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# Modeling Sleep Modes Gains with Random Graphs

Luca Chiaraviglio, Delia Ciullo, Marco Mellia, Michela Meo  
Electronics Department, Politecnico di Torino, Italy  
Email: {chiaraviglio,ciullo,mellia,meo}@tlc.polito.it

**Abstract**—Nowadays two main approaches are being pursued to reduce energy consumption of network devices: the use of sleep modes in which devices can be put in low-power state, and the adoption of energy proportional approaches where the device architecture is designed to make energy consumption proportional to the actual load. In this paper, we formulate a theoretical model based on random graph theory to estimate the potential gains that can be achieved by adopting sleep modes in networks where energy proportional devices are deployed. Is sleep mode still a winning approach in these scenarios? We consider a simple model of the energy consumption of network devices: a fixed cost represents the static consumption and a variable cost describes the linear proportionality to the traffic load. Our results show that sleep modes are effective also in presence of load proportional solutions, even if traffic has to be routed on longer paths, unless the static power consumption component is of the same order of magnitude of the load proportional component.

## I. INTRODUCTION

In networking, one of the main causes of energy waste is the fact that most of the devices do not consume energy proportionally to the work they do, but they consume much even when they are under-utilized. On the contrary, network usage and traffic follow the typical human being activity patterns, with significant differences between peak and off-peak values and with long periods of low traffic. The network results thus highly under-utilized for long periods of time causing a large energy waste. Many solutions are being studied to reduce this waste, or, equivalently, to make the network consumption proportional to the traffic load [1]. The proposed approaches can be divided into two main categories: i) *Energy proportional approaches* adopt solutions that work on the individual devices and try to achieve energy proportionality by adapting the speed (and capacity) of the devices to the actual load; and ii) *Sleep mode approaches* investigate solutions that involve the network as a whole and approximate load proportionality by carefully distributing the traffic in the network so that some devices are fully utilized and others become idle and are put in sleep modes. The latter are motivated by the fact that energy consumption of current devices is practically independent on the load. Clearly, the two solutions can be merged so that energy proportional devices are present and can be put in sleep mode to possibly save more energy.

A natural question is then which approach is more effective. Given the Internet topology characteristics and traffic, and given a model of the energy consumed by a device as a function of its load, is it better to purely rely on device energy proportionality capability, or, on the contrary, is it always better to couple it with sleep mode solutions? And,

also, which is the minimum energy proportionality that would make sleep mode ineffective? The answer to these questions is the ambitious goal of this paper.

When a device is switched-off the traffic passing through it has to be rerouted on different, typically longer, paths; thus, the beneficial saving achieved by switching off the device is mitigated by the increase of the consumption of the devices that remain on, due to the higher load they have to sustain.

To investigate this trade-off we model: i) the device energy consumption as a function of load, and ii) the load that the devices that are powered on have to sustain for a given traffic demand. We focus on network links, whose energy consumption we model by a *variable* part that is proportional to the traffic that flows through the device and a *constant* amount that includes the fraction of the node energy cost due to the link. Node energy cost is thus simply proportional to the number of its links that are active. The network model and its topological characteristics are represented by a random graph; leveraging then on random graph theory, the load on network links is computed from the knowledge of the shortest path among node pairs. Thus, the energy consumption of all network links is easily computed.

Given a random graph representing a network, the use of sleep modes is modeled by considering a new graph in which some nodes are removed according to some policy. The energy consumption due to the links of nodes that remain on can be evaluated from the topological characteristics of the new graph. The same problem has been recently faced in [2] using simple simulations. In this work we present some modeling results that corroborate the intuition of [2] but allow to derive more general insights.

We present an extensive sensitivity analysis to show the impact of the model parameters. In particular, we include both small-world and power-law models that are claimed to accurately reflect Internet topology properties [3]. Our results show that when the variable part of the cost model is small with respect to the constant part, as is typical of today devices, sleep modes are convenient. However, for future devices, whose consumption will probably be more load proportional, sleep modes might not be convenient anymore. Still, the variable part of the energy cost has to be of the same order of magnitude of the constant component to make sleep mode inefficient. Interestingly, when power-law graphs are considered, the degree of load proportionality required to make sleep mode not convenient anymore is higher than for simple random graphs. This suggests that, given the today technological constraints that make the constant energy consumption of networking

devices quite large, sleep mode enabled networks will allow to save more energy than purely energy proportional approaches for long time.

## II. METHODOLOGY

In this section we provide a general overview of the methodology we use to evaluate sleep mode gains.

The network is composed by access and transport devices. Access devices are the possible sources and destinations of traffic, therefore they can never be powered off. On the contrary, some of transport devices can be turned off if their traffic can be supported by other devices that remain on.

We adopt the following assumptions: i) traffic is uniformly exchanged among all access nodes; ii) traffic is routed on the shortest paths among nodes; iii) the set of devices to be switched off is given a-priori, e.g., it has been previously pre-computed; iv) we model the network power consumption focusing on links only, as in [2], and the contribution of nodes in terms of power consumption is accounted in the link power model; v) link power consumption is composed by a fixed amount of power, and a variable part that scales linearly with the current traffic flowing on the link; vi) the same power consumption model is applied to all the links in the network.

### A. Basic formulation and metrics

Let the transport network topology be described by an undirected graph  $G(n, l)$ , with  $n$  the set of nodes, with cardinality  $N = |n|$ , and  $l$  the set of links, with cardinality  $L = |l|$ . The average node degree is  $K = \frac{2L}{N}$ . The link capacity is denoted by  $R_l$ .  $T$  is the total traffic flowing in the transport network from access nodes.

The  $i$ -th link consumption is modeled by a constant part,  $C_F$ , and a variable part that is proportional to the link load  $\rho_i$  through a parameter  $\alpha$ : the link consumption is  $C_F + \alpha\rho_i$ . The average total network consumption,  $C$ , can be computed as

$$C = \sum_{i \in l} (C_F + \alpha\rho_i) = L(C_F + \alpha\bar{\rho}) = N\frac{K}{2}(C_F + \alpha\bar{\rho}) \quad (1)$$

where  $\bar{\rho}$  is the mean link load.

We define the *constant cost equivalent load* as

$$\nu = \frac{C_F}{\alpha} \quad (2)$$

$\nu$  is the amount of load that, added to a link, makes its energy consumption increase of a quantity  $C_F$ . Or, in other terms, whenever the load increases by an amount  $\nu$ , the energy consumption increases by  $C_F$ . The parameter  $\nu$  plays a crucial role in the evaluation of the sleep mode schemes, as we will show in Section IV.

The average link load can be computed as:

$$\bar{\rho} = \frac{Td}{N\frac{K}{2}R_l} \quad (3)$$

where  $d$  is the average shortest path length and  $N\frac{K}{2}R_l$  is the total capacity offered by the network. We call  $C$  the *all on*

*network consumption* and we take this value as a reference of the nominal consumption of the network.

We now intend to compute the network consumption when some nodes enter sleep mode. We assume that the scheme according to which the nodes are put in sleep mode has been preliminary planned so that, when the nodes are powered off, the network is still connected and no QoS target is violated; for example, the scheme might work during off-peak hours when many devices are under-utilized. Clearly, when a node is powered off, all links connected to it are switched off too.

Let  $p \in (0, 1)$  be the fraction of nodes that are switched off. We model the network resulting from the sleep mode scheme through the new random graph in which we randomly eliminate a fraction  $p$  of nodes; we assume that  $p < p_c$ , where  $p_c$  is the critical probability after which the network becomes disconnected. In this regime, random node elimination makes the new graph maintain the same structure of the original graph; refer to [6] for details. The total number of nodes after a random removal of nodes becomes  $N' = N(1 - p)$ , and the new average degree is  $K' = K(1 - p)$ . The average *network consumption in sleep mode*  $C'$  is now:

$$C' = N'\frac{K'}{2}(C_F + \alpha\bar{\rho}') = N\frac{K}{2}(1 - p)^2(C_F + \alpha\bar{\rho}') \quad (4)$$

with

$$\bar{\rho}' = T\frac{d'}{N'\frac{K'}{2}R_l} = T\frac{d'}{N\frac{K}{2}(1 - p)^2R_l} \quad (5)$$

where  $d'$  is the average shortest path length in the new graph.

Our aim now is to compare the energy consumption of the all on scheme,  $C$ , to the one of the network with sleep modes,  $C'$ . To this purpose, we define the ratio  $E = C'/C$  as the *energy reduction ratio*. Intuitively, the use of sleep modes for network devices saves energy when  $E < 1$ .

By evaluating  $E$  and comparing  $C$  and  $C'$  in (1) and (4), it is possible to evaluate when sleep modes are convenient:

$$C > C' \\ N\frac{K}{2}(C_F + \alpha\bar{\rho}) > N\frac{K}{2}(1 - p)^2(C_F + \alpha\bar{\rho}') \quad (6)$$

$$\left(C_F + \frac{\alpha Td}{N\frac{K}{2}R_l}\right) > (1 - p)^2 \left(C_F + \frac{\alpha Td'}{N\frac{K}{2}R_l(1 - p)^2}\right)$$

that is equivalent to

$$\nu > \frac{T}{(2p - p^2)N\frac{K}{2}R_l}(d' - d) \quad (7)$$

This equation defines the region in which a sleep mode approach is convenient.

*Lemma 1:* If  $\alpha = 0$  then  $C' < C$  and  $E < 1$ .

*Proof:* With  $\alpha = 0$ , (6) simplifies to  $N\frac{K}{2}C_F > N\frac{K}{2}(1 - p)^2C_F$ , which is always verified for any  $p \in (0, 1)$ . ■

*Lemma 2:* For graphs in which  $d' > d$ , if  $C_F = 0$  then  $E > 1$ .

*Proof:* If  $C_F = 0$ , (6) becomes:  $d > d'$ , i.e., it is verified if the average shortest path after some node switch off,  $d'$ , is smaller than the initial average shortest path  $d$ , which is never true. Consequently,  $E > 1$ . ■

Lemma 1 states that if devices do not implement load proportionality, sleep mode is always convenient. Conversely, Lemma 2 states that if the fixed power consumption is zero, sleep mode is never convenient.

### III. NETWORK MODELS

In the literature, several models have been proposed to represent the Internet topology. However, deciding which model fits better the current Internet is an open problem [4]. Therefore in this paper, instead of focusing on a single model, we analyze different models, showing that common properties about energy consumption can be inferred in all cases. In general, the Internet network satisfies the following properties: (i) small-world property, according to which the average number of hops between each node pair is quite limited, (ii) local clustering, according to which the Internet is divided in different and highly connected zones, (iii) heavy-tailed distributions of per node link number, meaning that, in general, most of the nodes have few links while few nodes have a large number of links.

We consider three well-known graph models: Erdős and Rényi, Power Law and Watts-Strogatz.

In the Erdős and Rényi (ER) model [5] nodes are connected by links according to a given probability, and the resulting degree distribution follows a Poisson distribution. The properties of this model are well-known in the literature and have been extensively studied. In particular, the ER model shows the *small-world* property, according to which the diameter of the graph scales typically as  $\log N$ . However, the local clustering and heavy-tailed properties are not met.

In the Power Law (PL) model [6] the node degree distribution,  $P(k)$ , follows a power-law distribution, i.e.,  $P(k) \sim k^{-\gamma}$ . The intuition is that some nodes behave like *hubs*, and have many more connections than others.

The Watts-Strogatz (WS) model [7] starts from a regular lattice in which each node is linked to a fixed number of neighbors. Then, as a second step, additional edges are inserted between randomly chosen pairs of nodes<sup>1</sup>. The resulting graph is an interpolation between ordered lattices and purely random graphs. This model matches both *small-world* and the local clustering properties, but the degree distribution is not heavy-tailed.

#### A. The Erdős-Rényi model

The average shortest path of an Erdős-Rényi graph is given by:  $d = \frac{\log(N)}{\log(K)}$ , as reported by [3]. With our energy consumption model, the all on consumption of the network is:

$$C = N \frac{K}{2} C_F + \frac{\alpha T \log(N)}{R_l \log(K)} \quad (8)$$

<sup>1</sup>In the original WS model presented in [8] shortcuts are rewired from the lattice. However, the resulting graph is affected by a not negligible probability to be disconnected. Therefore, we adopt the modification of the WS model proposed by [7], in which shortcuts are additionally inserted as new links. In this way, the resulting graph is always connected.

After randomly removing a fraction  $p$  of the nodes, the average shortest path becomes:  $d' = \frac{\log(N(1-p))}{\log(K(1-p))}$  and, from (4), the network consumption becomes:

$$C' = N \frac{K}{2} (1-p)^2 C_F + \frac{\alpha T \log(N(1-p))}{R_l \log(K(1-p))} \quad (9)$$

Note that Lemma 2 is verified for the ER model. Indeed, in this case  $d < d'$  is always verified. To prove this, we need to verify the inequality:  $\frac{\log(N)}{\log(K)} - \frac{\log(N(1-p))}{\log(K(1-p))} < 0$ . Simplifying the inequality, at the denominator we have  $\log(K) \log(K(1-p))$ , that is always larger than 0, being each argument of the logarithmic terms larger than 1 (for connected networks the average node degree must be larger than 1). The numerator becomes  $\log(N) \log(K(1-p)) - \log(N(1-p)) \log(K)$ , or, also,  $\log(1-p)(\log(N) - \log(K))$ ; that is always smaller than 0. Therefore,  $d < d'$  is verified and consequently  $E > 1$ .

#### B. Power-law model

We consider a graph in which the distribution of the degree  $k$  follows a power law (PL), i.e.,  $P(k) \sim k^{-\gamma}$ . In this case, the average shortest path can be computed as in [3]:

$$d \approx 1 + \frac{\log(N/K)}{\log[(\langle K^2 \rangle - K)/K]} \quad (10)$$

where  $\langle K^2 \rangle$  is the second moment of the degree distribution.

In particular, we consider a Pareto distribution for the degree since it is one of the most widely used and studied power laws in the literature. The Pareto distribution is described by the parameters  $(a, k_m)$ , where  $k_m$  is the minimum possible value of  $K$ , and  $a$  is a positive parameter. For this distribution we have:  $K = ak_m/(a-1)$  and  $\langle K^2 \rangle = ak_m^2/(a-2)$ . Similarly to what done for the previous graph model, using (1), (3) and (10), we can compute the all on network consumption  $C$ . Since, as reported in [6], a power law graph remains power law even after a random removal of nodes, the average shortest path of the network with sleep modes is:

$$d' \approx 1 + \frac{\log(N/K)}{\log\left[\left(\frac{\langle K^2 \rangle - K}{K}\right)(1-p)\right]} \quad (11)$$

from which we can compute the network consumption with sleep mode.

Using a rationale similar to the one adopted for the ER model, it can be shown that also the PL model satisfies Lemma 2.

#### C. The Watts-Strogatz model

The Watts-Strogatz (WS) model interpolates between ordered lattices and purely random graphs [9]. Starting from a lattice of  $N$  vertices in which each vertex is symmetrically connected to its  $K_L$  nearest neighbors, randomness is introduced by independently adding shortcuts between randomly chosen pairs of nodes. We denote by  $x$  the number of links that are randomly added [7]. The average vertex degree  $K$  is given by  $K = K_L + 2x/N$ .

For the shortest path length, depending on the value of  $x$ , two regimes are possible: for very small values of  $x$ , i.e.,  $x \ll N/K_L$ , the average shortest can be approximated as

$$d \simeq \frac{N}{K_L/2} \frac{\log(2x)}{4x} \quad (12)$$

while for large values of  $x$ , i.e.,  $x \gg N/K_L$ , the WS graph is similar to a purely random graph and the average shortest path can be approximated by

$$d \simeq \frac{\log(N)}{\log(K_L)} \quad (13)$$

However, since actual network topologies can be represented by a graph with a value of  $x$  that is in between these two extreme cases, we propose a new model to compute  $d$ . We use the following expression,

$$d \approx \frac{\log(N)}{\left[ \phi^\tau \log(K_L) + (1 - \phi^\tau) \log\left(\frac{2x}{N}\right) \right]} \quad (14)$$

with  $\phi = \frac{2x/N}{K_L}$  and  $\tau$  derived by interpolation from simulation results, the best fitting being  $\tau = 0.1$  for the considered scenarios. The intuition suggests that for lower values of  $x$ , the average shortest path tends to increase proportionally with  $x$ , i.e.,  $d \approx \frac{\log(N)}{\log(K_L)}$ , since the random component of the graph decreases with  $x$ . On the contrary, when  $x \approx \frac{NK_L}{2}$  the term  $\frac{\log(N)}{\log(K_L)}$  becomes predominant.

During the sleep mode regime, the average shortest path becomes:

$$d' \approx \frac{\log(N(1-p))}{\left[ \phi^\tau \log(K_L(1-p)) + (1 - \phi^\tau) \log\left(\frac{2x}{N}(1-p)\right) \right]} \quad (15)$$

As before, using (6), (14) and (15), we derive the cost  $C$  and  $C'$  for the WS graph in this regime.

Similarly to the previous models, it can be shown that also the WS model satisfies Lemma 2.

**Model Validation:** We validated the proposed models with comparison against simulations in a large number of scenarios. Due to lack of space, we report here only the results for the WS model since, in this case, we propose a new model for the computation of the average shortest path. The complete simulations for the ER and PL graphs are detailed in the Appendix. In brief, we found that the models accurately match results obtained by simulation, with an error always smaller than 4%.

To validate the WS model, we consider networks with a given value of  $N$  and of the node degree  $K$ , but different values of  $x$ . By varying  $x$  we make the random and local components of the degree vary. In particular, since the average degree is  $K = K_L + 2x/N$ , when  $x$  increases, the random component increases and the local component  $K_L$  decreases as  $K_L = K - 2x/N$ . We denote the degree of randomness by  $P_x = \frac{2x/N}{K}$ , which represents the fraction of the degree that is given by randomly chosen links.

Top plot of Fig. 1 reports the average shortest path  $d$  for the case  $N = 5,000$ ,  $K = 20$ , and degree of randomness  $P_x$  varying between 0.1 and 0.5. For each value of  $P_x$ , we

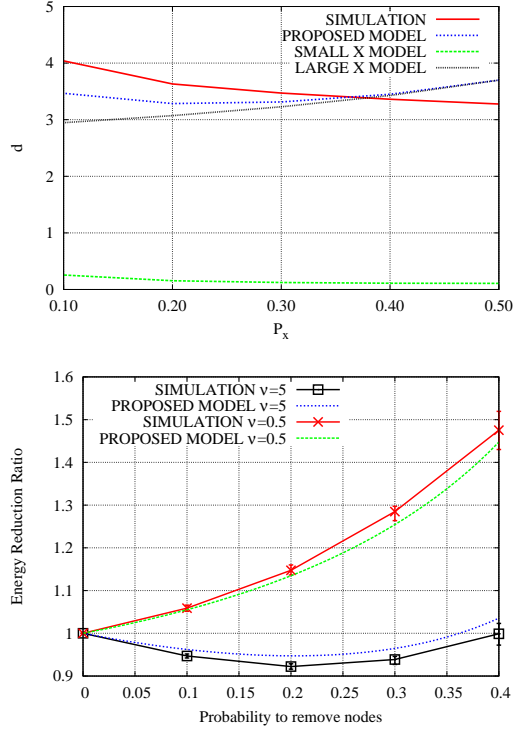


Fig. 1. Validation of the proposed model to compute  $d$  for WS graphs: (top) average shortest path for different models and by simulation, considering  $N = 5,000$ ,  $K = 20$ ,  $x \in [5,000-25,000]$ , (bottom) energy reduction ratio versus  $p$  for the proposed model and the simulation, considering  $N = 10,000$ ,  $K = 6$ ,  $x = 10,000$ .

average results over 20 independent runs in which different random seeds are used for adding the shortcuts. The figure reports  $d$  computed from: (12) that corresponds to the model for small values of  $x$ , (13) that is the model for large values of  $x$ , our proposed model (14), and simulation results. Clearly, the model for small values of  $x$  does not match the measured  $d$  for the considered scenarios; the model for large values of  $x$  matches the measured  $d$  only when  $\frac{2x}{N} \approx K_L$ , otherwise  $d$  is underestimated. Our model presents the best matching, since it is fitted for these scenarios. We have also validated the model in other scenarios, including scenarios with sleep modes. A complete set of results is reported in the Appendix.

Bottom plot of the figure shows the energy reduction ratio,  $E$ , computed by simulation and with the proposed model, for the case of a network with sleep modes,  $N = 10,000$  nodes,  $K = 6$  and  $x = 10,000$ ; the fraction of nodes that are switched off varies between 0 and 0.4. Again, observe how accurate the proposed model is. In the following we therefore adopt our model for computing  $C$  and  $C'$ .

#### IV. MODELS COMPARISON

In this section, we compare the effectiveness of the approaches based on sleep modes under the different network models proposed in the previous section.

For the numerical results, unless otherwise specified, we use the set of parameters reported in Tab. I. In particular, we

TABLE I  
PARAMETERS VALUES

N	$10^4$
K	6
$R_l$	100
T	$(0.5R_l)KN$

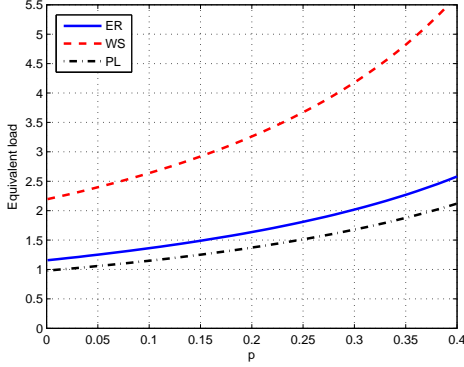


Fig. 2. Equivalent load  $\nu$  versus the switch-off probability  $p$  for the ER, PL and WS model.

set the average degree  $K$  to 6. This reflects the results of some measurement studies about Internet topology (see [10] for an overview), according to which  $K$  ranges between 4 and 8. We assume that the total traffic scales linearly with the number of nodes  $N$  and the current degree  $K$ . This is equivalent to assume that the average network load is 0.5, i.e., the network has to support a moderate load so that the variable energy cost  $\alpha\bar{p}$  is not negligible with respect to the fixed power consumption  $C_F$ . For the PL model, we set  $a = 3$  and  $K_m = 4$ , so that  $K = 6$ ; for the WS model, we set  $K_L = 4$  and  $x = 10,000$  ( $P_x \simeq 0.33$ ) and the average vertex degree is  $K = K_L + 2x/N = 6$ .

We first evaluate the energy reduction ratio  $E$  for different values of the switch off probability  $p$ . In particular, from (7) we compute the minimum value of  $\nu$  for which  $E$  becomes smaller than 1. This transition represents a breakeven point for which sleep mode saves energy. Fig. 2 reports the breakeven curve for the considered models. Intuitively, if the value of  $\nu$  falls above the breakeven curve, sleep mode is convenient; otherwise, the network consumes a higher amount of energy when devices are switched off. In all cases, as  $p$  increases the minimum value of  $\nu$  increases too, meaning that sleep mode with a large number of off devices (large  $p$ ) is convenient only when the constant part of the energy cost is high with respect to the variable part. In particular, for the ER model the breakeven curve ranges between 1.2 and 2.6. The PL model breakeven curve is below the ER model one, meaning that energy cost of the traffic demand increase can be balanced by lower fixed cost. The WS breakeven curve is above the ER model one. This is due to the fact that the average shortest path length increases faster than in the ER model. Interestingly, when power-law graphs are considered,  $\nu$  is smaller, meaning that  $\alpha$  required to make sleep mode not convenient anymore

is smaller than for simple random graphs, i.e., a less efficient load proportionality factor is required.

#### A. Impact of Technology Constraints

To assess the impact of technology constraints, we compute the energy reduction ratio  $E$  for different values of  $\nu$ . Fig. 3 reports the values of  $E$  for the three proposed models; the different curves correspond to different values of  $p$ .

In all cases, the breakeven point for which  $E = 1$  occurs when  $C_F$  and  $\alpha$  are of the same order of magnitude ( $\nu \approx 1$ ). Two different regimes are possible: i) sleep mode is not convenient ( $E > 1$ ), and ii) sleep mode is convenient ( $E < 1$ ). In the first regime, the higher the probability to switch off devices is, the higher the energy loss is, being the WS the worst case. In the second regime, instead, sleep mode leads to high energy savings for all models, and the savings strongly increase with  $p$ . If  $\nu \approx 1$ , the highest savings can be obtained by the ER and PL models; if  $\nu \gg 1$ , all models obtain similar savings.

Notice that with today technology, we are in the right part of the figures (sleep mode is always convenient), while in the future, the values of  $\nu$  will probably decrease, meaning that sleep mode will become less convenient.

#### B. Impact of Network Properties

We then consider the impact of the network properties on the possible energy savings. In particular, we start by setting  $K = 6$ , while we vary  $N$  in  $[10^2, 10^5]$ . For the WS model we set  $x = N$ . Fig. 4 reports the energy reduction ratio  $E$  for the considered models, for  $\nu = 3$  and  $p = 0.3$ . Again, the highest saving is obtained by the PL model. In all the cases,  $E$  increases (sleep mode effectiveness reduces) with the number of nodes. Under the WS model, sleep mode is not convenient for large values of  $N$ , namely  $N > 10^3$ . Intuitively, in the limit  $N \rightarrow \infty$ , no finite cost  $C_F$  can balance the increase of the variable cost.

Finally, we consider the impact of the average node degree  $K$  and select values of  $K$  that mimic those used in [10] to represent the average vertex degree of the Internet graph. We set  $\nu = 3$  and  $p = 0.3$ . Moreover, for the WS model we fix  $x = 10,000$  and we vary  $K_L \in [2, 8]$ . Fig. 5 shows  $E$  versus  $K$ . For all the models, the energy reduction ratio is decreasing as  $K$  is increasing. Indeed, for large values of  $K$ , the number of possible paths between any two nodes is large and, once some nodes are powered off, it is easy for the network to find alternative paths; or, in other terms, the increase of the average shortest path due to sleep modes is limited if  $K$  is large. For low values of  $K$ , sleep mode is not convenient. For example, for the WS model  $E$  is around 1.7 when  $K = 4$ , meaning that the sleep mode wastes an additional 70% of power with respect to an always on solution. Conversely, when  $K$  increases, the additional cost due to the increase of path length is smaller, so that sleep-mode is more efficient.

Finally, notice that the energy reduction ratio for the ER and PL models is consistently lower than the WS one. This is due to the better path length properties the former two graphs have.

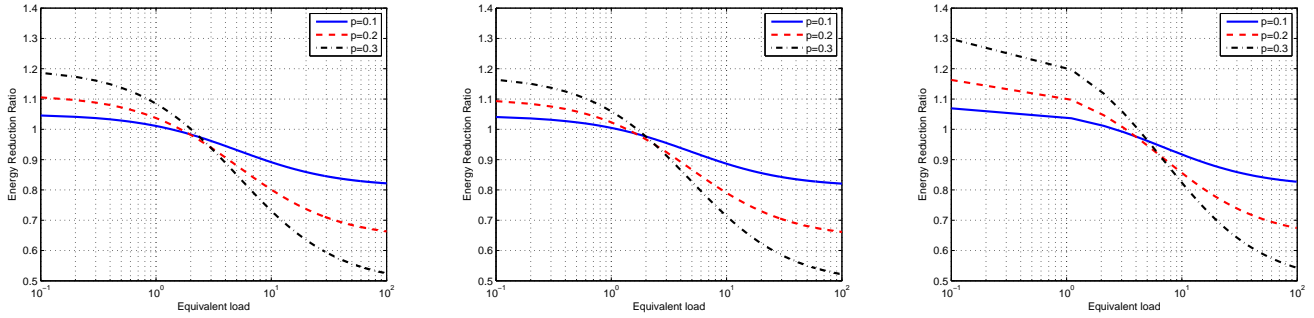


Fig. 3. Energy reduction ratio  $E$  versus equivalent load  $\nu$  for different values of  $p$ , from left to right: ER, PL, WS model.

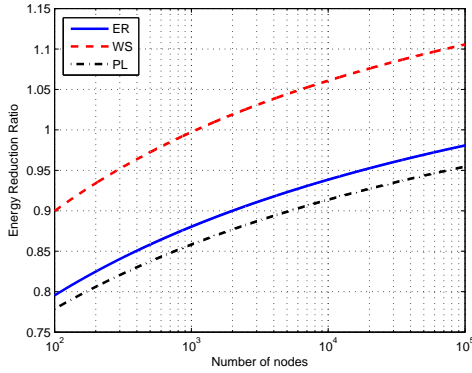


Fig. 4. Energy reduction ratio  $E$  versus the number of nodes  $N$  for  $\nu = 3$  and  $p = 0.3$ .

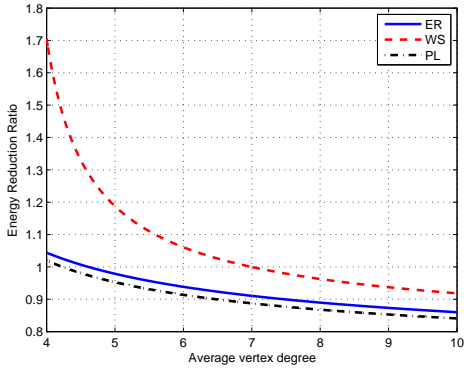


Fig. 5. Energy reduction ratio  $E$  versus average vertex degree  $K$  for  $\nu = 3$  and  $p = 0.3$ .

## V. CONCLUSION

In this paper, we have proposed an analytical framework for the evaluation of the potential energy saving that can be achieved applying sleep modes to the devices of a complex network, like the Internet. We have modeled the network device power consumption by means of a simple function composed of a constant cost and a variable cost proportional to the device load; leveraging on random graph theory, we have then computed the overall power consumption of a network equipped with load proportional devices. We have then evaluated the total power consumption of a network in which all devices are power on, and in which a fraction of

devices are put in sleep mode to save additional energy. By comparing the two figures, we can assess when the sleep mode adoption is still convenient. Indeed, if the load proportionality cost is larger than the constant cost, the adoption of sleep mode increases the total power consumption due to the extra load devices that are still on have to carry.

Our results suggest that with today technology, with device consumption that varies very little with the load, the use of sleep modes is the most effective in reducing the network energy consumption; however, with future devices whose consumption will very likely be more load proportional, the effectiveness of sleep mode approaches will reduce. Interestingly, highly connected networks, with large node degree and high randomness, tend to make the use of sleep modes more convenient.

Finally, we are currently investigating what can happen if variable power costs follow sub- or super-linear costs, as it can happen depending on the future device technology.

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## APPENDIX

In the following we report further simulation results that validate our analytical model for each of the considered graph models.

### A. Erdős-Rényi model

Fig. 6 shows the energy reduction ratio,  $E$ , computed by simulation and with the proposed model (eq. 8, 9). As previously observed for the WS model, we can note how accurate our model is.

### B. Power-law model

Fig. 7 shows the energy reduction ratio,  $E$ , computed by simulation and with the proposed model in Section III-B. For the PL model, we can observe that our model is even more accurate than in the case of ER and WS models.

### C. Watts-Strogatz model

Fig. 8 shows the average shortest path after the switch-off  $d'$  for the case  $N = 5,000$ ,  $K = 20$ , and degree of randomness  $P_x$  varying between 0.1 and 0.5. As previously observed in Fig. 1 (top), our model presents the best matching, since it is fitted for these scenarios.

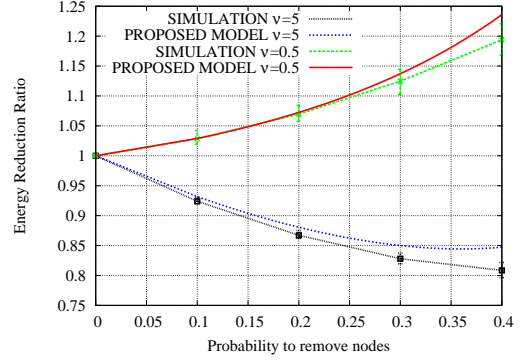


Fig. 6. ER model: energy reduction ratio versus  $p$  for the proposed model and the simulation, considering  $N = 10,000$  and  $K = 6$ .

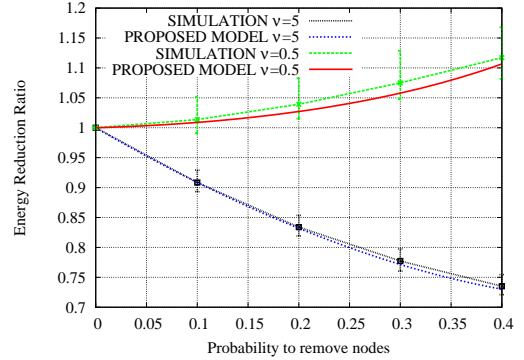


Fig. 7. PL model: energy reduction ratio versus  $p$  for the proposed model and the simulation, considering  $N = 10,000$  and  $K = 6$ .

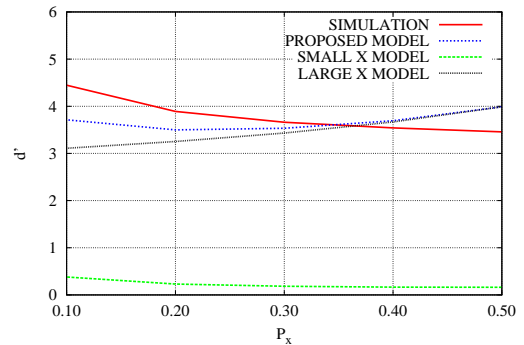


Fig. 8. Validation of the proposed model to compute  $d'$  for WS graphs: average shortest path for different models and by simulation, considering  $N = 5,000$ ,  $K = 20$ ,  $x \in [5,000 - 25,000]$