Stratified zig-zags on vortex pairs using vertically shifted perturbations

STRATIFIED ZIG-ZAGS ON VORTEX PAIRS USING VERTICALLY SHIFTED PERTURBATIONS

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Summary By vertically shifting the perturbations on two counter-rotating vortices, it is shown that the zig-zag patterns seen experimentally develops on a much faster timescale than if the perturbations are on the same horizontal plane. With the vertical shift, the density overturning by the two vortices generates horizontal density gradients that can force the horizontal baroclinic vorticity production terms in the incompressible Boussinesq, Navier-Stokes equations. The new horizontal vorticity that develops then generates vertical shears that horizontally advect the vertical vorticity in different directions at different values of z, yielding the observed zig-zags. The final geometry, before dissipation becomes important, is characterised by anti-parallel pairs of horizontal vortices which then reconnect. Possible relationships to how a horizontal energy cascade develops in stratified flows will be discussed.

Background The horizontal, mid-latitude kinetic energy spectra in the Earth's atmosphere are characterised by two regimes. At the largest scales, $k^{-1} > 600$km, there is a k^{-3} spectrum and and over the mesoscales, $k^{-1} < 400$km, a $k^{-5/3}$ regime. While the k^{-3} regime was originally interpreted as a forwards enstrophy cascade [1], it is now interpreted as a saturated backwards energy cascade.

Explaining the forwards cascade has been more difficult. It was originally proposed as a two-dimensional backwards energy cascade, but observational data has shown that it is predominantly a forwards energy cascade [2]. Since the dynamics of fully three-dimensional turbulence cannot be invoked, a new mechanism will be needed. Proposed mechanisms include storm generation, Kelvin-Helmholtz instabilities between layers, and the dynamics of homogeneous stratified turbulence, for which several large numerical simulations consistent with a downscale cascade of energy have been performed [3, 4].

This presentation will address this transition problem: What controls the transition from large-scale nearly two-dimensional vortex dynamics at the larger scales to this new stratified dynamical regime? The seminal experiment [5] shows that counter-rotating vortices in a stratified fluid are unstable to the formation of zig-zags. The zig-zags are large horizontal excursions of the originally vertical vortices in the direction perpendicular to the propagation of the original vortex pair. Amazingly, the pairs maintain their coherence even while being strongly distorted.

This experiment was then followed by a series of linear stability papers and a few direct numerical simulations using the three-dimensional, incompressible Boussinesq-Navier-Stokes equations. For the initial perturbations chosen, the primary instability did not yield the zig-zags. In the latest paper [6], it is shown that the primary stratification terms completely oppose the stretching terms. Only with the inclusion of higher-order terms could zig-zags form, and numerically, zig-zags are retarded. The most successful numerical experiment [7] did yield zig-zags, but only after the order of 50-70 characteristic timescales. What we found perplexing is how difficult it was for the instabilities to form zig-zags, but not difficult for the experiments.

The conclusion of the proposed presentation will be that there is a faster instability mechanism on pairs of vertical vortices than those already proposed. The primary difference in the new initial condition is that the perturbations are not in the same vertical positions. This shift in the vertical allows a stronger horizontal density perturbation to form in a shorter time. The vertical layering observed in many calculations is reproduced and is identified with anti-parallel pairs of horizontal vortices propagating in different directions at different levels. The new configuration is then primed for developing a horizontal energy cascade that would start with the reconnection of these horizontal pairs.

With this perturbation, following the formation of the first weak horizontal temperature gradients, the barotropic (stretching) and baroclinic (from horizontal temperature derivatives) vorticity production terms never cancel and it took only about 10 characteristic timescales for significant bends to form. Once the bends form, strong horizontal anti-parallel vorticity forms in the stream-wise direction at the tips of the bends, which then pulls these bends into the zig-zags. The following figures demonstrate the process in our 3D Boussinesq-Navier-Stokes calculations.

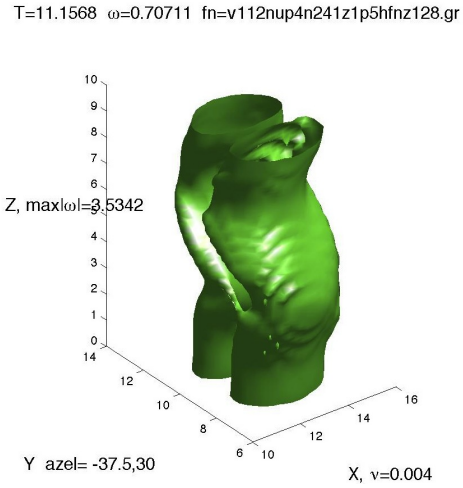

Figure 1: Vorticity isosurface of two originally vertical counter-rotating vortices at $t = 7.9$, about $16t_e$, where $t_e = 2\pi a^2/\Gamma = 0.5$ is the initial characteristic timescale used by [7] with a the radius of the initial vortices and Γ its circulation. Froude number $F_{h0} = 0.34$ and $Re_0 = 2400$. This calculation used $128 \times 512 \times 256$ mesh points for the domain shown. A new calculation with $Re_0 = 4000$ with twice the mesh points in each direction is giving similar results.

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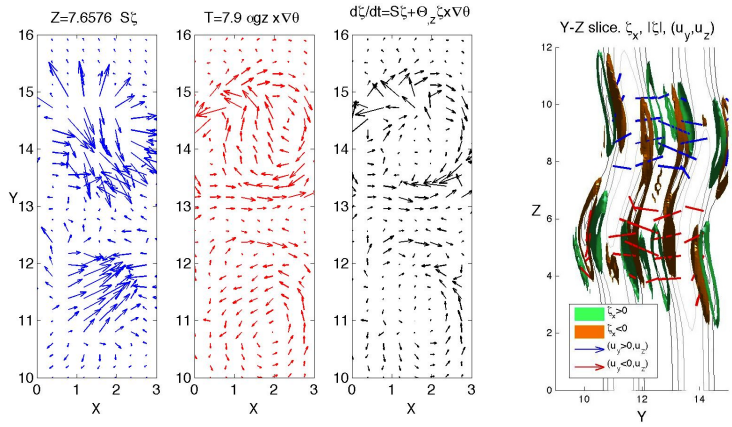
Figure 2: Left: Vortex stretching, baroclinic, then total vorticity production vector at $t = 7.9$ where one of the zig-zag tips ($z = 5.5$ below) will appear at $t = 44$. Horizonal ζ_x baroclinic production dominates at this stage. Right: Vertical slice (with x inverted) showing different signs of ζ_x that have formed and the vertical shear (red versus blue arrows) that will drag out the zig-zags.



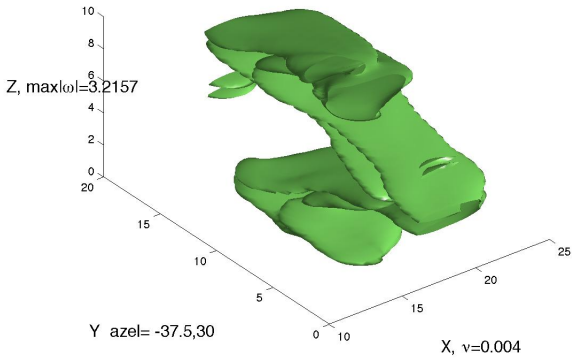


Figure 3: $t = 44$ vorticity isosurface in a 3D orientation. This shows the zig-zag crumpled vortex sheet with a relatively constant width in x and going from one side in y to the other at different z levels.

Discussion. In order to understand any possible relation to a cascade of energy to the small scales, detailed analysis of the spectral transfer terms, enstrophy production and production of scalar gradient variance. This work is in progress. Preliminary estimates of the velocity derivative skewness, a measure of enstrophy production by stretching normalised by the enstrophy, is about -0.25, half the value in fully developed unstratified turbulence. This would be consistent with about half the enstrophy production coming from the baroclinic terms.

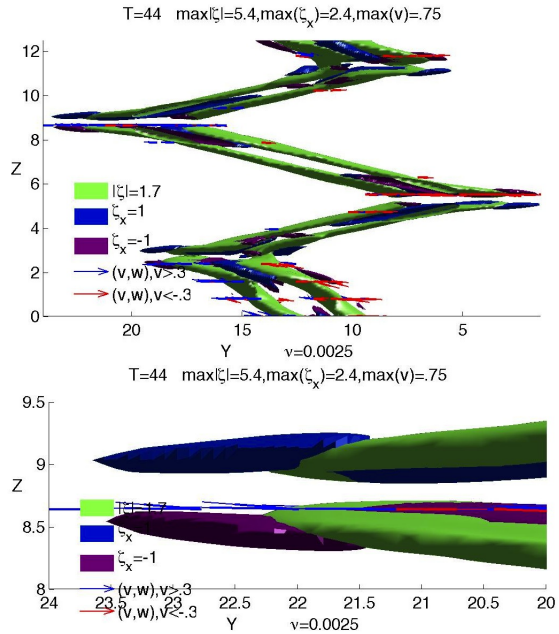


Figure 4: $t = 44$. $|\zeta| = 0.25|\zeta|_m$ vorticity isosurfaces. Top: Full domain from the x direction. Enstrophy production by vortex stretching $\boldsymbol{\zeta S \zeta}$ was along the vortex sheets between the tips. Baroclinic enstrophy production $\alpha g \hat{\boldsymbol{z}} \cdot (\boldsymbol{\zeta} \times \nabla\theta)$ was primarily on the tips, yielding the $\pm\zeta_x$ (blue/purple) pairs on the tips that continue to pull the zig-zags out. Bottom: Blow-up of the left tip of the zig-zag. Note the anti-parallel pair (blue up and purple down) of hozizontal ζ_x vorticity that is pulling this double tip to the left (larger y).

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