

# 3D human face description: landmarks measures and geometrical features

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## Abstract

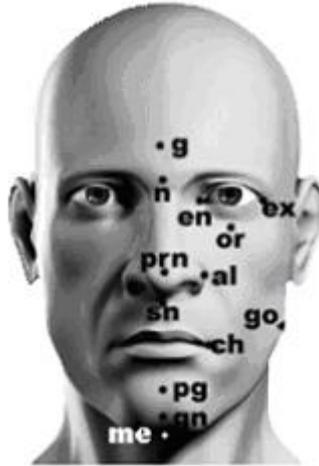
Morphometric measures and geometrical features are widely used to describe faces. Generally, they are extracted punctually from landmarks, namely anthropometric reference points. The aims are various, such as face recognition, facial expression recognition, face detection, study of changes in facial morphology due to growth, or dysmorphologies. Most of the time, landmarks were extracted with the help of an algorithm or manually located on the faces. Then, measures are computed or geometrical features are extracted to perform the scope of the study. This paper is intended as a survey collecting and explaining all these features, in order to provide a structured user database of the potential parameters and their characteristics. Firstly, facial soft-tissue landmarks are defined and contextualized; then the various morphometric measures are introduced and some results are given; lastly, the most important measures are compared to identify the best one for face recognition applications.

## 1. Introduction

Face study has been carried out in these decades for many applications: maxillofacial surgery, delict investigation, authentication, historical research, telecommunications or even games. Recognition is surely the largest branch of this diversified field, embracing subfields such as citizens identification, recognition of suspects, corporate usages in access control and on line banking. Since a new trend emerged to measure and evaluate 3D facial models, for the past decades three dimensional facial data were obtained mostly by direct anthropometric measurements. Anatomical landmarks have been used for over a century by anthropometrists interested in qualifying cranial variations. A great body of work in craniofacial anthropology is that of Leslie Farkas (Farkas, 1994; Farkas, 1996) who established a database of anthropometric norms by measuring and comparing more than 100 dimensions (linear, angular and surface contours) and proportions in hundreds of people over a period of many years. These measurements include 47 landmark points to describe the face (Čarnický *et al.*, 2006).

Nowadays the information in which researchers are interested are more complete and dynamic. The interest is to use facial landmarks as reference points of the subjects and extract geometrical features from them, in order to keep information of how the examined face is. Their uses are various and may depend on the research area.

The attention to facial landmark is due to the fact that they are points which all faces join and that have a particular biological meaning. Hard-tissue landmarks lie on the skeletal and may be identified only through lateral cephalometric radiographs; soft-tissue landmarks are on the skin and can be identified on the 3D point clouds generated by the scanning or on images. This study only deals with soft-tissue landmarks; the most famous ones are shown in Figure 1. Actually, the set of facial landmarks is much larger than this. In fact, there are approximately 60 identifiable soft-tissue points on human face, but they may change depending on the application they are used for.



**Figure 1. Anthropometric soft-tissue landmarks: g-glabella, n-nasion, en-endocanthion, ex-exocanthion, or-orbital, prn-pronasal, sn-subnasal, al-alae, ch-cheilion, pg-pogonion, gn-gnathion, go-gonion, me-menton (Calignano, 2009).**

One of the most important application that deal with facial landmarks is **face recognition**, whose large applications are: citizenship identification in borders, passports, I.D. documents, Visas; criminal identification in database screening, surveillance, alert, mob control and anti-terrorism; corporate usages in access control and time attendance in luxurious buildings, sensitive offices, airports, pharmaceutical factories; utility laptop, desktop, web, airport/sensitive console log-on and file encrypt; on line banking; gaming in casinos and watch-lists; hospitality industries such as hotel and resort CRM's; important sites like power plants and military installations. The purposes are various, but belong to two big branches: face verification, or authentication, to guarantee secure access, and face identification, or recognition of suspects, dangerous individuals and public enemies by Police, FBI and other safety organizations (Jain *et al.*, 2005).

Much research are carried out on this topic. In his various publications, Rohr *et al.* proposed multi-step differential procedures for subvoxel localization of 3D point landmarks, addressing the problem of choosing an optimal size for a region-of-interest (ROI) around point landmarks (Frantz *et al.*, 1998; Frantz *et al.*, 1999). They introduced an approach for the localization of 3D anatomical point landmarks based on deformable models. To model the surface at a landmark, they used quadric surfaces combined with global deformations (Frantz *et al.*, 2000; Alker *et al.*, 2001). Then proposed a method based on 3D parametric intensity models which are directly fitted to 3D images, introducing an analytic intensity model based on the Gaussian error function in conjunction with 3D rigid transformations as well as deformations to efficiently model anatomical structures (Wörz *et al.*, 2006). Finally introduced a novel multi-step approach to improve detection of 3D anatomical point landmarks in tomographic images (Frantz *et al.*, 2005). Romero *et al.* presented a comparison of several approaches that use graph matching and cascade filtering for landmark localization in 3D face data. For the first method, they apply the structural graph matching algorithm relaxation-by-elimination using a simple distance-to-local-plane node property and a Euclidean-distance arc property. After the graph matching process has eliminated unlikely candidates, the most likely triplet is selected, by exhaustive search, as the minimum Mahalanobis distance over a six dimensional space, corresponding to three node variables and three arc variables. A second method uses state-of-the-art pose-invariant feature descriptors embedded into a cascade filter to localize the nose tip. After that, local graph matching is applied to localize the inner eye corners (Romero *et al.*, 2009). Then described and evaluated their pose-invariant pointpair descriptors, which encode 3D shape between a pair of 3D points. Two variants of descriptor are introduced: the first is the point-pair spin image, which is related to the classical spin image of Johnson and Hebert, and the second is derived from an implicit radial basis function (RBF) model of the facial surface. These descriptors can effectively encode edges in graph based representations of 3D shapes. Here they show how the descriptors are able to identify the nose-tip and the eye-corner of a human face

simultaneously in six promising landmark localisation systems (Romero *et al.*, 2009). Ruiz *et al.* (Ruiz *et al.*, 2008) presented an algorithm for automatic localization of landmarks on 3D faces. An Active Shape Model (ASM) is used as a statistical joint location model for configurations of facial features. The ASM is adapted to individual faces via a guided search whereby landmark specific Shape Index models are matched to local surface patches. Similarly, Sang-Jun *et al.* (Sang-Jun *et al.*, 2008) applied the Active Shape Models to extract the position of the eyes, the nose and the mouth. Salah *et al.* (Salah *et al.*, 2006) proposed a coarse-to-fine method for facial landmark localization that relies on unsupervised modeling of landmark features obtained through different Gabor filter channels. D'Hose *et al.* (D'Hose *et al.*, 2007) presented a method for localization of landmarks on 3D faces using Gabor wavelets to extract the curvature of the 3D faces, which is then used for performing a coarse detection of landmarks.

A connected but quite different field is **face detection**, which consists in identifying one or more faces in an image, where many other objects can be present. Most of the literature concerning face detection investigates face detection in two-dimensional (2D) images. Colombo *et al.* (Colombo *et al.*, 2006) presented an innovative method that combines a feature-based approach with a holistic one for 3D face detection. Salient face features, such as the eyes and nose, are detected through an analysis of the curvature of the surface. Each triplet consisting of a candidate nose and two candidate eyes is processed by a PCA-based classifier trained to discriminate between faces and non-faces. Nair *et al.* (Nair *et al.*, 2009) presented an accurate and robust framework for detecting faces, localizing landmarks and achieving fine registration of face meshes based on the fitting of a facial model. Face detection is performed by classifying the transformations between model points and candidate vertices based on the upper-bound of the deviation of the parameters from the mean model. Landmark localization is performed on the segmented face by finding the transformation that minimizes the deviation of the model from the mean shape. Jesorsky *et al.* (Jesorsky *et al.*, 2001) presented a shape comparison approach to achieve fast and accurate face detection that is robust to changes in illumination and background. The proposed method is edge-based and works on greyscale still images. Takács *et al.* (Takács *et al.*, 1997) described a general approach for the detection of faces and landmarks based on biologically motivated image representation and classification schemes. The optimal set of face, eye pair, nose and mouth feature models, respectively, is found by an enhanced SOFM approach using cross-validation and corrective training. Yow *et al.* (Yow *et al.*, 1997) identified that a feature-based approach was able to detect faces efficiently over large viewpoint and illumination variations. They enhanced the approach by proposing the use of active contour models to detect the face boundary, and subsequently use it to verify face candidates. Rodrigues *et al.* (Rodrigues *et al.*, 2005) studied the importance of multi-scale keypoint representation, i.e. retinotopic keypoint maps which are turned to different spatial frequencies. They showed that this representation provided important information for Focus-of-Attention (FoA) and object detection. In particular, they showed that hierarchically-structured saliency maps for FoA can be obtained, and that combinations over scales in conjunction with spatial symmetries can lead to face detection through grouping operators that deal with keypoints at the eyes, nose and mouth, especially when non-classical receptive field inhibition is employed.

A similar application is **facial expression recognition**, a branch of recognition which deals with identifying different facial expressions. Unlike face recognition, a little work has been done to study the usefulness of facial data for recognizing and understanding facial expressions. Some researchers worked on this topic. In their various papers, Tang *et al.* (Tang *et al.*, 2008; Tang *et al.*, 2008) performed person and gender independent facial expression recognition based on properties of the line segments connecting certain 3D facial feature points. They proposed an automatic feature selection method based on maximizing the average relative entropy of marginalized class-conditional feature distributions and apply it to a complete pool of candidate features composed of normalized Euclidean distances between 83 facial feature points in the 3D space. Soyel *et al.* (Soyel *et al.*, 2007; Soyel *et al.*, 2008; Soyel *et al.*, 2009) described a pose invariant three-dimensional

facial expression recognition method using distance vectors retrieved from 3D distributions of facial feature points to classify universal facial expressions. Their works are based on the theories of Paul Ekman, a psychologist who has been a pioneer in the study of emotions and their relation to facial expressions. His theory is that the expressions associated with some emotions were basic or biologically universal to all humans. He devised a list of 6 basic emotions from cross-cultural research: anger, disgust, fear, happiness, sadness and surprise (Ekman, 1992; Ekman, 1999). For his precious and unique work, Ekman has been considered one of the 100 most eminent psychologists of the twentieth century. Nowadays, many authors involved in studies of facial expressions used his theory to concentrate their researches on expressions referred to emotions considered basic.

Another field in which facial landmarks are applied is the **study of facial morphology**. The purposes are various, such as the analysis of facial abnormalities, dysmorphologies, growth changes, aesthetic or purely theoretical. The discipline that deals with this kind of studies is Anthropometry, which is directly connected to **maxillofacial surgery**, namely aesthetic, plastic and corrective. Facial landmarks do not only appear in the applications of this discipline, but even belong to it. In fact, they were defined by surgeons in order to have a common name for every specific part of the face. A pioneer in Anthropometry is surely Leslie G. Farkas, who used anatomical landmarks to provide an essential update on the best methods for the measurement of the surfaces of the head and neck (Farkas, 1994). He gathered a set of anthropometric measurements of the face in different ethnic groups (Farkas *et al.*, 2005). Then examined the effects on faces of some syndromes, such as Treacher Collins's (Kolar *et al.*, 1985), Apert's (Kolar *et al.*, 1985), cleft lips, nasal deformity (Kohout *et al.*, 1998) and children's cleft palate (Farkas *et al.*, 1972). He studied the changes of the head and face during the growth (Farkas *et al.*, 1992) and also researched on facial beauty and neoclassical canons in face proportions (Dawei *et al.*, 1997; Le *et al.*, 2002; Farkas *et al.*, 2000; Farkas, 1995).

There are two quite different applications which face landmarks are used for. The first one is **face correction**. It consists in detecting and correcting imperfections in group photos, such as close eyes, inappropriate, unflattering and goofy faces. Dufresne (Dufresne) presented a method for diagnose and correct these issues. Face and facial landmarks are detected by an implementation of Bayesian Tangent Shape Model search. Then trained an SVM classifier to identify unflattering faces. Bad faces are then warped to match nearest neighbour faces from the good face set.

The other application is the **performance evaluation of technical equipments**. If the examined equipment is able to identify facial landmark correctly, his performance is considered effective. Enciso *et al.* (Enciso *et al.*, 2004) investigated on methods for generating 3D facial images such as laser scans, stereo-photogrammetry, infrared imaging and CT and focused on validation of indirect three-dimensional landmark location and measurement of facial soft-tissue with light-base techniques. They also evaluated precision, repeatability and validation of a light-based imaging system. Aung *et al.* (Aung *et al.*, 1995) analysed the development of laser scanning techniques enabling the capture of 3D images, especially for surface measurements of the face. They used a laser optical surface scanner to took 83 facial anthropometric measurements, using 41 identifiable landmarks on the scanned image. Then demonstrated that the laser scanner can be a useful tool for rapid facial measurements in selected anatomical parts of the face. In fact, accurate location of landmarks and operator skill are important factors to achieve reliable results.

Once landmarks are extracted from faces, manually or automatically, they become useful if it is possible to extrapolate the precious information their particular position give them. Gupta *et al.* (Gupta *et al.*, 2007; Gupta *et al.*, 2010) indeed investigated the effect of the choice of facial fiducial points on the performance of their proposed 3D face recognition algorithm. They repeated the same steps with distances between arbitrary face points, instead of the anthropometric fiducial points. These points were located in the form of a  $5 \times 5$  rectangular grid positioned over the primary facial features of each face. They chose these particular facial points as they measure distances between the significant facial landmarks, including the eyes, nose and the mouth regions, without requiring localization of specific fiducial points. They showed that in their algorithms, when anthropometric

distances are replaced by distances between arbitrary regularly spaced facial points, their performances decrease substantially. As a matter of fact, landmarks have both a geometrical and biological meaning on the human face and for this reason the extraction of measures and features from their links become necessary for providing a complete face description. Next section faces this task.

## 2. Features types: classification

Facial landmarks lie in zones of the faces which have peculiar geometric and anthropometric features. These features were extrapolated from the faces by various authors in many different ways, depending on the usages they were assigned to. The scope is to extract accurate geometric information of the examined face and allow the comparison with other faces from which the same corresponding information were previously extracted. For face recognition applications, the computation of the Euclidean or geodesic distances between landmarks is a method widely used. They are considered measures, rather than real features. Particularly, in anthropometry applications, these measures, are called *morphometric*. They are generally distances or angles and their property is that one measure involves more than one landmark. As a matter of fact, both Euclidean and Geodesic distances refer to two points, while angles involve three landmarks. But the information obtained by these reference points may be more geometric in nature, keeping for instance specific data of curvature or shape.

### 2.1 Euclidean distance

The *Euclidean distance* or *Euclidean metric* is the “ordinary” distance between two points that one would measure with a ruler, and is given by the Pythagorean formula. It is shown in Figure 2.



Figure 2. Euclidean distance between *pronasal* and *right exocanthion*.

By using this formula as distance, Euclidean space becomes a metric space. The Euclidean distance between points  $P$  and  $Q$  is the length of the line segment connecting them ( $\overline{PQ}$ ). In Cartesian coordinates, if  $P = (p_1, p_2, \dots, p_n)$  and  $Q = (q_1, q_2, \dots, q_n)$  are two points in Euclidean  $n$ -space, then the distance from  $P$  to  $Q$ , or from  $Q$  to  $P$  is given by:

$$d(P, Q) = d(Q, P) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \dots + (q_n - p_n)^2} = \sqrt{\sum_{i=1}^n (q_i - p_i)^2}.$$

In three-dimensional Euclidean space, the distance is:

$$d(P, Q) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2}.$$

The Euclidean distance between landmarks is used by most authors as a morphometric measure. Once landmarks are obtained from a facial image or a three-dimensional face, they select some significant distances between them and compute the corresponding Euclidean distances. Then these distances are used to compare faces for face recognition purposes or to perform studies on face morphometry, as said above. The Euclidean-distance-based morphometric measures are chosen depending on the application.

There is wide previous work on this topic. Gupta *et al.* (Gupta *et al.*, 2007; Gupta *et al.*, 2010) presented three-dimensional **face recognition** algorithms, which employ Euclidean distances between these anthropometric fiducial points as features along with linear discriminant analysis classifiers. Prabhu *et al.* (Prabhu *et al.*) addressed the problem of automatically locating the facial landmarks of a single person across frames of a video sequence. By calculating the mean of the Euclidean distances between the coordinates of each of the 79 landmarks that were fitted by the tracking method to those that were manually annotated, they obtained the fitting error for a particular frame. Similarly, Zhao *et al.* (Zhao *et al.*, 2010) formed a vector by 11 Euclidean distances between facial expression sensitive landmarks. Moreno *et al.* (Moreno *et al.*) performed an *HK* segmentation, i.e. based on the signs of the mean (*H*) and Gaussian (*K*) curvatures, for isolating regions of pronounced curvature on 420 3D facial meshes. After the segmentation task, a feature extraction is performed. Among them, Euclidean distances between some fiducial points were computed. Gordon (Gordon, 1992) presented a face recognition system which uses features extracted from range and curvature data to represent the face. She extracted high level features which mark salient events on the face surface in terms of points, lines and regions. Since the most basic set of scalar features describing the face correspond measurements of the face, she firstly computed the Euclidean distance of: left eye width, left eye width, eye separation, total width (span) of eyes, nose height, nose, width, nose depth and head width. Likewise, Lee *et al.* (Lee *et al.*, 2005) calculated with Euclidean distance relative lengths of extracted facial feature points. Efraty *et al.* (Efraty *et al.*, 2009; Efraty *et al.*, 2010) studied the silhouette of face profile and introduced a new method for face recognition that improves robustness to rotation. They achieved this by exploring the feature space of profiles under various rotations with the aid of a 3D face model. Based on the fiducial points on the profile silhouette, they extracted a set of rotation-, translation- and scale-invariant features which are used to design and train a hierarchical pose-identity classifier. Euclidean distance was chosen by him as one type of measurements between landmarks. Daniyal *et al.* (Daniyal *et al.*, 2009) represented the face geometry with inter-landmark distances within selected regions of interest to achieve robustness to expression variations. The proposed recognition algorithm first represents the geometry of the face by a set of Euclidean Inter-Landmark Distances (ILDs) between the selected landmarks. These distances are then compressed using Principal Component Analysis (PCA) and projected onto the classification space using Linear Discriminant Analysis (LDA). Soyel *et al.* (Soyel *et al.*, 2007; Soyel *et al.*, 2008) used six different Euclidean distances between feature points to form a distance vector for **facial expression recognition**. They are: openness of eyes, height of eyebrows, openness of mouth, width of mouth, stretching of lip and openness of jaw. During the recognition experiments, a distance vector is derived for every 3D model and the whole procedure is repeated numerous times. Ras *et al.* (Ras *et al.*, 1996) introduced stereophotogrammetry as a three-dimensional registration method for quantifying facial **morphology** and detecting changes in facial morphology during growth and development. They used six sets of automatically extracted 3D landmarks coordinates to calculate the Euclidean distances between *exocanthion* and *chelion*, *chelion* and *pronasale*, *exocanthion* and *pronasale* for both sides of the face. Changes in facial morphology due to growth and development were analysed with an analysis of variance of these distances. The last field in which Euclidean distances between landmarks were applied is **performance evaluation of technical equipments**. Enciso *et al.* (Enciso

*et al.*, 2004) used a digitizer to obtain landmarks and then directly measured Euclidean distances between them. These distances were compared with the indirect homologous distances measured on the scans with our computer tools. Aung *et al.* (Aung *et al.*, 1995) firstly carried out direct Euclidean-distance-based anthropometric measurements of the face using standard anthropometric landmarks as defined by Farkas. The subject was then laser scanned with the optical surface scanner and the laser scan measurements were done using selected landmarks identifiable on the laser scan image. The same number of corresponding sets of measurements from the direct and indirect methods were then compared, in order to evaluate laser scanner performance.

## 2.2 Geodesic distance and arc-length distance

A *geodesic* is a generalization of the notion of a “straight line” to curved spaces. In the presence of a metric, geodesics are defined to be (locally) the shortest path between points in the space. The term “geodesic” comes from Geodesy, the science of measuring the size and shape of Earth; in the original sense, a geodesic was the shortest route between two points on the Earth's surface, namely, a segment of a great circle. More generally, on a sphere, the images of geodesics are the great circles. The shortest path from point  $A$  to point  $B$  on a sphere is given by the shorter arc of the great circle passing through  $A$  and  $B$ . If  $A$  and  $B$  are antipodal points (like the North pole and the South pole), then there are “infinitely many” shortest paths between them. It is shown in Figure 3.



**Figure 3. Geodesic distance between *pronasal* and right *exocanthion*.**

Formally, geodesics can then be defined as curves whose osculating planes contain the normals to the surface. The parametrized curves  $\gamma : I \rightarrow \mathfrak{R}^2$  of a plane along which the field of their tangent vectors  $\gamma'(t)$  is parallel are precisely the straight lines of that plane. The parametrized curves that satisfy an analogous condition for a surface are called geodesics. More precisely, a nonconstant, parametrized curve  $\gamma : I \rightarrow S$  is said to be geodesic at  $t \in I$  if the field of its tangent vectors  $\gamma'(t)$  is parallel along  $\gamma$  at  $t$ ; that is,

$$\frac{D\gamma'(t)}{dt} = 0;$$

$\gamma$  is a parametrized geodesic if it is geodesic for all  $t \in I$ . Immediately  $|\gamma'(t)| = \text{const.} = c \neq 0$  is obtained. Therefore, the arc-length  $s = ct$  may be introduced as a parameter, and it is possible to conclude that the parameter  $t$  of a parametrized geodesic  $\gamma$  is proportional to the arc length of  $\gamma$ . A

parametrized geodesic may admit self-intersections. However, its tangent vector is never zero, and thus the parametrization is regular.

The notion of geodesic is clearly local. The previous considerations allow to extend the definition of geodesic to subsets of  $S$  that are regular curves. A regular connected curve  $C$  in  $S$  is said to be a geodesic if, for every  $p \in S$ , the parametrization  $\alpha(s)$  of a coordinate neighborhood of  $p$  by the arc length  $s$  is a parametrized geodesic; that is,  $\alpha'(s)$  is a parallel vector field along  $\alpha(s)$ . Every straight line contained in a surface satisfies this definition. From a point of view exterior to the surface  $S$ , the definition is equivalent to saying that  $\alpha''(s) = kn$  is normal to the tangent plane, that is, parallel to the normal to the surface. In other words, a regular curve  $C \subset S (k \neq 0)$  is a geodesic if and only if its principal normal at each point  $p \in C$  is parallel to the normal to  $S$  at  $p$ . The above property can be used to identify some geodesics geometrically.

The great circles of a sphere  $S^2$  are geodesics. Indeed, the great circles  $C$  are obtained by intersecting the sphere with a plane that passes through the centre  $O$  of the sphere. The principal normal at a point  $p \in C$  lies in the direction of the line that connects  $p$  to  $O$  because  $C$  is a circle of centre  $O$ . Since  $S^2$  is a sphere, the normal lies in the same direction, which verifies our assertion. For the case of the sphere, through each point and tangent to each direction there passes exactly one great circle, which, as we proved before, is a geodesic. Therefore, by uniqueness, the great circles are the only geodesics of a sphere.

For the right circular cylinder over the circle  $x^2 + y^2 = 1$ , it is clear that the circles obtained by the intersection of the cylinder with planes that are normal to the axis of the cylinder are geodesics. That is so because the principal normal to any of its points is parallel to the normal to the surface at this point. On the other hand, the straight lines of the cylinder (generators) are also geodesics. To verify the existence of other geodesics on the cylinder  $C$  we shall consider a parametrization

$$x(u, v) = (\cos u, \sin u, v)$$

of the cylinder in a point  $p \in C$ , with  $x(0, 0) = p$ . In this parametrization, a neighborhood of  $p$  in  $C$  is expressed by  $x(u(s), v(s))$ , where  $s$  is the arc length of  $C$ . Then,  $x$  is a local isometry which maps a neighborhood  $U$  of  $(0, 0)$  of the  $uv$  plane into the cylinder. Since the condition of being a geodesic is local and invariant by isometries, the curve  $(u(s), v(s))$  must be a geodesic in  $U$  passing through  $(0, 0)$ . But the geodesics of the plane are the straight lines. Therefore, excluding the cases already obtained,

$$u(s) = as, \quad v(s) = bs, \quad a^2 + b^2 = 1.$$

It follows that when a regular curve  $C$  (which is neither a circle or a line) is a geodesic of the cylinder it is locally of the form

$$(\cos as, \sin as, bs),$$

and thus it is a helix. In this way, all the geodesics of a right circular cylinder are determined.

Observe that given two points on a cylinder which are not in a circle parallel to the  $xy$  plane, it is possible to connect them through an infinite number of helices. This fact means that two points of a cylinder may in general be connected through an infinite number of geodesics, in contrast to the situation in the plane. Observe that such a situation may occur only with geodesics that make a "complete turn", since the cylinder minus a generator is isometric to a plane (Do Carmo, 1976).

In metric geometry, a Geodesic is a curve which is everywhere locally a distance minimizer. More precisely, a curve  $\gamma : I \rightarrow M$  from an interval  $I$  of the reals to the metric space  $M$  is a geodesic if there is a constant  $v \geq 0$  such that for any  $t \in I$  there is a neighborhood  $J$  of  $t$  in  $I$  such that for any  $t_1, t_2 \in J$  we have

$$d(\gamma(t_1), \gamma(t_2)) = v|t_1 - t_2|.$$

This generalizes the notion of geodesic for Riemannian manifolds. However, in metric geometry the geodesic considered is often equipped with natural parametrization, i.e. in the above identity  $v = 1$  and

$$d(\gamma(t_1), \gamma(t_2)) = |t_1 - t_2|.$$

If the last equality is satisfied for all  $t_1, t_2 \in I$ , the geodesic is called a minimizing geodesic or shortest path. In general, a metric space may have no geodesics, except constant curves. At the other extreme, any two points in a length metric space are joined by a minimizing sequence of rectifiable paths, although this minimizing sequence need not converge to a geodesic.

Some authors used geodesic distance between facial landmarks. First of all, Bronstein *et al.* (Bronstein *et al.*, 2003; Bronstein *et al.*, 2004; Bronstein *et al.*, 2005; Bronstein *et al.*, 2005; Bronstein *et al.*, 2006) proposed to model facial expressions as isometries of the facial surface. The facial surface is described as a smooth compact connected two-dimensional Riemannian manifold (surface), denoted by  $S$ . The minimal geodesics between  $s_1, s_2 \in S$  are curves of minimum length on  $S$  connecting  $s_1$  and  $s_2$ . The geodesics are denoted by  $C_S^*(s_1, s_2)$ . The geodesic distances refer to the lengths of the minimum geodesics and are denoted by

$$d_S(s_1, s_2) = \text{length}(C_S^*(s_1, s_2)).$$

A transformation  $\psi : S \rightarrow Q$  is called an *isometry* if

$$d_S(s_1, s_2) = d_Q(\psi(s_1), \psi(s_2))$$

for all  $s_1, s_2 \in S$ . In other words, an isometry preserves the intrinsic metric structure of the surface. The isometric model, assuming facial expressions to be isometries of some neutral facial expression, is based on the intuitive observation that the facial skin stretches only slightly. All expressions of a face are assumed to be “intrinsically” equivalent (i.e. have the same metric structure), and “extrinsically” different. Broadly speaking, the intrinsic geometry of the facial surface can be attributed to the subject’s identity, while the extrinsic geometry is attributed to the facial expression. The isometric model tacitly assumes that the expressions preserve the *topology* of the surface. This assumption is valid for most regions of the face except the mouth. Opening the mouth changes the topology of the surface by virtually creating a hole. Based on this model, expression-invariant signatures of the face were constructed by means of approximate isometric embedding into flat spaces. They applied a new method for measuring isometry-invariant similarity between faces by embedding one facial surface into another. Promising **face recognition** results are obtained in numerical experiments even when the facial surfaces are severely occluded. Gupta *et al.* (Gupta *et al.*, 2007; Gupta *et al.*, 2007; Gupta *et al.*, 2010) worked on the same assumption, namely that different facial expressions could be regarded as isometric deformations of the face surface. These deformations preserve intrinsic properties of the surface, one of which is the geodesic distance between a pair of points on the surface. Based on these ideas they presented a preliminary

study aimed at investigating the effectiveness of using geodesic distances between all pairs of 25 fiducial points on the surface as features for face recognition. Instead of choosing a random set of points on the face surface, they considered facial landmarks relevant to measuring anthropometric facial proportions employed widely in facial plastic surgery and art. They calculated geodesics using the Dijkstra's shortest path algorithm by defining 8 connected nearest neighbors about each point. Twenty-five fiducial points, as depicted in were manually located on each face. Three face recognition algorithms were implemented. The first employed 300 geodesic distances (between all pairs of fiducial points) as features for recognition. The fast marching algorithm for front propagation was employed to calculate the geodesic distance between pairs of points. The second algorithm employed 300 Euclidean distances between all pairs of fiducial points as features. The normalized L1 norm where each dimension was divided by its variance, was used as the metric for matching faces with both the Euclidean distance and geodesic distance features.

The *arc-length* is the length of an irregular arc segment and is also called “rectification of a curve”. Thanks to its definition, it is strictly connected to the geodesics. Efraty *et al.* (Efraty *et al.*, 2009; Efraty *et al.*, 2010) were interested in profile-based face recognition. They defined five types of measurements based on the properties of the profile between two landmarks. One of them was exactly the arc-length between landmarks. Nevertheless, Aung *et al.* (Aung *et al.*, 1995), who used facial landmarks to **evaluate the performance of a laser scanner**, argued that tangential or arc measurements were slightly more complex and needed careful positioning of the image before accurate measurements could be made.

### 2.3 Ratios of distances

The ratios of geometric features are common in nature, the golden ratio  $\Phi = \frac{1 + \sqrt{5}}{2}$  being the most familiar. Many artists utilize the golden ratio to make their painting and sculpture more appealing. Scientists believe that some human faces are more attractive because their features are related by the golden ratio. It was demonstrated that the perception of face beauty is not based entirely on cultural influences and that length of the internal features can cause different perceptions of beauty. The ratios of face features play a crucial role in the classification of faces. In the past 20 years, researchers and practitioners in anthropology and aesthetic surgery have analyzed faces from a different perspective. They use a set of canonical points on a human face that are critical for face reconstruction. These points, and distances between them are used to represent a face. In fact, artists developed a set of neoclassical canons (ratios of distances) to represent faces as far back as the Renaissance period All these observations motivate researchers to explore the role of ratios of the distances between face landmarks in face recognition (Shi *et al.*, 2006). Generally, in face study, ratios are defined for the Euclidean distances or geodesic distances among landmarks. These ratios are often normalized distances, obtained by dividing a distance between points by face width. Shi *et al.* (Shi *et al.*, 2006) investigated how well-normalized Euclidean distances (special ratios) can be exploited for **face recognition**. Exploiting symmetry and using principal component analysis, they reduced the number of ratios to 20. They are free from translation, scaling and 2D rotation of face images. The normalized distances for a face are then defined as

$$r(l_i, l_j) = \frac{d(l_i, l_j)}{d(l_a, l_b)}, \quad \forall l_i, l_j \in \{l_1, \dots, l_N\},$$

where  $\{l_1, \dots, l_N\}$  are the landmarks,  $N$  is their cardinality,  $l_a$  and  $l_b$  are two landmarks whose Euclidean distance is defined as a benchmark distance. Together with Euclidean distances and geodesic distances, Gupta *et al.* (Gupta *et al.*, 2007; Gupta *et al.*, 2007) used ratios. They presented an anthropometric three-dimensional (Anthroface 3D) face recognition algorithm, which is based on a systematically selected set of discriminatory structural characteristics of the human face derived

from the existing scientific literature on facial anthropometry. Anthropometric cranio-facial proportions are ratios of pairs of straight-line and/or along-the-surface distances between specific cranial and facial fiducial points. For example, the most commonly used nasal index  $NI$  is the ratio of the horizontal nose width to the vertical nose height. Lee *et al.* (Lee *et al.*, 2005) used relative ratios between feature points to perform face recognition. Tang *et al.* (Tang *et al.*, 2008; Tang *et al.*, 2008) performed **facial expression recognition**. They devised a set of features based on properties of the line segments connecting certain facial feature points on a 3D face model. Among them, normalized distances were extracted. Mao *et al.* (Mao *et al.*) took care of **studying facial change due to growth**. They formulated a new inverse flatness metric, the ratio of the geodesic distance and the Euclidean distance between landmarks, to study 3D facial surface shape. With this ratio, they were able to analyze curvature asymmetry, which cannot be detected by studying the Euclidean distance alone. They also attempted to combine it with the conventional Euclidean inter-landmark distances based symmetric method to express facial symmetry in terms of both surface flatness and also the geometric symmetry of landmark positions (captured by the Euclidean distances), to give a better overall description of three-dimensional facial symmetry. If  $GD_{i,j}$  is the geodesic distance between points  $i$  and  $j$ , and  $ED_{i,j}$  is the Euclidean distance, then the ratio of the geodesic to Euclidean distance

$$R = \frac{GD_{i,j}}{ED_{i,j}}$$

is employed in their work to analyze surface flatness, since it can reflect the inverse flatness of the geodesic curve that samples the surface on which the two end points  $(i, j)$  lie. Therefore, this ratio is capable of capturing obvious differences in facial curvature.

## 2.4 Curvature and shape

Punctual values of curvature and shape are precious information about facial surface behaviour. Although their valuable contribution, they are not used as often as distances. That is because they are not as easily tractable and extractable from faces. The necessity to condensate and formalize their values becomes basic in this field, where generally surfaces are not real, but described by point clouds or meshes.

Several techniques have been developed to estimate the curvature information in the last two decades. From the mathematical viewpoint, the curvature information can be retrieved by the first and second partial derivatives of the local surface, the local surface normal and the tensor voting (Worthington *et al.*, 2000). An interesting curvature representation was proposed by Koenderink *et al.* (Koenderink *et al.*, 1992). It is based on the parametrization of the structure in two features maps, namely the Shape Index  $S$  and the Curvedness Index  $C$ . The formal definition of **Shape Index** can be given as follows:

$$S = -\frac{2}{\pi} \arctan \frac{k_1 + k_2}{k_1 - k_2}, \quad S \in [-1, 1], \quad k_1 \geq k_2,$$

where  $k_1$  and  $k_2$  are the principal curvatures. It describes the shape of the surface. Koenderink *et al.* proposed a partition of the range  $[-1, 1]$  in nine categories, which correspond to nine different surfaces. Nevertheless, Dorai *et al.* (Dorai *et al.*, 1995; Dorai *et al.*, 1996, Dorai *et al.*, 1997) employed a modified definition to identify the shape category to which each surface point on an object belongs. With their definition, all shapes can be mapped on the interval  $S \in [0, 1]$ , conveniently allowing aggregation of surface patches based on their shapes:

$$S = \frac{1}{2} - \frac{1}{\pi} \arctan \frac{k_1 + k_2}{k_1 - k_2}.$$

Dorai *et al.* addressed the problem of representing and recognizing arbitrarily curved 3D rigid objects when the objects may vary in shape and complexity, and no restrictive assumptions are made about the types of surfaces on the object. They proposed a new and general surface representation scheme for recognizing objects with free-form (sculpted) surfaces from range data.

$S$  does not give an indication of the scale of curvature present in the shapes. For this reason, an additional feature is introduced, the **Curvedness Index** of a surface:

$$C = \sqrt{\frac{k_1^2 + k_2^2}{2}}.$$

It is a measure of how highly or gently curved a point is and is defined as the distance from the origin in the  $(k_1, k_2)$ -plane. Whereas the Shape Index scale is quite independent of the choice of a unit of length, the curvedness scale is not. Curvedness has the dimension of reciprocal length. In practice one has to point out some fiducial sphere as the unit sphere to fix the curvedness scale.

Since principal curvatures may be computed punctually, then both  $S$  and  $C$  may be too. This advantage allow to extract shape and curvedness information from landmarks or fiducial points, guaranteeing a formalization for these features.

Few authors used Shape and Curvedness Indexes for **recognition**. Worthington *et al.* (Worthington *et al.*, 2000) investigated whether regions of uniform surface topography can be extracted from intensity images using shape-from-shading and subsequently used for the purposes of thirty object recognition. They drew on the constant Shape Index maximal patch representation of Dorai *et al.*. Song *et al.* (Song *et al.*, 2005) described a 3D face recognition method using facial Shape Indexes. Given an unknown range image, they extracted invariant facial features based on the facial geometry. For face recognition method, they defined and extracted facial Shape Indexes based on facial curvature characteristics and perform dynamic programming. Shin *et al.* (Shin *et al.*, 2006) described a pose invariant three-dimensional face recognition method using distinctive facial features. They extracted invariant facial feature points on those components using the facial geometry from a normalized face data and calculated relative features using these feature points. They also calculated a Shape Index on each area of facial feature point to represent curvature characteristics of facial components. Calignano (Calignano, 2009) used Shape and Curvedness Indexes for a morphological analysis methodology for soft-tissue landmarks automatic extraction. Nair *et al.* (Danial *et al.*, 2009; Nair *et al.*, 2009) dealt with **face recognition**, **face detection** and landmark localization. In isolation-of-candidate-vertices-phase, in order to characterize the curvature properly of each vertex on the face mesh they computed two feature maps, namely the Shape Index and the Curvedness Index. The low-level feature maps were computed after Laplacian smoothing that reduced outliers arising from the scaling process. The smoothed and decimated mesh is only used for the isolation of the candidate vertices. Zhao *et al.* (Zhao *et al.*, 2010) analysed **facial expressions**. To describe local surface properties, they computed Shape Index of all points on the local grids and concatenate into vector  $SI$ . They choose Shape Index because it has been proven to be an efficient feature to describe local curvature information and is independent of the coordinate system. The Shape Index is computed on each vertex on local grids and the feature  $SI$  is constructed by concatenating those values into a vector.

Other parameters and methodologies were used to extract shape and curvature information from facial landmarks or fiducial points. Moreno *et al.* (Moreno *et al.*) performed **face recognition** using 3D surface-extracted descriptors. Averages and variances of the **mean and Gaussian curvatures**, evaluated in points belonging to the various regions which face surface was divided by, were extracted. Gordon (Gordon, 1992) defined a set of features which describe the nose ridge and

are based on measurement of curvature. They are: maximum Gaussian curvature on the ridge line, average minimum curvature on the ridge above the tip of the nose, Gaussian curvature at the bridge, Gaussian curvature at the base. The maximum Gaussian curvature will occur approximately at the tip of the nose, and provides some description of local shape at that point. The average minimum curvature between the bridge and the tip of the nose is meant provide a simple measure of the curvature along the ridge. Xu *et al.* (Xu *et al.*, 2004) developed an automatic face recognition method combining the global geometric features with local shape variation information. The scattered 3D point cloud is first represented with a regular mesh. Then the local shape information is extracted to characterize the individual together with the global geometric features. They firstly defined a metric to describe the 3D shape of the principle areas with a 1D vector and then used the Gaussian-Hermite moments to analyze the shape variation. Efraty *et al.* (Efraty *et al.*, 2009; Efraty *et al.*, 2010) computed for each pair of landmarks the mean curvature of the region between landmarks and the  $L_2$ -norm of curvature along the contour between landmarks (proportional to bending energy). Wang *et al.* (Wang J. *et al.*, 2006) dealt with **facial expression recognition**. They proposed an approach to extract primitive 3D facial expression features from the triangle meshes of faces. They performed principal curvature analysis, which produced a set of attributes that describes the surface property at each vertex. Among them, *principal curvatures*, representing the maximum and the minimum degrees of bending of a surface, and *steepness* are included. Using these geometric attributes, they were able to classify every vertex into a category.

## 2.5 Other features

Other geometrical features were extracted from face landmarks. Ras *et al.* (Ras *et al.*, 1996) studied **facial morphology** and computed *angles* between fiducial points. Particularly, the angles *exocanthion-chelion-pronasal*, *exocanthion-pronasal-exocanthion*, *pronasal-exocanthion-chelion* and between the two planes formed by *exocanthion*, *chelion* and *pronasal* of both sides were calculated. Changes in facial morphology due to growth and development were analyzed with an analysis of variance with the angles. Lee *et al.* (Lee *et al.*, 2005) performed **face recognition** calculating relative angles among facial feature points. Moreno *et al.* (Moreno *et al.*) computed angles, *regions*, *areas of regions* and *centroids of regions*. Zhao *et al.* (Zhao *et al.*, 2010), for **face recognition** purposes, used the *multi-scale LBP* operator, a powerful texture measure used widely in 2D face analysis. It extracts information which is invariant to local gray-scale variations of the image with low computational complexity. They also computed a landmark *displacement* vector. The displacement of a landmark means to capture the change of the landmark location when an expression appears on a neutral face. It is informative because it represents the difference between the face with an expression and the neutral one. Similarly, Sun *et al.* (Sun *et al.*, 2008) derived the displacement vector between each individual frame and the initial frame, namely the neutral expression one. Dufresne (Dufresne) utilized the *vectors* between selected facial points as features for 2D **face correction**. He showed that simply measuring the width and height of the mouth does not indicate what pose that mouth is in, i.e. smiling, scowling or smirking. Vectors were selected due to being particularly expressive. That is, a human could understand the expression if only these vectors were presented to them. Tang *et al.* (Tang *et al.*, 2008) extracted *slopes* of the line segments connecting a subset of the 83 facial feature points for **facial expression recognition** purposes. Daniyal *et al.* (Daniyal *et al.*, 2009) analyzed the performance of different landmark combinations (*signatures*) to determine a signature that is robust to expressions for the purpose of **face recognition**. The selected signature is then used to train a Point Distribution Model for the automatic localization of the landmarks. As a validation, Jesorsky *et al.* (Jesorsky *et al.*, 2001) used *relative error* for **face detection**. Relative error is based on the distances between the expected and the estimated eye position, so it must not be considered as a normalized distance. Other authors extracted *depth* and *texture* features from landmarks or face zones. A texture coding provides information about facial regions with little geometric structures, such as hair, forehead and eyebrows, while a depth coding provides information about regions where there is little texture such

as the chin, jawline and cheeks. Particularly, Wang *et al.* (Wang Y. *et al.*, 2002) extracted shape and texture features from defined feature points for **face recognition** purposes. BenAbdelkader *et al.* (BenAbdelkader *et al.*, 2005) worked on face coding for recognition and identification. They designed a pattern classifier for three different inputs: depth map, texture map, and both depth and texture maps. Hüsken *et al.* (Hüsken *et al.*, 2005) included both texture and shape as typical 2D and 3D representations of faces.

### 3. Results and conclusions

Depending on the application field, these measures were judged by the researchers as valid, effective and suitable to a face description. Since the fields which all these geometrical features are used for are really various, it is out of the scope of this paper to add here the results these measures give in their applications. However, it is possible to give an overview of how functional are the most important features, namely *Euclidean* and *geodesic distances*, in recognition usage, i.e. the main field. These evaluations are given by those authors who employed both the two measures and compare the obtained results. Bronstein *et al.* (Bronstein *et al.*, 2003; Bronstein *et al.*, 2004; Bronstein *et al.*, 2005; Bronstein *et al.*, 2005; Bronstein *et al.*, 2005), which used geodesic distances, obtained promising face recognition results on a small database of 30 subjects even when the facial surfaces were severely occluded. They also demonstrated that the approach has several significant advantages, one of which is the ability to handle partially missing data. This is exactly the contrary of what was proven by Gupta *et al.* in his first study (Gupta *et al.*, 2007), who tested both Euclidean and geodesic distance. The two algorithms based on Euclidean or geodesic distances between anthropometric facial landmarks performed substantially better than the baseline PCA algorithm. The algorithms based on geodesic distance features performed on a par with the algorithm based on Euclidean distance features. Both were effective, to a degree, at recognizing 3D faces. In this study the performance of the proposed algorithm based on geodesic distances between anthropometric facial landmarks decreased when probes with arbitrary facial expressions were matched against a gallery of neutral expression 3D faces. This suggests that geodesic distances between pairs of landmarks on a face may not be preserved when the facial expression changes. This was contradictory to Bronstein *et al.*'s assumption regarding facial expressions being isometric deformations of facial surfaces. In conclusion, geodesic distances between anthropometric landmarks were observed to be effective features for recognizing 3D faces, however they were not more effective than Euclidean distances between the same landmarks. The 3D face recognition algorithm based on geodesic distance features was affected by changes in facial expression. Lately, Gupta *et al.* (Gupta *et al.*, 2010) gained other results. He obtained that, for expressive faces, the recognition rates of the algorithm that was based on both the Euclidean and geodesic facial anthropometric distances were also generally higher than those of the algorithm that was based on only Euclidean distances. This suggests that facial geodesic distances may be useful for expression invariant 3D face recognition and further strengthens Bronstein *et al.*'s proposition that different facial expressions may be modeled as isometric deformations of the facial surface.

An exhaustive set of morphometric measures and geometrical features extractable from facial landmarks were here presented and explained. The most popular ones are certainly Euclidean and geodesic distance, which were used by many authors, also as benchmarking elements of comparison. The application which involve them the most is recognition, with its various subfields, such as face recognition, facial expression recognition and face detection. Landmarks are the starting point for this study, being exactly the reference points which information are extracted from. This is due to the fact that from various evaluations it resulted necessary use fiducial points. As a matter of fact, most of the work concerning 3D facial morphometry refers exactly to landmarks.

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