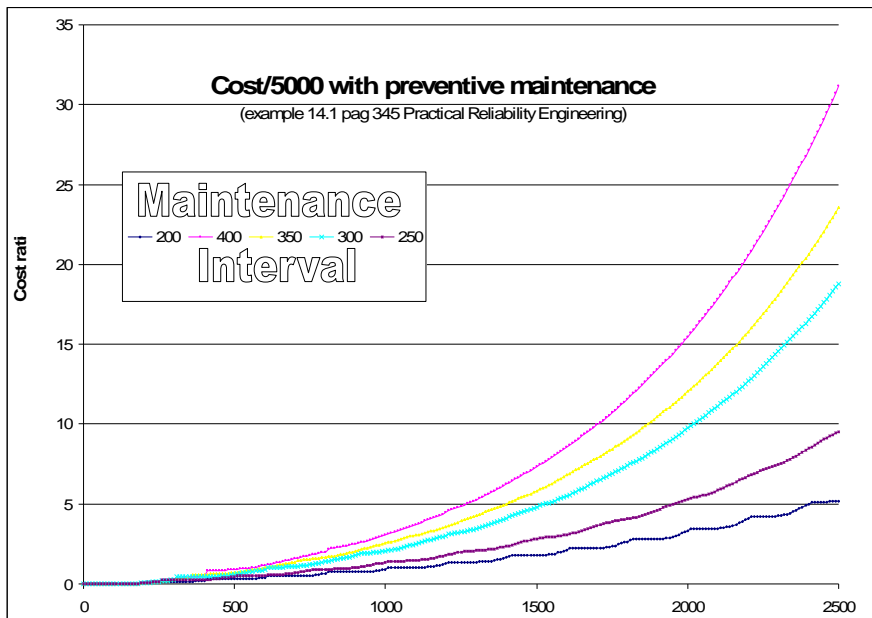
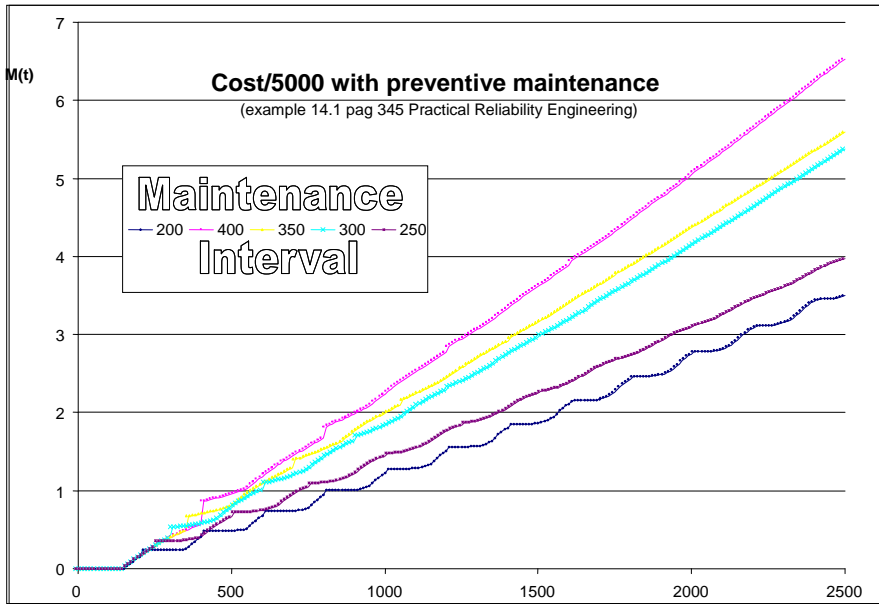


Fig. 24. Costs of preventive maintenance

“A flexible cable on a robot assembly line has a time-to-failure distribution which is Weibull, with  $\gamma=150$  h,  $\beta=1.7$ ,  $\eta=300$  h. If a failure occurs whilst in use the cost of stopping the line and replacing the cable is \$5000. The cost of replacement during scheduled maintenance is \$500. If the line runs for 5000 hours a year and scheduled maintenance takes place every week (100 hours), what would be the annual expected cost replacement at one-weekly or two-weekly intervals?”

The solution provided in the book, for 5000 h, is

|              |      |      |      |
|--------------|------|------|------|
| interval (h) | 100  | 200  | 400  |
| cost/5000    | 5.00 | 3.66 | 7.75 |

therefore 200 h interval is considered the best.

Actually it is better to replace every 150 h: no failures and only preventive replacements. We made a digression to preventive maintenance problems because their analogy with inventory management; the table shows it

|               |                                       |   |                          |
|---------------|---------------------------------------|---|--------------------------|
| $M_0(t, t_p)$ | Mean number of preventive replacement |   | Mean number of orders    |
| $M_1(t, t_p)$ | Mean number of failures               |   | Mean number of Stockouts |
| $c_0$         | cost of preventive replacement        | A | cost of orders           |
| $c_1$         | cost of one failure                   | p | cost of a Stockout       |

It is apparent that Reliability Integral Theory will help us in dealing with inventory management.

### 6. 6<sup>th</sup> step: the case of random demand with “NONconstant” demand rate and NOT steady state of the stochastic process

We assume the following hypotheses: 1) *delivery takes a constant Lead Time L, after the order*, 2) any order incurs a fixed order cost, A, 3) there is no interaction between different products, 4) *demand is random*, 6) *demand rate is NOT constant at every instant of time*.

In the five steps before we considered the process in its steady state, i.e. after a long time [ $t \rightarrow \infty$ ]; now we consider the case of finite time t, as done for preventive maintenance.

Letting again  $P_i(t) = P[\text{process in the state } i, \text{ at time } t]$ , be the probability that process in the state i, at time t, we can depict the system with the following transition diagram for the case  $R=2$  and  $Q=4$ .

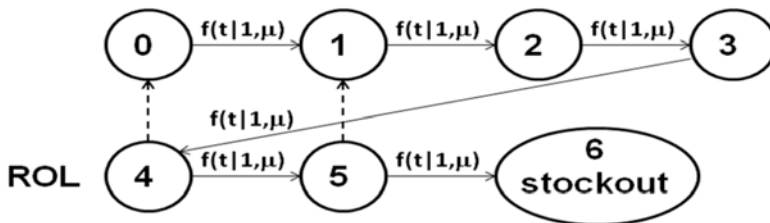


Fig. 25. System inventory states (random demand, variable rate); case  $Q=4$ ,  $R=2$  [Factory Physics]

The equations are now integral equations, as in the Reliability Integral Theory; the integral equations have the same structure, whatever is the analytic form of the density  $f(t | 1, \mu)$ , that we write simply  $f(t)$ ; let  $g(s)$  be the probability density of the time to replenishment (so dealing with random Lead Time). Following the Galetto's Reliability Integral Theory ideas we can write, being  $R_i(t-s)$  the probability of no stockout [service level] if the system enters state  $i$ , at time  $s$ , and

$$\bar{F}(t) = 1 - F(t) \quad \bar{G}(t) = 1 - G(t) \quad \bar{W}(t) = \bar{F}(t)\bar{G}(t) \tag{20}$$

$$\begin{aligned} R_0(t) &= 1 - F(t) + \int_0^t f(s)R_1(t-s)ds & R_1(t) &= 1 - F(t) + \int_0^t f(s)R_2(t-s)ds \\ R_2(t) &= 1 - F(t) + \int_0^t f(s)R_3(t-s)ds & R_3(t) &= 1 - F(t) + \int_0^t f(s)R_4(t-s)ds \\ R_4(t) &= \bar{F}(t)\bar{G}(t) + \int_0^t f(s)\bar{G}(s)R_4(t) + \int_0^t g(s)\bar{F}(s)R_0(t) & R_5(t) &= \bar{F}(t)\bar{G}(t) \end{aligned} \tag{21}$$

This can be written in matrix form as

$$R(t) = \bar{W}(t) + \int_0^t B(s)R(t-s)ds \tag{22}$$

where  $b_{i,k}(s)ds$  is the matrix of probability transition from state  $i$  to state  $k$ , in the interval  $s \rightarrow s+ds$ .

The Service Level  $S(t)$ , i.e. no stockout for the interval  $0 \rightarrow t$ , is  $R_0(t)$ . The probability  $P_i(t)$  of being in state  $i$ , at time  $t$ , is  $P_i(t) = R_i(t) - R_{i+1}(t)$ .

The integral of  $R_i(t)$ , from  $0$  to  $\infty$ , provides the Mean Time To StockOut  $MTTSTO_i$ , when the system begins in state  $i$ .

The instantaneous Inventory Level  $I(s)$ , for any time  $s$  of the interval  $0 \rightarrow t$ , and random Lead Time (for replenishment) is  $I(s) = \sum_0^5 (R + Q - i)P_i(s)$  with cost of Inventory, for the

interval  $0 \rightarrow t$   $I(t) = h \int_0^t I(s, L)ds$ . Since the probabilities  $P_i(t) \rightarrow 0$ , as  $t \rightarrow \infty$ , because it is certain

that a stockout happens,  $I(\infty) = h \int_0^\infty I(s, L)ds$  is a finite quantity.

Putting all together we have the relationship that provides the cost  $C(t, L)$  of the system; the cost rate, usually used in books, is here  $C(t, L)/t$ ; it is in the following figure 26. If we compute the cost Mean Cost To StockOut  $MCTSTO_0$ , from state  $0$  to the state of stockout, the cost rate is  $g = MCTSTO_0 / MTTSTO_0$ . [compare this with the findings in RCM...]

This result is valid for any distribution of the time to sell product, from exponential, to Erlang, to ..., ....

As we can see, for the distribution  $f(t | 1, \mu)$  considered before, the cost rate increases and, after 12 months decreases. The cost rate is lower in the case  $Q=4$  and  $R=2$ , while the service is better for  $Q=2$  and  $R=4$ , as we found with the steady state analysis.

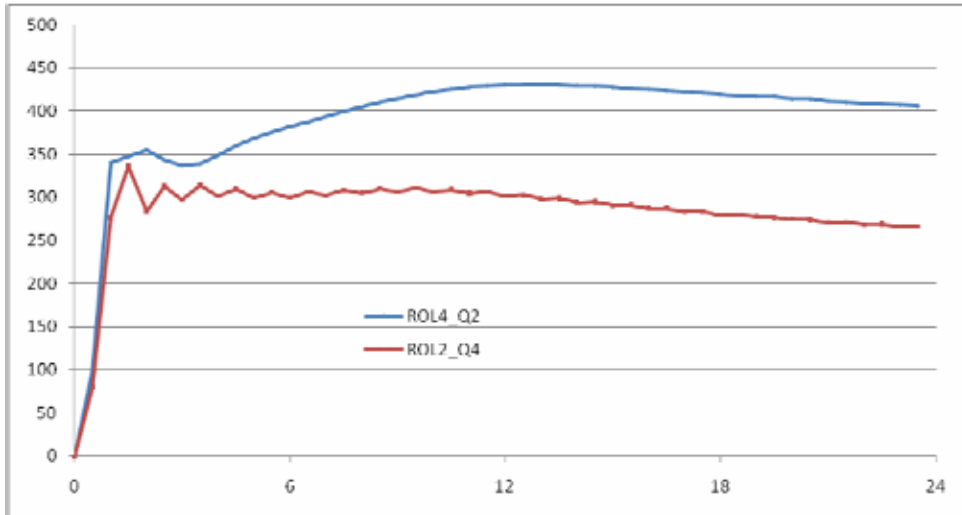


Fig. 26. Cost rate of two inventory systems (random demand at variable rate): Q=4, R=2 vs Q=2, R=4 [Factory Physics]

|                                 |               |               |
|---------------------------------|---------------|---------------|
| R=                              | 4             | 2             |
| Q=                              | 2             | 4             |
| Cost rate (\$/year) at 6 months | <b>381.54</b> | <b>299.29</b> |
| Service Level at 6 months       | <b>0.914</b>  | <b>0.691</b>  |

If the service level 0.69, due to the consequences of downtime, is not adequate, we have to choose...

It is therefore very important the interval in which we want to optimise. [fig. 1, 32-35]

This subject has been pointed out many times by F. Galetto, in relation with the Preventive Maintenance. We showed it because it has an amazing similarity with inventory management.

The best solution for the cost rate is Q=4 and R=2, while for the service level is Q=2 and R=4 (as before).

We end this paragraph by noting that F. Galetto's ideas considered here for the "lost sales" model, or "type 1 Service" model, **as can be found in books and papers**, whose costs involved are (being  $\lambda$  the rate of demand, always considered as constant !!!!):

- $A\lambda/Q$ , the "average" order cost per year
- $(1-F_{LT}(R))p\lambda/Q$ , the cost of stockout [p is the cost per stockout, in euro]  $F_{LT}(R)$  is the CD of demand during the lead time LT, **generally ASSUMED as Normally distributed in books and papers**
- $h(R + Q/2 - \lambda LT)$ , "average" inventory cost per year

are much more suitable for the analysis of practical problems.

The promise of books and papers, "If stockouts are permitted and lead time is  $LT > 0$ , if demand is random, the formula to be minimised is the

average cost rate =  $A\lambda/Q + h(R + Q/2 - \lambda LT) + (1 - F_{LT}(R))p\lambda/Q$  (where R is the trigger quantity for launching the order Q, p is the penalty for stockouts and  $F_{LT}(d)$  is the probability distribution of the demand d, during the lead time LT". actually is based on an inconsistent formula, because it is founded on an **intuitive [not proved] extension** of the formula for <<"constant" demand, no stockouts permitted and lead time zero>>.

Notice that this formula is the same as that in case of "everything known" and constant!!!!!!!!!!!!!!.

No scientific proof of the formula is ever provided!!! Understanding that the formula is wrong is very easy.

The probability of stockout depends on the competition of two stochastic processes: the demand versus the replenishment. The replenishment density g(t) is not present in any formula you can find **in books and papers.**

One can make a Gedanken Experiment, using the Bernoulli's Theorem (equation of motion in hydrodynamics) to guess: let's consider a physical system of three water containers with the same base area, as in the figure 27; the water flows from the discharge tube of the middle container; when the water level of the middle container falls below R level, the pump is triggered and the water flows (from the lower container) into the upper container till the level Q is achieved: at this time the water is immediately dropped into the middle container. IF this happens before the 0 level in the middle container is reached, the probability of water "stockout" is zero! One needs many cycles to experience water "stockout"!!!! [depending on the random factors, pump delivery and discharge tube, in order to take into account the random demand and the random replenishment time]

The formula average cost rate =  $A\lambda/Q + h(R + Q/2 - \lambda LT) + (1 - F_{LT}(R))p\lambda/Q$  does not "agree" with figure 27, and therefore is wrong: you can get all that using F. Galetto ideas, as given in his books and papers. [Brandimarte 2004]

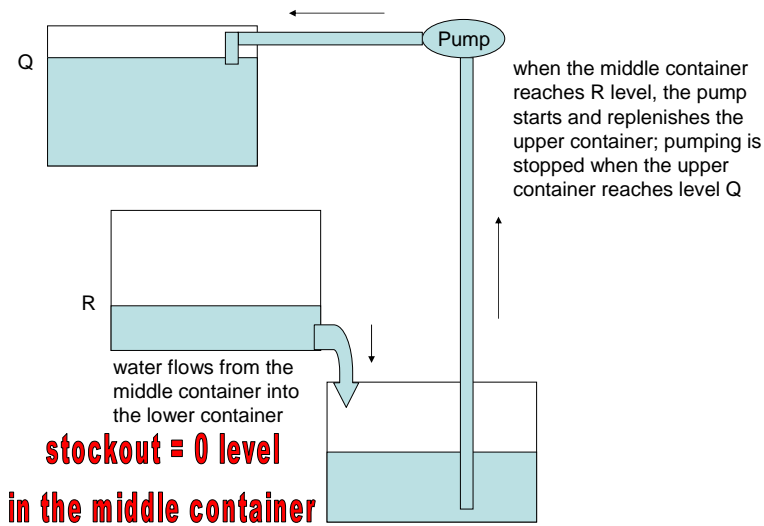


Fig. 27. the water tanks

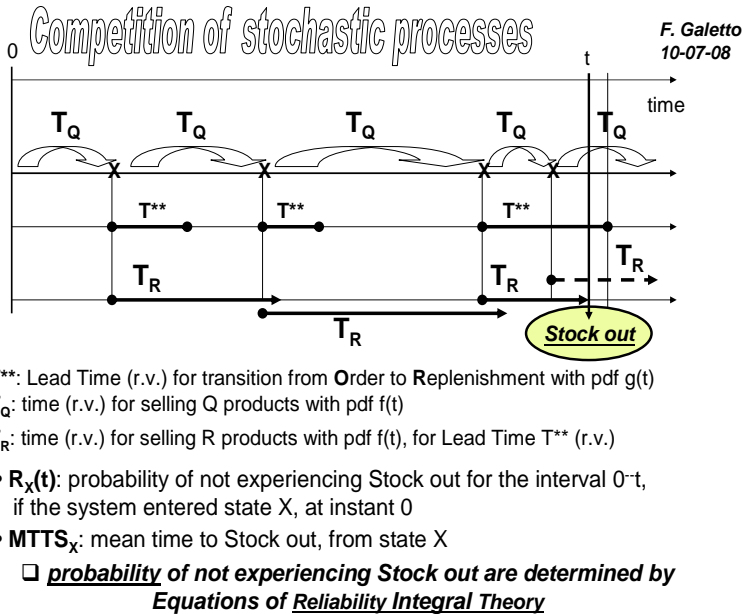


Fig. 28. the competition of the two stochastic processes, selling and replenishment

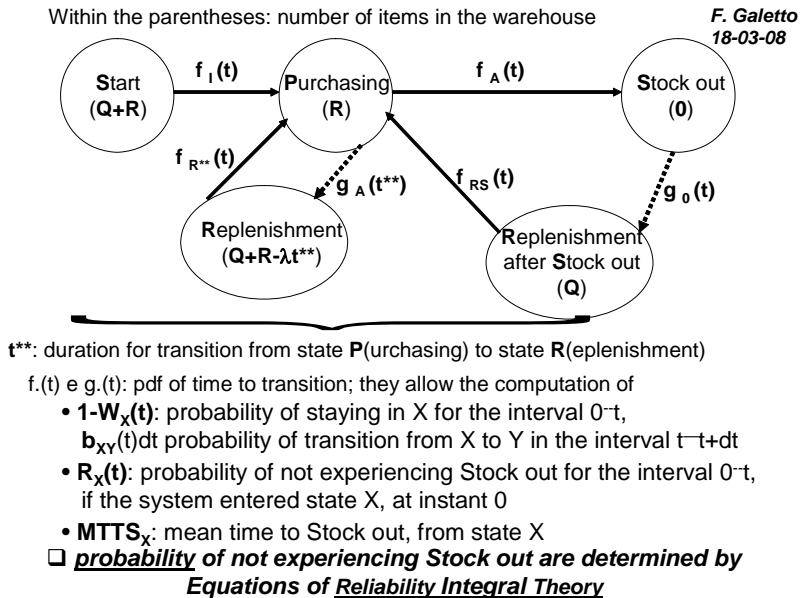


Fig. 29. Transitions between the inventory system due to selling and replenishment competition (stochastic processes)

**7. Some others cases found in documents**

Let's consider several examples, with wrong solutions, from various sources.  
 For **example** Brandimarte and Zotteri, in their book, found  $Q=111, R=143$  and type 1 Service level= $95.73\%$ , using the following data:  $A=50, h=2, p=500$ , Normal Distribution during  $LT=6$  months, with pdf  $N(\mu,\sigma)=N(100, 25)$ . They say

We can find the optimal solution  $Q^* = 111$  e  $R^* = 143$  with a type I service level  $F(R) = 95.73\%$  by simply iterating the above process.

Using the same data, and the right ideas (as sketched in fig. 27), F. Galetto finds  $Q=80, R=92$  and type 1 Service level= $95.73\%$ : **lower costs for the same service level, in the steady state!!!!** This result comes from the right formulae.

If we consider a finite horizon, and the density  $f(t | 1, \mu)$ , the system faces stockout if  $T_R < LT$ ; if, on the contrary, if  $T_R > LT$ , then the inventory, expired the time  $LT$ , raises its quantity by  $Q$ , and a new cycle starts. [see Fig. 27]

The findings, for the three methods, are [Lead time assumed constant]:

| <i>Method</i>   | <i>Formula of the books</i> | <i><math>g=MCTSTO_0/MTTSTO_0</math></i> | <i>using <math>f(t   1, \mu)</math></i> |
|-----------------|-----------------------------|---|---|
| Q and R         | <b>1:</b> $Q=100, R=141$    | <b>2:</b> $Q=100, R=133$                | <b>3:</b> $Q=100, R=121$                |
| cost (6 months) | 603,6                       | 567,8                                   | 492,50                                  |

The scientific method provides **"Lower costs for the same service level"!!** Many other examples could be provided. How many students, all over the world, are learning wrong methods and will take wrong decisions?

**Even though the Holy Spirit**  
 should have provided the  
**LOGISTIC formulae**

$$C_{tot} = A \cdot \frac{E(d)}{Q} + h \cdot (R + Q/2 - E(d) \cdot LT) + p \cdot \frac{E(d)}{Q} \cdot (1 - F_{d_{lr}}(R))$$

$$C_{tot} = A \cdot \frac{E(d)}{Q} + h \cdot (R + Q/2 - E(d) \cdot LT) + p_u \cdot \frac{E(d)}{Q} \cdot n(R)$$

**NEVERTHELESS**  
**they MUST be PROVED**




Fig. 30. Logistics and Holy Spirit

A **further example** is taken from the Politecnico di Milano course, they say *"Buyers Product Company distributes an item known as a Tie Bar, which is U-bolt used on truck equipment. The monthly demand is 11 with standard deviation 3.1."*, and they find  $Q=11, R=19$  and type I Service level= $94.8\%$ , using the following data:  $A=10, h=0.022, p=69$ , Normal Distribution

during  $LT=1.5$  months, with pdf  $N(\mu, \sigma)$ . They use a different formula of the one used by Brandimarte and Zotteri, and by Hoop and Spearman but they, as well, do not consider the “pump” of fig. 27. Using the same data, and the right ideas (as sketched in fig. 27), F. Galetto finds  $Q=6$ ,  $R=23$  and type I Service level=98.2%: **lower costs for better service level!!!!** This result comes from the right formulae.

As a another **example**, A. Caridi, in his book, provides an example where *The weekly demand is Normal distributed with mean 50 and standard deviation 5.*; he finds  $Q=294$ ,  $R=177$  and Service level=99.9%, using the following data:  $A=30$ ,  $h=1.8$ ,  $p=525.4$ , Normal Distribution during  $LT=3$  weeks, with pdf  $N(\mu, \sigma)$ . He uses a different formula of the one used by Brandimarte and Zotteri, and by Hoop and Spearman but he, as well, does not consider the “pump” of fig. 27. Using the same data and the right ideas (as sketched in fig. 27), F. Galetto finds  $Q=190$ ,  $R=133$  and Service level=99.91%: **lower costs for the same service level!!!!** This result comes from the right formulae.

As a **last example** Law and Kelton, in their book *Simulation Modelling and Analysis*, say “Consider a company which sells a single product and would like to decide how many items to have in inventory for each of the  $n$  months. The times between demands are IID exponential random variable with a mean of 0.1 month. The sizes of the demands  $D$  are IID random variables (independent of when the demands occur) with probability  $P(D=1)=P(D=4)=1/6$   $P(D=2)=P(D=3)=1/3$ . At the beginning of each month, the company reviews the inventory level and decides how many items to order from its supplier.” They compare in example 9.2 two inventory policies: ( $Q=40$ ,  $R=20$ ) and ( $Q=80$ ,  $R=20$ ) carrying out 10 + 10 simulations, and find that the second policy is better. They do not compute the Service level. F. Galetto makes a comparison (only for comparison purpose), using the following data:  $A=32$ ,  $h=1$ ,  $p=100$ , Normal Distribution during  $LT$ : the result is opposite, the first one is better; moreover there is a policy ( $Q=5$ ,  $R=6$ ) much better than both!!! This finding was so peculiar that F. Galetto decided to read thoroughly the book and found in example 9.4 the comparison of 5 alternative policies: ( $Q, R$ )= (40, 20), (80, 20), (60, 40), (100, 40), (100, 60): Law and Kelton “selected policy 2, ( $Q=80$ ,  $R=20$ ) as being the lowest-cost configuration”. F. Galetto finds that also doubling all the cost involved, the policy ( $Q=5$ ,  $R=6$ ) is much better than all the previous policies!!! That sounded still strange and F. Galetto went on: he found this statement in example 12.1 “... it appears that the smaller values of both  $Q$  and  $R$  would be preferable, since lower monthly costs are desired”: therefore it is rather surprising that, just few pages before, Law and Kelton chose the policy ( $Q=80$ ,  $R=20$ ) as better than ( $Q=40$ ,  $R=20$ ). Law and Kelton used simulations, but they, as well, do not consider the “pump”.

### **How many students, all over the world, are learning wrong methods and will take wrong decisions?**

These examples show very clearly Deming statements:

*“The result is that hundreds of people are learning what is wrong. .... I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.” “It is a hazard to copy”, “It is necessary to understand the theory of what one wishes to do or to make.”*

It is important to notice that **you do not need the right formula to understand that a wrong formula is wrong**: an example is the following “*The General Triangle is used for practical applications of trigonometry in determining distances that cannot be measured directly. Such a problem may be solved by making the required distance one side of a triangle, measuring other sides or angles of the triangle, and then applying the Carnot Theorem (cosine law): if  $\alpha, \beta, \gamma$  are the three angles of a triangle, and  $a, b, c$  the respective opposite sides, it may be proved that  $a^2=b^2+c^2-2bc \cos\alpha$*

[Carnot Theorem (cosine law)]. You need a lot of ingenuity to solve the problem if you know only the Pythagorean Theorem, which states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. Anyway you do not need the knowledge of the Carnot Theorem (cosine law) to understand that it wrong to apply the Pythagorean Theorem to General Triangles”.

Any formula that does not consider the fig. 27 is wrong! How many professors do not use “pump” in their optimisation?

The statement of the Nobel prize M. Gell-Mann is relevant: “Once that such a misunderstanding has taken place in the publication, it tends to become perpetual, because the various authors simply copy one each other.”...>> {“The Quark and the Jaguar: Adventures in the Simple and the Complex” [W. Freeman and Company, N. Y., 1994]} [fig.1, 5, 32 -35]

**8. Reliability centred maintenance and spare parts management**

The ideas we presented in the paragraph “6<sup>th</sup> step: the case of random demand with “NONconstant” demand rate and NOT steady state of the stochastic process” can be used for the fields of Reliability Centred Maintenance and Spare Parts Management.

They were presented by F. Galetto in the paper “A general model for system cost-effectiveness” at a joint conference of EOQC&IAQ 1975 [European Organisation for Quality Control & International Academy for Quality], in Venice (Italy).

At that time F. Galetto used a Time Homogeneous Markov Process, as we did previously for constant demand rate (exponential distribution).

Let’s indicate with  $\lambda_{ik}$  the transition rate from state i to state k,  $p_{ik}$  the steady transition probability from state i to state k,  $m_i$  the mean time that the system stays in state i before making a transition,  $e_{ik}(0, s)$  the earning [or a cost] of the system due the transition from state i to state k for the interval 0–s,  $d_{ik}(s)$  the earning [or a cost] of the system due the transition from state i to state k at the instant s,  $v_i(t)$  the total expected profit [or cost] of the system for the interval 0–t, if it starts in state i at time 0. If the system makes its 1<sup>st</sup> transition out of the state i before the instant t it earns a profit

$$\sum_{k=0}^N \lambda_{ik} \int_0^t e^{-\lambda_i s} [e_{ik}(0, s) + d_{ik}(s) + v_k(t - s)] ds$$

If the system makes its 1<sup>st</sup> transition out of the state i after the instant t it earns a profit

$$\sum_{k=0}^N p_{ik} e^{-\lambda_i t} e_{ik}(0, t)$$

Putting all together we have the system of integral equations [notice the similarity with what done before for reliability]

$$v_i(t) = \sum_{k=0}^N p_{ik} e^{-\lambda_i t} e_{ik}(0, t) + \sum_{k=0}^N \lambda_{ik} \int_0^t e^{-\lambda_i s} [e_{ik}(0, s) + d_{ik}(s) + v_k(t - s)] ds \tag{23}$$

When  $t \rightarrow \infty$ , there is an asymptotic solution  $v_i(t) = v_i + gt$ , where g is the gain rate; finding the steady state probability of being in state i,  $\phi_i$ , and  $r_i$ , the reward for each entrance into state i, F. Galetto proved that

$$g = \sum_{i=0}^N (r_i / m_i) \varphi_i \tag{24}$$

For the use of this formula we have to allow the system to enter the “down states” and return to “up states”.

Letting  $\Phi_{ij}(t)$  the transition probability for the transition  $i \rightarrow j$ , when  $t \rightarrow \infty$ , we can put the system in an equivalent vector and matrix form (for a number of states such that inverse matrix exists)

$$\bar{v}(t) = [I - P]^{-1} \bar{r} + \Phi(t) [I - P]^{-1} \bar{r} \tag{25}$$

The vector  $\bar{r} = \sum_{k=0}^N \lambda_{ik} \int_0^\infty e^{-\lambda_i s} [e_{ik}(0, s) + d_{ik}(s)] ds$  is a constant vector.

Constant transition rates are not necessary, as shown by F. Galetto in the paper “*CLAUDIA (Cost Life Analysis)*”, *21st EOQC Conf*, Varna (Bulgaria); we indicate with  $b_{ik}(s)$  the instantaneous transition probability from state  $i$  to state  $k$  in the interval  $s-s+ds$ ,  $p_{ik}$  the steady transition probability from state  $i$  to state  $k$ ,  $1-W_i(t)$  the probability that the system stays in state  $i$  for the interval  $0-t$ . We have

$$v_i(t) = \sum_{k=0}^N p_{ik} \bar{W}_i(t) e_{ik}(0, t) + \sum_{k=0}^N \int_0^t b_{ik}(s) [e_{ik}(0, s) + d_{ik}(s) + v_k(t-s)] ds \tag{26}$$

and again  $\bar{v}(t) = [I - P]^{-1} \bar{r} + \Phi(t) [I - P]^{-1} \bar{r}$  (for a number of states such that inverse matrix exists), and the vector  $\bar{r}$  suitably defined.

If we partition the states in two sets, the up states [where the system performs as it must] and the down states [where the system does not perform as it should], we can partition the vectors and the matrices accordingly.

$$\bar{v}(t) = \begin{bmatrix} \bar{v}_1(t) \\ \bar{v}_2(t) \end{bmatrix} = \begin{bmatrix} I_{11} - P_{11} & P_{12} \\ P_{21} & I_{22} - P_{22} \end{bmatrix}^{-1} \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \end{bmatrix} + \begin{bmatrix} \Phi_{11}(t) & \Phi_{12}(t) \\ \Phi_{21}(t) & \Phi_{22}(t) \end{bmatrix} \begin{bmatrix} I_{11} - P_{11} & P_{12} \\ P_{21} & I_{22} - P_{22} \end{bmatrix}^{-1} \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \end{bmatrix} \tag{27}$$

F. Galetto proved that the steady state system availability  $A_{SS} = MUT/(MUT+MDT) = MUT/MTBF$  where the Mean Up Time is the dot product of the two vectors  $z\alpha * MTTF$  and the Mean Down Time is  $z\beta * MTTR$ .

Letting  $t \rightarrow \infty$ , the limit of the vector  $v_1(t)$  is the vector *SEPUF*, System Expected Profit Until Failure and the limit of the vector  $v_2(t)$  is the vector *SEPUR*, System Expected Profit Until Repair (inverse matrices DO exist!)

$$\bar{v}_1(t \rightarrow \infty) = SEPUF = [I_{11} - P_{11}]^{-1} \bar{r}_1 \quad \bar{v}_2(t \rightarrow \infty) = SEPUR = [I_{22} - P_{22}]^{-1} \bar{r}_2 \tag{28}$$

Using the same probability vectors  $z\alpha$  and  $z\beta$  we can compute the system UEG, Up Expected Gain,  $UEG = z\alpha * SEPUF$  and the system DEG, Down Expected Gain,  $DEG = z\beta * SEPUR$ ; the system CEG, Cycle Expected Gain is  $CEG = DEG + UEG$  and finally the gain rate  $g = CEG / MTBF$ .

Let’s now derive the famous Wilson lot-size formula, in a very general context; we assume only that, if different types of distributions are used, they have the same mean  $m$ , the mean

time to sell one product; the “time to sell a new unit (time between demands)” is a random variable “identically” [but not exponentially] distributed and we indicate the probability density of the time between transitions as  $f(t)$  [related to the “rate”  $\lambda(t)$ , with cumulative distribution  $F(t)$ ]; its mean is  $m$ . We compute the gain rate  $g$ . When  $t \rightarrow \infty$ , the steady state probability of being in state  $i$ ,  $\varphi_i = 1/Q$  and  $r_i$  the reward for each entrance into state  $i$ , are  $r_0 = A + Qhm$  and  $r_i = (Q-i)hm$ ; therefore, after some algebra  $g = [A / m + hQ(Q + 1) / 2] / Q$ . We can depict, again, the system as before

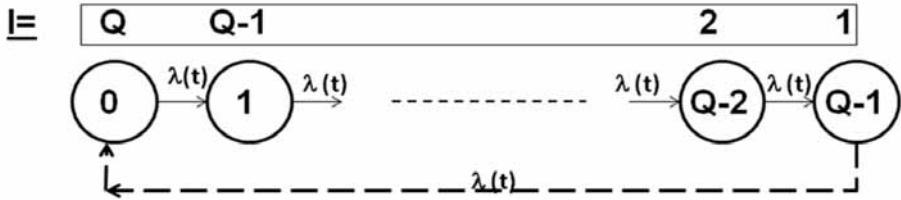


Fig. 31. System inventory states, with random demand and variable demand rate; state  $i$  means  $i$  products dispatched

### 8. Conclusion

We presented some cases where professors were completely failing to comply with the ideas of the Standard ISO 9001:2008, in the 1<sup>st</sup> place, NOT preventing the generation of wrong methods, in the 2<sup>nd</sup> place, NOT realising that methods were wrong (nonconformities), in the 3<sup>rd</sup> place, spreading around nonconformities (wrong methods), in the 4<sup>th</sup> place, NOT taking the needed corrective actions for the nonconformities (wrong methods), in the 5<sup>th</sup> place, punishing people who realised that those methods were wrong!!!!

Since Universities MUST prepare flexible graduates who can think integratively, with their own mind, prevent and solve problems, be life-long learners using their intelligence and intellectual honesty, Universities **must** teach future managers to be Intellectually Honest and to act in a scientific way. To get that professors must understand that “Quality of methods for Quality is important”: some methods are really wrong. [Galetto 1989, Vienna ]

If the people mentioned here (also the ones that I do not know [referees are unknown to me]) are willing to find me wrong and prove that they are right, I am ready to meet all of them together, and to have a profound discussion.

If they accept the challenge, they MUST remember that “Many wrongs don't make a right”. Will they accept? Only God knows it ...

“A prophet is not without honour, but in his own country, and among his own kin, and in his own house.” (ST Mark)

Unfortunately professors and referees do not believe in the ideas of Deming, and therefore

***The result is that hundreds of people are learning what is wrong. .... I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.***

Papers with errors are not scientific, even though the authors say: “We thank the referees and the editor for careful reading and helpful comments that improved our paper.”

There are too many people, wrong-headed waffling about Quality, who do not take corrective actions when I signal them their nonconformities:

... they fly into rages and they continue teaching wrongly ...

Why there are so many professionals, wrong-headed waffling about Quality and related methods, and with very little knowledge of Reliability, Design of Experiments, Statistics, Quality Management, Mathematics, Logic, ...?

If we want to achieve QUALITY, MANAGERS (now students) NEED TO BE EDUCATED ON QUALITY  $\varepsilon Q_{GE}^{IO}$  by Quality Professors, EDUCATED on Quality. (fig. 32 and 33)

I could, at last, paraphrase ST John "And there are also many other things, the which, if they should be written everyone, I suppose that even the world itself could not contain the books that should be written."

Will someone want to see the truth? Only God knows that ...

The personal conclusion is left to the Intellectually Honest reader to whom is offered the **Quality Tetralogy**: Prevent, Experiment, Improve, Plan, SCIENTIFICALLY to avoid disquality, to eliminate disquality, to achieve Quality, to assure Quality, using Intellectual Honesty, as shown in the figure 34 (also fig. 35 for items, with fig. 32 and 33).

Quality Tetralogy is much better than ISO 9004:2008 because Quality Tetralogy takes into account explicitly the need for scientific behaviour either of people or of organisations that really want to make Quality. Moreover it shows clearly that prevention is very important for Quality, where Good Management is strongly related to Good Knowledge for Business Excellence. [fig.1, 5, 32, 33, 34, 35]

**Brain is the most important asset: let's not forget it, IF we want that our students be better that their professors.** [fig.1, 5, 32, 33, 34, 35]

Disquality Vicious Circle MUST be avoided, in all Companies and Universities.

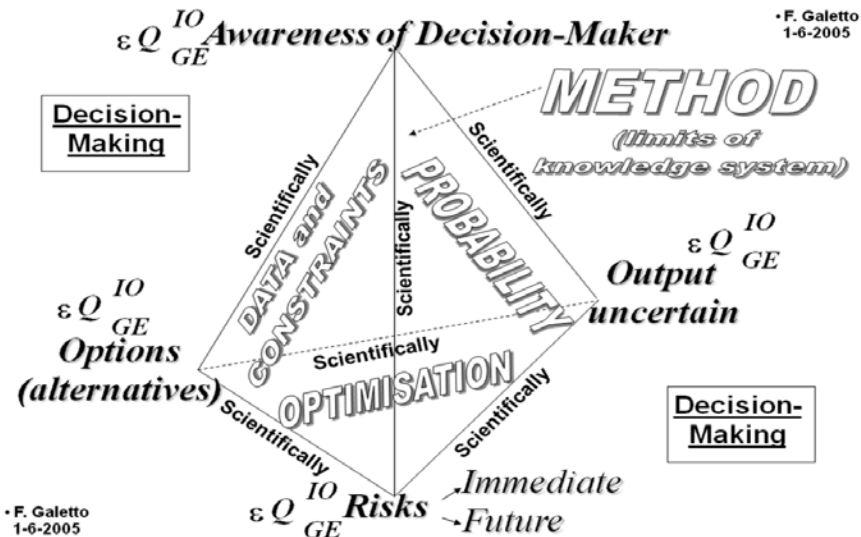


Fig. 32. The Decision-making Tetrahedron



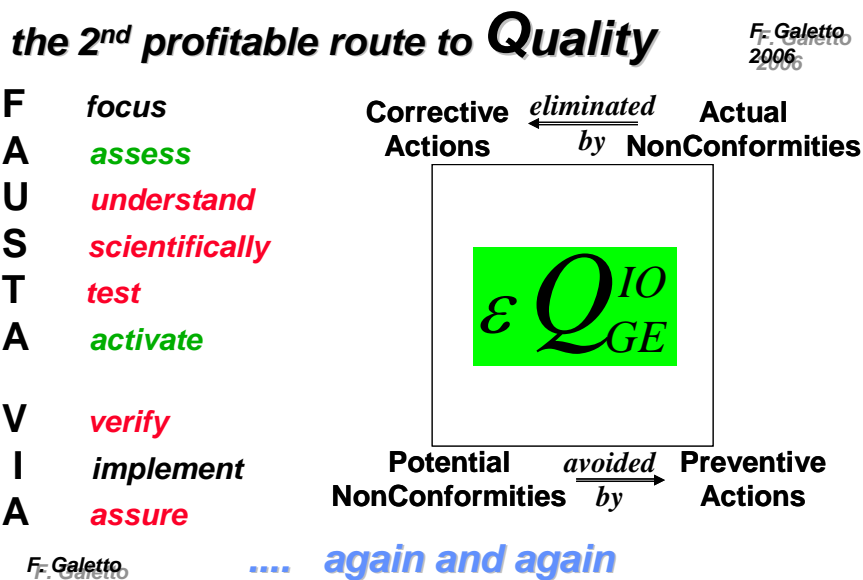


Fig. 35. Preventive and Corrective Actions, and FAUSTA VIA

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## Biography

Fausto GALETTO was born in Sanguinetto (Verona, Italy, 1942). He received an Electronics Engineering degree and a Mathematics degree from Bologna University in 1967 and 1973 respectively. Since 1992 he has been Professor of "Industrial Quality Management" at Politecnico of Turin.

From 1998 to 2001 he has been Chairman of the Working Committee "AICQ-Università" (Quality and University) of the Italian Association for Quality, taking care of Quality in Courses about Quality in Universities. He has written three books and more than 170 papers on Reliability, Quality Management and Quality Methods (DOE, Applied Statistics, Testing, Process Control, ...). He was formerly Reliability Engineer with CGE (General Electric, 2 years) From 1975 to 1982 he was Reliability Manager with Fiat Auto. Then he was Director of the Quality Dept, (comprising the Reliability, Production Quality Control, and After Sales Depts.) with Philco Italiana for 3 years. Since 1985 he has been Director of the Quality/Reliability Dept. at Iveco-Fiat, for 5 years. Since 1990 he has been Quality Management consultant. Since 1980 he has been Lecturer with the Italian Organisation for Quality Control (AICQ) and with COREP. Co-ordinator of: Reliability Working Group of CUNA (until 1989), Scientific and Technical Committee of QUALITAL (1989), Vice-Chairman of Automotive Sec. of AICQ (85-90)

It seems he is one of the very few who take care of "Quality of Quality Methods used for making Quality".

## APPENDIX: Excerpts from ISO Standards. Quality, nonconformities, preventive actions and corrective actions

In the author's opinion, the first step to Quality achievement is to define *logically* and *correctly* what Quality is; in order to provide a practical and managerial definition, since 1985 F. Galetto was proposing the following one:

**Quality is the set of characteristics of a system that makes it able to satisfy the NEEDS of the Customer, of the User and of the Society.**

This definition highlights the importance of the *needs* of the three actors: **the Customer, the User and the Society.**

**Prevention** is the fundamental idea present: you possibly satisfy the needs only by **preventing the occurrence** of any problem against the needs. Teachers must teach that to "present" students (*future* Managers). In order to teach Quality to students, professors have to understand what Quality entails. "Peer Reviewers" should consider the needs of the readers: papers with wrong ideas, errors, false statements, ... do satisfy the NEEDS?

The basic ideas for Quality are the following: any serious organisation that finds a nonconformity, must analyse it and **take corrective actions.**

Are acting that way Universities, professors, "Peer Reviewers", magazines, journals, ...?

Anybody should have to behave like stated in the Standard ISO 9001:2000. Unfortunately, referees generally are not “Quality experts” and do not know the *ISO 9001:2008. Quality management systems – Requirements*: **any Good Quality Manager knows very well that making Quality involves prevention of potential nonconformities and correction of actual nonconformities (problems)** [while many professors and referees do not]:

*ISO 9001:2000, Quality management systems - Requirements*

**8.3 Control of nonconforming product**

*The organization shall ensure that product that does not conform to product requirements is identified and controlled to prevent its unintended use or delivery. The controls and related responsibilities and authorities for dealing with nonconforming product shall be defined in a documented procedure.*

*The organization shall deal with nonconforming product by one or more of the following ways:*

- a. *by taking action to eliminate the detected nonconformity,*
- b. *by authorizing the use, release or acceptance con concession by a relevant authority and, where applicable, by the customer;*
- c. *taking action to preclude its original use or application.*

*Records of the nature of nonconformities and any subsequent actions taken, including le concessions obtained., shall be maintained (see 4.2.4)*

*When nonconforming product is corrected, it shall subject to re-verification to demonstrate conformity to requirements.*

*When nonconforming product is detected after delivery or use, the organization shall take action appropriate to the effects, or potential effects, of the nonconformity.*

**8.4 Analysis of data**

*The organisation shall determine, collect and analyse appropriate data to demonstrate the suitability and, effectiveness of the quality management system and to evaluate where continual improvement of the effectiveness of the quality management system can be made. This shall include data generated as a result of monitoring and measurement and from other relevant sources.*

*The analysis of data shall provide information relating to*

- a. *customer satisfaction (see 8.2.1),*
- b. *conformity to product requirements (see 7.2.1),*
- c. *characteristics and trends of processes and products including opportunities for preventive action, and*
- d. *suppliers.*

.....

**8.5 Improvement**

**8.5.1 Continual Improvement**

*The organization shall continually improve the effectiveness of the quality management system, though the use quality policy, quality objectives, audit results, analysis of data, corrective and preventive actions and management review.*

**8.5.2 Corrective action**

*The organization shall take action to eliminate the cause of nonconformities in order to prevent recurrence. The corrective actions shall be appropriate to the effects of the nonconformities encountered.*

*A documented procedure shall be established to define requirements for*

- a. *reviewing nonconformities (including customer complaints),*
- b. *determining the cause of nonconformities,*
- c. *evaluating the need for action to ensure that nonconformities do not recur,*
- d. *determining and implementing the action needed,*
- e. *records of the results of action taken (see 4.2.4) and,*
- f. *reviewing corrective action taken.*

### **8.5.3 Preventive action**

*The organization shall determine action to eliminate the causes of potential nonconformities in order to prevent occurrence. Preventive actions shall be appropriate to the effects of potential problems.*

*A documented procedure shall be established to define requirements for*

- a. *determining potential nonconformities and their causes,*
- b. *evaluating the need for action to prevent occurrence of nonconformities,*
- c. *records of the results of action taken (see 4.2.4) and,*
- d. *reviewing preventive action taken.*

Particularly important is noting that the principle g) **Factual approach to decision making:** "Effective decisions are based on the analysis of data and information." of the ISO Standards 9004:2000 still does not care of Quality of Quality Methods. Professionals do the same. They do not consider the need for "Quality Decision" in Decision Making. From that it is apparent that ***facts and figures are useless, if not dangerous, without a sound theory.*** (F. Galetto), or better, using Deming's own words "**Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality**".

Unfortunately many organisations do not comply with the Decision Making Tetrahedron (figure 29) in their decision making: in my 40 years experience I saw many time wrong decisions, based on wrong methods.