Tutorial lecture on Modeling and simulation of high-speed interconnects: approaches, challenges, and solutions

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Modeling and Simulation of High-Speed Interconnects: Approaches, Challenges and Solutions

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Overview – part I

- Electrical interconnects
  - A showcase: chip, package, board, system
  - Why interconnects are so critical
  - Characterization via Scattering parameters
  - Frequency-domain simulation
- Transient simulation of terminated interconnects
  - Impulse responses and convolution methods
  - Macromodeling approaches
    - Fitting, passivity, and synthesis
  - Direct discretization + Model Order Reduction
- Case study
  - Coupled Signal/Power Integrity analysis of a complex board
Overview – part II

- Accuracy and robustness of interconnects simulation
  - Fundamental properties of data and models
    - Stability
    - Causality
    - Passivity
  - Impact on frequency- and time-domain simulations
  - Best practices for accurate and robust simulations
  - Case studies

Interconnects: showcase
Interconnects: showcase

 Courtesy D. Kaller, IBM Boeblingen, Germany
Interconnects: showcase
Interconnects: showcase

The objective

Interconnect network
- Signal and Power
- Linear
- Many ports
- Complex geometry
- Electrically large

Terminations
- Nonlinear
- Complex

Terminations
- Nonlinear
- Complex

Terminations
- Nonlinear
- Complex
Interconnects: why so critical?

- Interconnect network
- Signal and Power
- Linear
- Many ports
- Complex geometry
- Electrically large

The ideal interconnect

\[ v_1 = v_2 \]
\[ i_1 = -i_2 \]
**Inductive effects**

\[ v_1 - v_2 = L \frac{di_1}{dt} \]
\[ i_1 = -i_2 \]

**Capacitive effects**

\[ v_1 = v_2 \]
\[ i_1 + i_2 = C \frac{dv_1}{dt} \]
Inductive and capacitive effects

\[ i_1 + v_1 = i_2 - v_2 \]

\[ v_1 - v_2 = L \frac{di_1}{dt} \]

\[ i_1 + i_2 = C \frac{dv_2}{dt} \]

Ideal transmission line

\[ i(z) + L \frac{\partial i(z + dz)}{\partial z} = \frac{\partial v(z + dz)}{\partial z} \]

\[ -C \frac{\partial i(z + dz)}{\partial z} = \frac{\partial v(z)}{\partial z} \]
Inductive and capacitive coupling

\[ i_1 \rightarrow H \rightarrow + \]

\[ + \quad v_m \quad - \]

\[ q_m \quad E \]

Lossless Multiconductor Transmission Lines

Vectors collecting voltages and currents

\[ - \frac{\partial \mathbf{v}(z,t)}{\partial z} = \mathbf{L} \frac{\partial \mathbf{i}(z,t)}{\partial t} \]

\[ - \frac{\partial \mathbf{i}(z,t)}{\partial z} = \mathbf{C} \frac{\partial \mathbf{v}(z,t)}{\partial t} \]

Matrix-valued per-unit-length parameters
Frequency domain

\[ X(s) = \int_{0^-}^{\infty} x(t)e^{-st} \, dt \quad \text{One-sided Laplace transform} \]

\[ X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} \, dt \quad \text{Two-sided Laplace transform} \]

\[ X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} \, dt \quad \text{Fourier transform} \]

Lossless MTLs – frequency domain

\[- \frac{\partial v(z,t)}{\partial z} = L \frac{\partial i(z,t)}{\partial t} \quad \text{Partial Differential Equations} \]

\[- \frac{\partial i(z,t)}{\partial z} = C \frac{\partial v(z,t)}{\partial t} \]

\[- \frac{dV(z,s)}{dz} = sL I(z,s) \quad \text{Ordinary Differential Equations} \]

\[- \frac{dI(z,s)}{dz} = sC V(z,s) \]
DC metal losses

\[ I_{DC} \rightarrow \ell \rightarrow V_{DC} \]

\[ R = \frac{\ell}{\sigma S} \]

AC metal losses

Time-harmonic (AC) excitation

Current density distribution in 2 wires

Relative colormap

Frequency

Resistance
Computation of PUL parameters

Frequency-dependent effective cross-section

\[ R(\omega) = \frac{\ell}{\sigma S(\omega)} \approx K\sqrt{\omega} \]

Per-unit-length parameters depend on internal and external EM fields

Need a 2D electromagnetic field solver
Volume-based (FEM, FD)
Surface-based (MoM)

Frequency is a parameter in the solution

Frequency-dependent Per-Unit-Length parameters

![Graphs of frequency-dependent Per-Unit-Length parameters](image)
Lossy Multiconductor Transmission Lines

\[ -\frac{dV(z,s)}{dz} = Z(s) I(z,s) \]
\[ -\frac{dI(z,s)}{dz} = \left[ G(s) + sC(s) \right] V(z,s) \]

3D interconnects

No simple analytical model!
Need full-wave (3D) EM solution

Same qualitative features of T-lines
Characterization depends on frequency
Frequency must be a parameter!
Transmission Lines modeling and simulation

\[
\begin{align*}
-\frac{d}{dz} V(z,s) &= Z(s) I(z,s) \\
-\frac{d}{dz} I(z,s) &= Y(s) V(z,s)
\end{align*}
\]

- Particular structure of TL equations
- Applicable to interconnects with uniform X-section only
- Dedicated modeling and simulation methods
  - Lumped RLGC sections
  - Pade’ approximations
  - Matrix Rational Approximations with and without delay extraction
  - Method of Characteristics
  - … many more…
- NOT discussed in this tutorial

Characterization of linear multiports

- Several linear algebraic equations expressing two output variables for two inputs are possible
- The matrix operator of these equations is composed of network functions for defined load conditions
Open-circuit impedances . . .

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]

\( Z_i(s) \) are network functions of the circuit defined by the element with open-circuit terminations.

Generic terminations: wave variables

\[ V = A + B \]
\[ I = \frac{1}{R_0}(A - B) \]
\[ A = \frac{1}{2}(V + R_0I) \]
\[ B = \frac{1}{2}(V - R_0I) \]

Port reference impedance (real and constant)
### Generic terminations: wave variables

\[ I(s) \quad \rightarrow \quad \bullet \]
\[ V(s) \quad \leftarrow \quad \bullet \]

\[ A(s) \quad \rightarrow \quad \bullet \]
\[ B(s) \quad \leftarrow \quad \bullet \]

\[ A = \frac{1}{2} \left( V + R_0 I \right) \]
\[ B = \frac{1}{2} \left( V - R_0 I \right) \]

Imposing \( A \) amounts to connecting

\[ V = -R_0 I + 2A \]

### Scattering network functions

Relate the \( B \) wave variables to the imposed \( A \) variables

\[ R_0 \]
\[ 2A \]

\[ 2A_1 \]
\[ A_1(s) \quad \rightarrow \quad \bullet \]
\[ B_1(s) \quad \leftarrow \quad \bullet \]

\[ 2A_2 \]
\[ A_2(s) \quad \rightarrow \quad \bullet \]
\[ B_2(s) \quad \leftarrow \quad \bullet \]

The resulting network functions refer to the circuit defined by the element with all ports terminated into the reference impedance

\[ B_p = V_p - A_p, \quad p = 1, 2, ... \]
Scattering network functions

\[
\begin{bmatrix}
    B_1 \\
    B_2
\end{bmatrix} =
\begin{bmatrix}
    S_{11}(s) & S_{12}(s) \\
    S_{21}(s) & S_{22}(s)
\end{bmatrix}
\begin{bmatrix}
    A_1 \\
    A_2
\end{bmatrix}
\]

Scattering matrix

Real Rational Functions

\[
S_y(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{b_0 + b_1 s + b_2 s^2 + \cdots + b_n s^n}
\]

- \( S_y(s) \) are network functions
- For lumped multiports: real rational functions
- The poles of the scattering functions are the poles of the multiport element terminated by the reference impedances
S matrix of a 2 terminal element

\[ B = V - A = 2A \frac{Z}{Z + R_0} - A = \frac{Z - R_0}{Z + R_0} A \]

S of a shunt capacitor

\[
S(s) = \begin{bmatrix}
-\frac{sCR_0}{2 + sCR_0} & \frac{2}{2 + sCR_0} \\
\frac{2}{2 + sCR_0} & -\frac{sCR_0}{2 + sCR_0}
\end{bmatrix}
\]
Physical interpretation of wave variables

Wave variables represent traveling waves on ideal matched transmission lines.
S matrix from Z matrix

\[ S(s) = \left( Z(s) - R_0 1 \right) \left( Z(s) + R_0 1 \right)^{-1} \]

A PCB example

One-port structure, characterized using a full-wave field solver

- Board size: 25cm X 25cm
- 2 Metal layers thickness: 0.03 mm
- Relative permittivity: 4
- Tandelta = 0.005
- Dielectric thickness: 0.2 mm

Nominal capacitance

\[ C = \frac{\varepsilon_r \varepsilon_0 A}{d} \approx 11.0675 \text{nF} \]

Impedance is infinite at DC!

\[ Z(j\omega) \approx -j \frac{1}{\omega C} \]
A PCB example

Impedance matrix entries

Frequency [GHz]

Impedance matrix entries, magnitude [dB]

Impedance matrix entries, phase (degrees)
A PCB example

Board size: 25cm X 25cm
2 Metal layers thickness: 0.03 mm
Relative permittivity: 4
Tandelta = 0.005
Dielectric thickness: 0.2 mm

Nominal capacitance

\[ C = \frac{\varepsilon_r \varepsilon_0 A}{d} \approx 11.0675 \text{nF} \]

\[ Z(j\omega) \approx -j\frac{1}{\omega C} \]
A PCB example

Observations

- Analytic models are excellent within their range of validity
- Analytic models cannot reproduce with good accuracy the broadband response of complex structures
- Impedance/admittance representations may be ill-defined
- Scattering representations are always well defined and normalized
- As far as the “ports” are well-defined, any electromagnetic structure can be described via S-parameters
More wave variables

Voltage waves

\[ A = \frac{1}{2} (V + R_0 I) \]
\[ B = \frac{1}{2} (V - R_0 I) \]

Power waves

\[ A = \frac{1}{2 R_0} (V + R_0 I) \]
\[ B = \frac{1}{2 R_0} (V - R_0 I) \]

Current waves

\[ A = \frac{1}{2} (V + R_0 I) \]
\[ B = \frac{1}{2} (V - R_0 I) \]

Energy

Generalized Average Power entering port

\[ P = \Re \left\{ \frac{1}{2} V(s) I^*(s) \right\} = \frac{1}{2} |A|^2 - \frac{1}{2} |B|^2 \]

Power waves only
Why S-parameters

- S-parameters are always defined
- Impedance or admittance may not
- S-parameters are normalized
  - Good numerical properties in simulation
- S-parameters are easily measured
  - Even at very high frequency, good reliability
- Standard format for S-parameters
  - Touchstone files from measurement hardware
  - All field solvers provide S-parameters on output
- Tabulated frequency data
  - Intrinsic IP protection for vendors
  - Do not disclose design details, but only I/O electrical properties
- Best way to represent broadband EM/circuit interactions
  - The essence of Signal and Power Integrity
Examples

Via array
12 ports

Wiring harness
8 ports

High-speed channel
18 ports

Insertion loss (signal transmission)
Return loss (signal reflection)

Near-end Crosstalk
Far-end crosstalk

An alternative: Moder Order Reduction
Spatial discretization of Maxwell equations
(FDTD, FEM, MoM, PEEC, ...)

\[ H(s) \approx H_q(s) \]
The objective

Terminations
Nonlinear
Complex

Interconnect network
Signal and Power
Linear
Many ports
Complex geometry
Electrically large

Terminations
Nonlinear
Complex

Linear case: frequency-domain solution

\[ V(j\omega_k) = V_S(j\omega_k) - Z(j\omega_k)I(j\omega_k) \]

\[ B(j\omega_k) = S(j\omega_k)A(j\omega_k) \]

\[ A(j\omega_k) = \frac{1}{2} \left( Z_p^{1/2}V(j\omega_k) + Z_R^{1/2}I(j\omega_k) \right) \]

\[ B(j\omega_k) = \frac{1}{2} \left( Z_p^{1/2}V(j\omega_k) - Z_R^{1/2}I(j\omega_k) \right) \]
Linear case: frequency-domain solution

\[
\begin{pmatrix}
1 \\
Z_R^{-1/2} - S(j\omega_k)Z_R^{-1/2} \\
-Z_R^{1/2} - S(j\omega_k)Z_R^{1/2}
\end{pmatrix}
\begin{pmatrix}
V(j\omega_k) \\
V(j\omega_k) \\
I(j\omega_k)
\end{pmatrix}
= \begin{pmatrix}
V_S(j\omega_k) \\
0
\end{pmatrix}
\]

Far end voltages

25-Ohm drivers
1pF receivers
Channel 1: active
Channels 2-9: quiet

Nonlinear terminations

Interconnect network
Signal and Power
Linear
Many ports
Complex geometry
Electrically large
Nonlinear terminations

Termination
Nonlinear
Dynamic
Time-varying

Interconnect
Linear
One port

\[ f(v, i; \frac{dv}{dt}; t) = 0 \]

\[ B(j\omega_k) = S(j\omega_k)A(j\omega_k) \]

Inverse Fourier/Laplace transform

\[ b(t) = h(t) * a(t) \]

Impulse response

\[ a(t) = \delta(t) \]

\[ b(t) = h(t) \]

\[ B(j\omega_k) = S(j\omega_k)A(j\omega_k) \]

Inverse Fourier/Laplace transform

\[ b(t) = h(t) * a(t) \]
Finite-width impulse response

\[ a(t) = \delta_{\Delta}(t) \approx \delta(t) \]

\[ b(t) = h_{\Delta}(t) \approx h(t) \]

Discretizing convolution

\[ b(t) = h(t) * a(t) \]
\[ = \int_{0}^{t} h(t - \tau)a(\tau)d\tau \]

\[ b(t_k) = (h * a)(t_k) \]
\[ = \int_{0}^{t_k} h(t_k - \tau)a(\tau)d\tau \]
\[ \approx \sum_{m=0}^{k-1} a(t_m)\Delta h_{\Delta}(t_k - t_m) \]
Discretizing convolution

\[ b(t_k) \approx \sum_{m=0}^{k-1} a(t_m) \Delta h(t_k - t_m) \]

Memory: Number of non-vanishing time-samples in the impulse response

An example: CPU-I/O channel
System-level simulation via convolution

Termination
Nonlinear
Dynamic
Time-varying

Interconnect
Linear
One port

\[ f(v_i; \frac{d}{dt}; t) = 0 \]

\[ b(t_k) \approx \sum_{m=0}^{k-1} a(t_m) \Delta h_m (t_k - t_m) \]

(e.g., backward Euler)

\[ \frac{dv}{dt} \bigg|_{t=t_k} \approx \frac{v(t_k) - v(t_{k-1})}{\Delta} \]

Need nonlinear solver

Use many past samples

May be very slow due to long memory in convolution

Very robust (when a good impulse response is available…)
Finding impulse responses

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega)e^{j\omega t} d\omega \]

Requires knowledge of \( S \)
- on a continuous frequency axis
- over an infinite bandwidth

Only a finite number of samples is available, over a finite bandwidth

\[ B(j\omega_k) = S(j\omega_k)A(j\omega_k) \]

Inverse Fourier/Laplace transform

\[ b(t) = h(t) * a(t) \]

**Inverse Fourier transform**

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega)e^{j\omega t} d\omega \]

\[ s \leftrightarrow j\omega \]

**Inverse Laplace transform**

\[ h(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} S(s)e^{st} ds \]

Complex s-plane

\[ \omega = \text{Im} s \]

\[ \omega_{\text{max}} \]

\[ \omega_s \]

DC

\[ \sigma \]

\[ \sigma_0 \]

\[ \text{Res} \]
Finding impulse responses

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(j\omega)e^{j\omega t} d\omega \]

Strategy 1:
Discrete Fourier Transform
(Fast Fourier Transform, FFT)

\[ \omega_m = \frac{2\pi m}{N\Delta} \]
\[ h(k\Delta) = \frac{1}{N} \sum_{m=0}^{N-1} S(j\omega_m) \exp\left(\frac{j2\pi m k}{N}\right) \]

Strategy 2:
Fit a parametric model allowing analytic Fourier/Laplace inversion

\[ S(j\omega) \approx \sum_{n=1}^{N} \frac{R_n}{j\omega - p_n} + S_\infty \]
\[ h(t) \approx \sum_{n=1}^{N} R_n \exp(p_n t) u(t) + S_\infty \delta(t) \]

Finding impulse responses via FFT

\[ h(k\Delta) = \frac{1}{N} \sum_{m=0}^{N-1} S(j\omega_m) \exp\left(\frac{j2\pi m k}{N}\right) \]

<table>
<thead>
<tr>
<th>Time</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampled</td>
<td>Periodic</td>
</tr>
<tr>
<td>Periodic</td>
<td>Sampled</td>
</tr>
</tbody>
</table>

Must be zero

\[ h(t) \]
\[ S(j\omega) \]
Finding impulse responses via FFT

FFT

IFFT
Finding impulse responses via fitting

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(j\omega)e^{j\omega t} d\omega \]

Parametric model fitting frequency samples

\[ S(j\omega) \approx \sum_{n=1}^{N} \frac{R_n}{j\omega - p_n} + S_\infty \]

Analytic inversion of Laplace transform

\[ h(t) \approx \sum_{n=1}^{N} R_n \exp(p_n t)u(t) + S_\infty \delta(t) \]

Rational fitting: why?

Circuit solvers understand circuits

Any lumped circuit has rational frequency responses (poles-residues, poles-zeros, ratio of polynomials)

\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty \]

Impedance, admittance, scattering
Rational fitting: why?

Circuit solvers understand circuits

Any lumped circuit has rational frequency responses (poles-residues, poles-zeros, ratio of polynomials)

$$ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty $$

Extraction of an equivalent circuit is an inverse problem (two-step)

Rational curve fitting

Model \( H(j\omega_k) \approx \hat{H}(j\omega_k) = \hat{H}_k \quad k = 1, \cdots, K \) Input data

$$ H(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_N s^N}{b_0 + b_1 s + b_2 s^2 + \cdots + b_N s^N} $$

3 alternative rational forms

$$ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + H_\infty $$

$$ H(s) = H_\infty \frac{(s - z_1)(s - z_2)\cdots(s - z_N)}{(s - p_1)(s - p_2)\cdots(s - p_N)} $$
Rational curve fitting: using polynomials

\[ \hat{H}(s) \approx \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_N s^N}{b_0 + b_1 s + b_2 s^2 + \cdots + b_N s^N} \]

Problem: nonlinear fitting equations

\[
\min \left\| \hat{H}(s) - \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_N s^N}{b_0 + b_1 s + b_2 s^2 + \cdots + b_N s^N} \right\|
\]

\[ s = j \omega_k, \quad k = 1, \ldots, K \]

Solution: linearization via weighting

\[
\left[ b_0 + b_1 s + b_2 s^2 + \cdots + b_N s^N \right] \hat{H}(s) \approx \left[ a_0 + a_1 s + a_2 s^2 + \cdots + a_N s^N \right]
\]
Rational curve fitting: using polynomials

\[
\left[ b_0 + b_1s + b_2s^2 + \cdots + b_Ns^N \right] \hat{H}(s) \approx \left[ a_0 + a_1s + a_2s^2 + \cdots + a_Ns^N \right]
\]

Problem: severe ill-conditioning (Vandermonde matrices)

\[
\begin{bmatrix}
\hat{H}_1 & \hat{H}_1s_1 & \cdots & \hat{H}_1s_1^N \\
\hat{H}_2 & \hat{H}_2s_2 & \cdots & \hat{H}_2s_2^N \\
\vdots & \vdots & \ddots & \vdots \\
\hat{H}_K & \hat{H}_Ks_K & \cdots & \hat{H}_Ks_K^N \\
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
\vdots \\
b_N \\
\end{bmatrix}
\approx
\begin{bmatrix}
1 & s_1 & \cdots & s_1^N \\
1 & s_2 & \cdots & s_2^N \\
\vdots & \vdots & \ddots & \vdots \\
1 & s_K & \cdots & s_K^N \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_N \\
\end{bmatrix}
\]

Rational curve fitting: generalization

\[
\hat{H}(s) \approx \frac{a_0 + a_1s + a_2s^2 + \cdots + a_Ns^N}{b_0 + b_1s + b_2s^2 + \cdots + b_Ns^N}
\]

“basis functions”

\[
\varphi_n(s) = s^n
\]

\[
\hat{H}(s) \approx \frac{\sum_{n=0}^{N} a_n \varphi_n(s)}{\sum_{n=0}^{N} b_n \varphi_n(s)}
\]

\[\varphi_n(s) \text{ rational } \Rightarrow \hat{H}(s) \text{ rational}\]
Vector Fitting

\[ \hat{H}(s) \approx \sum_{n=0}^{N} a_n \varphi_n(s) \]

Partial fractions

\[ \varphi_0(s) = 1 \]
\[ \varphi_n(s) = \frac{1}{s-q_n} \]

\[ \{q_n\} \text{ “starting poles”} \]

Arbitrary choice!

\[ \hat{H}(s) \approx \frac{a_0 + \sum_{n=1}^{N} \frac{a_n}{s-q_n}}{b_0 + \sum_{n=1}^{N} \frac{b_n}{s-q_n}} \]

Linearized (weighted) system: no more ill-conditioning!

\[ \left[ b_0 + \sum_{n=1}^{N} \frac{b_n}{s-q_n} \right] \hat{H}(s) \approx a_0 + \sum_{n=1}^{N} \frac{a_n}{s-q_n} \]

The VF “weight function”

\[ w(s) = b_0 + \sum_{n=1}^{N} \frac{b_n}{s-q_n} \]
Vector Fitting

\[
\begin{bmatrix}
\hat{H}_1 \varphi_0(s_1) & \cdots & \hat{H}_1 \varphi_N(s_1) \\
\hat{H}_2 \varphi_0(s_2) & \cdots & \hat{H}_2 \varphi_N(s_2) \\
\vdots & \ddots & \vdots \\
\hat{H}_K \varphi_0(s_K) & \cdots & \hat{H}_K \varphi_N(s_K)
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
\vdots \\
b_N
\end{bmatrix}
= \begin{bmatrix}
\varphi_0(s_1) & \cdots & \varphi_N(s_1) \\
\varphi_0(s_2) & \cdots & \varphi_N(s_2) \\
\vdots & \ddots & \vdots \\
\varphi_0(s_K) & \cdots & \varphi_N(s_K)
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_N
\end{bmatrix}
\]

\[
H = \text{diag}([\hat{H}_1 \cdots \hat{H}_K])
\]

\[
[\Phi - H\Phi] \begin{bmatrix} a \\ b \end{bmatrix} = 0 + \text{"nontriviality constraint"}
\]

Vector Fitting

\[
w(s) = b_0 + \sum_{n=1}^{N} \frac{b_n}{s - q_n}
\]

\[
w(s) = D_w + C_w (sI - Q_w)^{-1} B_w \quad \leftrightarrow \quad \begin{cases}
x = Q_w x + B_w u \\
y = C_w x + D_w u
\end{cases}
\]

\[
Q_w = \begin{bmatrix} q_1 & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & q_N \end{bmatrix} \quad B_w = \begin{bmatrix} 1 \\
\vdots \\
1 \end{bmatrix}
\]

\[
C_w = \begin{bmatrix} b_1 & \cdots & b_N \end{bmatrix} \quad D_w = b_0
\]
Vector Fitting

\[ w(s) = b_0 + \sum_{n=1}^{N} b_n \frac{s-q_n'}{s-q_n} = \frac{b_0(s-q_1')(s-q_2') \cdots (s-q_{N}')}{(s-q_1)(s-q_2) \cdots (s-q_N)} \]

zeros of VF
weight function
\[ \{q'_n\} = eig \left\{ Q_w - B_w D_w^{-1} C_w \right\} \]

"Pole relocation" process
\[ \{q_n\} \rightarrow \{q'_n\} \rightarrow \cdots \rightarrow \{p_n\} \quad "true ~poles" \]
\[ w(s) \rightarrow \text{constant} \]

Vector Fitting

Poles are known: compute residues
\[ \hat{H}(s) \approx \sum_{n=1}^{N} \frac{R_n}{s-p_n} + H_{\infty} = \sum_{n=1}^{N} R_n \phi_n(s) + H_{\infty} \]

Another linear least-squares system…
State-space realizations

\[ H(s) = \sum_{n=1}^{N} \frac{R_n}{s - P_n} + H_x \]

\[ Y(s) = H(s)U(s) \]

\[ H(s) = D + C(sI - A)^{-1}B \]

\[ \begin{cases} 
    x = Ax + Bu \\
    y = Cx + Du 
\end{cases} \]

Any rational macromodel can be cast in state-space form

Example

\[ H(j\omega) = \frac{s + 0.001}{s + 100} \]

Non-rational smooth function
Example

\[ H(j\omega) \]

1 pole

\[ 1 \text{ pole} \]

2 poles

\[ 2 \text{ poles} \]
Example

\[ H(j\omega) \]

- Real, Exact
- Imag, Exact
- Real, Approx
- Imag, Approx

3 poles

4 poles
Example

\[ H(j\omega) \]

- Real, Exact
- Imag, Exact
- Real, Approx
- Imag, Approx

5 poles

Stripline + launches

Data: measured S-parameters

Scattering matrix entries, magnitude

Scattering matrix entries, phase

S(1,1), data
S(1,1), model
S(2,1), data
S(2,1), model
Stripline + launches

Macromodel: 60 poles

Scattering matrix entries, magnitude

Scattering matrix entries, phase

S(1,1), data
S(1,1), model
S(2,1), data
S(2,1), model

Frequency [GHz]

Scattering matrix entries, magnitude

Frequency [GHz]

Scattering matrix entries, phase (degrees)

LGA via field (20 ports)
High-speed connector (18 ports)

Scattering matrix entries, magnitude

Scattering matrix entries, phase (degrees)

Frequency [GHz]

S(1,1), data
S(1,1), model
S(1,2), data
S(1,2), model

Frequency [MHz]

S(1,1), data
S(1,1), model
S(3,1), data
S(3,1), model

High-speed connector, measured
Advanced VF formulations

- Time-domain Vector Fitting
  - Processes time samples instead of frequency samples
- Orthonormal Vector Fitting
  - Further improvement in matrix conditioning using orthonormal rational functions
- Z-domain (orthonormal) Vector Fitting
  - Works on discrete-time/frequency systems
- Fast Vector Fitting
  - Uses smart QR decomposition (compressions) for systems with many ports
- Eigenvalue-based Vector Fitting
  - Possibly with relative error minimization, for improved robustness
- Multivariate/Parameterized Vector Fitting
  - Allows closed-form inclusion of geometry-material parameters in the macromodel equations
- Delayed Vector Fitting
  - Uses modified basis functions for representing propagation delays in closed form
- More…

Passivity: why?

![Transient voltage graph](image)

- Non-passive model
- Passive model

Model

Passive loads
Passivity conditions (scattering)

1. \( S(\omega) = S^*(\omega) \)
   
   Guarantees real-valued impulse response.
   Always assumed by construction

2. \( \|S(\omega)\| \leq 1 \) or \( \max_i \sigma_i(S(\omega)) \leq 1 \)
   
   Energy condition: structure must not amplify signals.
   Sometimes called simply "passivity" condition

3. \( S(\omega) \) is causal
   
   No anticipatory behavior in time-domain.
   Note: causality is a prerequisite for passivity!

Passivity: a ping-pong match

Model: \( B = S^*A \)
Load: \( A = P^*B \)

The poor man’s illustration of passivity: iterate through signal reflections…

- Start with \( B=0 \) and \( A_0=1 \)
- Model hits signal: \( B_0 = S^*A_0 \)
- Load hits signal: \( A_1 = P^*B_0 = (P^*S)^*A_0 \)
- Model hits signal: \( B_1 = S^*A_1 = S^*P^*S^*A_0 \)
- Load hits signal: \( A_2 = P^*B_1 = (P^*S)^2*A_0 \)
- ... 
- And the winner is… \( A_N = (P^*S)^N A_0 \)
Passivity: a ping-pong match

Model: \[ B = S \cdot A \]
Load: \[ A = P \cdot B \]

One-port case

\[ A^N = (PS)^N A_0 \]

| \[ P \] < 1, | \[ S \] | < 1 \[ \Rightarrow \] \[ A^N \] remains bounded
| \[ P \] = 1, | \[ S \] | > 1 \[ \Rightarrow \] \[ A^N \] \[ \xrightarrow{N \to \infty} \] \[ \infty \] Blow-up!

\[ P \] is a reflection coefficient: for a passive load it does not exceed 1

Passivity requires that \[ |S(j\omega)| \leq 1 \] for all frequencies!

(not just the modeling bandwidth... all means really all, from 0 to \( \infty \))
Passivity: what?

\[
\begin{align*}
A_1(s) & \rightarrow A_2(s) \\
B_1(s) & \leftarrow B_2(s)
\end{align*}
\]

In case of matrices, math is more complicated…

… but visualization is simple and straightforward

Passivity constraints (scattering macromodels)

\[ \text{S}(s) \text{ is passive } \iff \{ \text{singular values of } \text{S}(j\omega) \} \leq 1, \forall \omega \]
Not all S-parameter models should be passive

Small-signal characterization of a FET-based amplifier

A passive interconnect model

All curves are below 1
Checking passivity (scattering)

\[ \{\text{singular values of } S(j\omega)\} \leq 1, \quad \forall \omega \]

Several techniques can be used

**Frequency sweep test**: most straightforward
- Choose a set of frequency samples
- Compute $S$ and its singular values, and check
  - Time-consuming for large models
  - May give wrong answers due to poor sampling

Equivalent purely algebraic conditions:
- Linear Matrix Inequalities (LMI)
- Algebraic Riccati Equations (ARE)
- Eigenvalues of Hamiltonian matrices
Passivity conditions (scattering)

\[
S(s) = D + C(sI - A)^{-1}B
\]

\[
\{\text{singular values of } S(j\omega) \leq 1, \ \forall \omega\}
\]

Checking passivity

\[
\{\text{singular values of } S(j\omega) \leq 1, \ \forall \omega\}
\]

Linear Matrix Inequality (LMI)

\[
\begin{pmatrix}
A^TP + PA + C^TC & PB + C^TD \\
B^TP + D^TC & D^TD - I
\end{pmatrix} \leq 0 \quad P = P^T, \ P > 0
\]

Real matrix \( P \) is the variable
Checking passivity

\{\text{singular values of } S(j\omega)\} \leq 1, \quad \forall \omega

Algebraic Riccati Equation (ARE)

\[ A^T P + PA + C^T C + \left( PB + C^T D \right) \left( I - D^T D \right)^{-1} \left( PB + C^T D \right)^T = 0 \]

\[ \text{P} = \text{P}^T \]

Real matrix \( P \) is the variable

Checking passivity

\{\text{singular values of } S(j\omega)\} \leq 1, \quad \forall \omega

Eigenvalues of Hamiltonian matrix

\[ M = \begin{pmatrix}
A - B \left( D^T D - I \right)^{-1} D^T C & -B \left( D^T D - I \right)^{-1} B^T \\
C^T \left( D D^T - I \right)^{-1} C & -A^T + C^T D \left( D^T D - I \right)^{-1} B^T
\end{pmatrix} \]

Real matrix \( M \) must have no imaginary eigenvalues
Theorem

\( j\omega_0 \) is an eigenvalue of \( M \) \( \Leftrightarrow \sigma = 1 \) is a singular value of \( S(j\omega_0) \)
Passivity enforcement

- Generate a **new passive macromodel**
- Apply **small correction** to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\quad \Rightarrow \quad \begin{align*}
\dot{x} &= Ax + Bu \\
y &= (C + \Delta C)x + Du
\end{align*}
\]

\[\Delta S = \Delta C(sI - A)^{-1}B\]

---

Passivity enforcement

Accuracy control

\[
\min \| \Delta S \|
\]

+ 

Passivity constraints

- LMI (Bounded/Positive Real Lemma)
- Perturbation of residues (QUAD, SOC,…)
- Perturbation of Hamiltonian eigenvalues (HAM)
Computational effort

![Graph showing computational effort for LMI, HAM, and SOC methods]

- **LMI**: $O(n^4) - O(n^6)$
- **SOC**: $O(np^2)$
- **HAM**: $O(np^2)$

Perturbation of Hamiltonian eigenvalues

Singular values of $H$

**HAM**:\[
\begin{align*}
\min \|x\|_2 \\
Qx = b
\end{align*}\]

- Few constraints
- Equality: fast solution
- Ham eigs: reliable
Perturbation of Residues

Singular values of $H$

$\sigma$

$1$

$0$

$0$

$\omega_1$

$\omega_2$

$\omega$

SOC:

\[
\begin{cases}
\min \| x \|_2 \\
R x \leq g
\end{cases}
\]

- Simple constraints
- Convex: reliable
- Fast solver available

Preserve accuracy of macromodel

Induced perturbation in the impulse responses

$\sum \int_0^\infty (\tilde{h}_{i,j}(t) - h_{i,j}(t))^2 dt = \| \Delta C \ W \Delta C^T \|_F^2$

Weighted norm of state matrix perturbation

$W$: controllability Gramian

$AW + WA^T = -BB^T$
Macromodel implementation

1. Synthesize an equivalent circuit in SPICE format
   No access to SPICE kernel
   Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   Laplace element, most efficient
3. Other languages for mixed-signal analyses
   Verilog-AMS, VHDL-AMS, …
   Equation-based

SPICE synthesis

Admittance representation
One-port, one-pole

\[ \begin{align*}
    \dot{x} &= a \cdot x + b \cdot v \\
    i &= c \cdot x + d \cdot v
\end{align*} \]
SPICE synthesis

Scattering representation
One-port, one-pole
\[ u = G_0 v + i, \quad y = G_0 v - i \]

\[ \begin{align*}
  &i \\
  &v \\
 \end{align*} \quad \begin{array}{ccc}
  &G_0 & c \cdot x & d \cdot u \\
  &\downarrow & \downarrow & \downarrow \\
  &v & b \cdot u & \downarrow \\
 \end{array} \quad \begin{align*}
  &-a^{-1} \\
  &\downarrow \\
  &x \\
 \end{align*} \]

Admittance representation
One-port, two-poles (complex)
\[ p_{1,2} = \alpha \pm j\beta \]

\[ \begin{align*}
  &i \\
  &v \\
 \end{align*} \quad \begin{array}{ccc}
  &\downarrow & \downarrow & \downarrow \\
  &b_1 \cdot v & c_1 \cdot x_1 & c_2 \cdot x_2 \\
 \end{array} \quad \begin{align*}
  &d \cdot v \\
  &\downarrow \\
  &\beta \cdot x_2 \\
 \end{align*} \quad \begin{array}{ccc}
  &\downarrow & \downarrow & \downarrow \\
  &b_2 \cdot v & 1 & \downarrow \\
 \end{array} \quad \begin{align*}
  &\beta \cdot x_1 \\
  &\downarrow \\
 \end{align*} \]
SPICE synthesis

Admittance representation

General state-space synthesis

\[ \begin{align*}
\dot{x} &= Ax + Bv \\
i &= Cx + Dv
\end{align*} \]

Recursive convolution

\[ S(s) = D + \sum_n \frac{R_n}{s - p_n} \quad h(t) = D\delta(t) + \sum_n R_n e^{p_n t} u(t) \]

Interconnect network
Signal and Power
Linear
Many ports
Complex geometry
Electrically large

\[ b(t) = Da(t) + \sum_n R_n \int_0^t e^{p_n (t-\tau)} a(\tau) d\tau = Da(t) + \sum_n R_n \tilde{b}_n(t) \]
Recursive convolution

\[ \tilde{b}(t_k) = \int_{0}^{t_k} e^{p(t_k - \tau)} a(\tau) d\tau \]

Discrete time

\[ t_k = t_{k-1} + \Delta \]

\[ = \int_{0}^{t_{k-1}} e^{p(t_k - \tau)} a(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k - \tau)} a(\tau) d\tau \]

\[ = e^{p\Delta} \int_{0}^{t_{k-1}} e^{p(t_k - \tau)} a(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k - \tau)} a(\tau) d\tau \]

\[ \approx e^{p\Delta} \tilde{b}(t_{k-1}) + \frac{1 - e^{p\Delta}}{p} a(t_k) \]

Requires only one sample in the past!

Convolution, equivalent circuit, or recursive convolution?

Benchmark: board with 13 ports (11 Signal+2 Power)

<table>
<thead>
<tr>
<th></th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard convolution</td>
<td>389 s</td>
</tr>
<tr>
<td>Equivalent circuit</td>
<td>180 s</td>
</tr>
<tr>
<td>Recursive convolution</td>
<td>5.8 s</td>
</tr>
</tbody>
</table>
Macromodeling of long channels

Connectors, Via fields, ...

TL segments

Models for lumped 3D interconnects

Connectors

Via fields

Lumped (rational) macromodels

\[ H(s) = \sum_{n} \frac{R_n}{s - p_n} \]

Broadband: possibly many circuit elements
Models for Transmission Lines

Transmission Line segments

Lumped macromodels
RLGC sections, Matrix Rational Approximations,…

Distributed delay-based macromodels
ToPLine, DEPACT, …

Building the global interconnect model

Fast simulation of long interconnects for Signal Integrity

Global interconnect model
- Complex (many elements)
- Slow in transient simulations

Terminal behavior is of interest only
Internal structure is unimportant
Modeling terminal behavior

Lumped model?
\[ P(s) = \sum_{n=1}^{\infty} P_n \]

T-Line model?
\[ V(z,s) + Z(s)I(z,s) = 0 \]
\[ \frac{dV(z,s)}{dz} = Y(s)V(z,s) \]

Long interconnects: lumped macromodeling

DATA
- 800 samples
- Bandwidth 0-40 GHz
- 2 ports interconnect
Long interconnects: lumped macromodeling

**DATA**
- 800 samples
- Bandwidth 0-40 GHz
- 2 ports interconnect

**MODEL**
- ~1 second to build the model
- 80 poles
- Error ~10^{-2}

Scattering matrix entries, magnitude

Scattering matrix entries, phase (degrees)
Long interconnects: lumped macromodeling

### DATA
- 5001 samples
- Bandwidth 0-40 GHz
- 2 ports LONG interconnect

### MODEL
- 5 min to build the model
- <1000 poles
- Error ~10^{-7}

Very inefficient model

![Scattering matrix entries, magnitude](image1)

![Scattering matrix entries, phase(degrees)](image2)

### A simple example

#### Ideal T-line

\[ Z_0 \cdot T \]

\[ S_{21}(s) = \frac{A e^{-sT}}{1 - \Gamma^2 e^{-2sT}} \]

\[ \Gamma = \frac{Z_0 - R_0}{Z_0 + R_0} \]

\[ = A e^{-sT} \left( 1 + \Gamma^2 e^{-2sT} + \Gamma^4 e^{-4sT} + \cdots \right) \]

\[ = Q_1(s) e^{-sT} + Q_3(s) e^{-3sT} + Q_5(s) e^{-5sT} + \cdots \]
Delayed Rational Approximation

\[ S_{ij}(s) \approx \sum_{m=1}^{M} Q_{ij}^{(m)}(s) e^{-s T_m} \]

\[ Q_{ij}^{(m)}(s) = \sum_{n} \frac{R_{ij,n}^{(m)}}{s - P_{ij,n}^{(m)}} + R_{ij,\infty}^{(m)} \]

DVF: Delayed Vector Fitting (scalar)

\[ H(s) \approx \sum_{m} Q_{m}(s) e^{-s T_m} \]  

\[ Q_{m}(s) = \sum_{n} \frac{R_{n}^{m}}{s - p_{n}} + R_{m,\infty} = \frac{\sum_{n} w_{n}^{m}}{s - a_{n}} + w_{m,\infty} + \sum_{n} r_{n}^{m} + r_{\infty} \]

"starting poles"
DVF: Delayed Vector Fitting (scalar)

\[ H(s) \approx \sum_m \frac{W_m}{s-a_n} e^{-sT_m} \]

\[
\left( \sum_n \frac{r_n}{s-a_n} \right) H(s) = \sum_m \left( \sum_n \frac{W_n}{s-a_n} \right) e^{-sT_m} \approx 0
\]

Linear least squares solution for \( \omega_k \), \( k = 1, \ldots, K \)

Comparison of VF and DVF

\[ \text{VF} \quad \begin{bmatrix} \Phi & -H\Phi \end{bmatrix} \xi = 0 \]

\[ \text{DVF} \quad \begin{bmatrix} E_1 \Phi & \cdots & E_M \Phi & -H\Phi \end{bmatrix} \xi = 0 \]

\( H = \text{diag}[H(j\omega_1) \cdots H(j\omega_K)] \) \quad Input data

\( \Phi_{k,n} = \varphi_n(j\omega_k) \) \quad Rational basis functions

\( \xi = \text{unknowns: } r_n, W_n \)

\( E_m = \text{diag}[e^{-j\omega_k T_m} \cdots e^{-j\omega_K T_m}] \) \quad Delays
DVF: pole relocation

\[ Q_m(s) = \sum_n \frac{W_n^m}{s - a_n} + W_{m,\infty} \]
\[ = \sum_n \frac{R_n^m}{s - p_n} + R_{m,\infty} \]

compute the zeros

Repeat the process with the new starting poles...
...until convergence

State-space form of DRM

\[ H(s) = \left[ \sum_m C_m e^{-s\tau_m} \right] (sI - A)^{-1} B + D \]

residues+delays poles constant
input-state mapping

\[ H(s) = C(sI - A)^{-1} B + D \quad \text{(delayless macromodel)} \]
State-space form of DRM

\[ H(s) = \left( \sum_{m} C_m e^{-s \tau_m} \right) (sI - A)^{-1} B + D \]

- residues + delays
- poles
- constant
- input-state mapping

Passivity condition: \( \max \sigma \{ H(j \omega) \} \leq 1, \forall \omega \)

(all singular values must not exceed one at any frequency)

Passivity enforcement via perturbation

\[ H(s) = \left( \sum_{m} C_m e^{-s \tau_m} \right) (sI - A)^{-1} B + D \]

\[ C_m \leftarrow C_m + \partial C_m \]

\[ \text{model perturbation} \]

Accuracy constraint: \( \min \| x \| \)

Passivity constraint: \( \max \sigma \{ H(j \omega) \} \leq 1, \forall \omega \)
DVF validation example

Nominal propagation delays

\[ T_1 = \ell_1 \sqrt{C \ell_1} = 16.7\text{ns} \]
\[ T_2 = \ell_2 \sqrt{C \ell_2} = 23.4\text{ns} \]

All delays can be computed analytically

Frequency Dependent Line Parameters (MoM solver)

Bandwidth: 0-10 GHz

Using standard VF (no delays) a comparable model requires:

\[ M = 5, \quad N = 10 \]
\[ \text{RMS error} \approx 1\text{E-3} \]

N=1000 poles

\[ M = 5, \quad N = 10 \]
\[ \text{RMS error} \approx 1\text{E-3} \]

N=1000 poles
DVF validation example

SPICE transient simulations
Input: single pulse (10 ns width, 1 ns rise/fall time)

Time required for SPICE simulations
- VF 140 s
- VF 135 s
- DVF 3.44 s
- DVF 3.16 s
Speedup factor = 40X

Example - IBM GX bus

Using standard VF (no delays) a comparable model requires
- N=110 poles
- N=120 poles
Example - IBM GX bus

SPICE transient simulations
Input: single pulse (1 ns width, 100 ps rise/fall time)

Time required for SPICE simulations
- VF 12.44 s
- VF 9.16 s
- DVF 2.58 s
- DVF 1.84 s

Speedup factor = 5X

More on transient SPICE simulation...
A case study: coupled Signal/Power Integrity

- Input: CAD model of board
  - Thirteen Ports identified on Board
    - Ports 1 – 11 correspond to Signal Lines
    - Ports 12 and 13 on power planes
- Step 1: characterization
  - Use frequency-domain field solver to characterize
    - Signal/Power distribution network including couplings
    - 13x13 S-parameter matrix, frequency sweep from 1MHz to 5GHz
- Step 2: macromodeling
  - Fit macromodel to S-parameter data obtained in Step 1
    - Vector Fitting with passivity enforcement
    - Synthesis of SPICE-compatible equivalent circuit
- Step 3: Time-domain analysis via SPICE
  - Choose suitable termination schemes
  - Timing, crosstalk, Signal/Power coupling, eye diagrams, ...

This case study courtesy of
- Georgia Institute of Technology, Atlanta GA, USA
- E-System Design, Inc.
  - Provided field solver Sphinx
- Politecnico di Torino, Italy
- IdemWorks s.r.l.
  - Provided passive macromodeling tool IdEM
Board cross-section

Layers L2 and L3
Port locations – L3 (Ref: L2): ports 1,7; 2,3; 8,9

Port locations – L4 (Ref: L5): ports 10,11
Power ports – L2 (Ref: L5): ports 12,13

Macromodel vs S-parameters from solver
Macromodel vs S-parameters from solver
Macromodel vs S-parameters from solver

Coupling to PDN port

SPICE: excitation on signal lines
Response on a signal line, 700MHz

Response on a signal line, 1.3GHz
Coupling to power ports, 700MHz

Coupling to power ports, 1.3GHz
Xtalk and substrate coupling, 700MHz

Xtalk and substrate coupling, 1.3GHz
SPICE: excitation on PDN – core switching

Decoupling capacitors

- Cap 1 (C=1nF, R=10mOhm, L=51.65pH)
- Cap 3 (C=1nF, R=10mOhm, L=14.97pH)
- Cap 2 (C=1nF, R=10mOhm, L=51.65pH)
- Cap 4 (C=1nF, R=10mOhm, L=14.97pH)

Port 12
Port 13
PDN response

Port 13: With and Without Caps

Eye diagram simulation: setup 1
Eye diagram simulation: setup 2

Eye diagram simulation: setup 3
Eye diagram results, 1.4 Gb/s

No decoupling caps

With decoupling caps

Single active line

+ aggressors

+ core switching

Eye diagram results, 2.6 Gb/s

No decoupling caps

With decoupling caps

Single active line

+ aggressors

+ core switching
Conclusions of part I

- Electrical interconnects are still (and will) remain a challenge in system-level simulations
- Many approaches for characterization, modeling and simulation
- Full-wave solvers always required
- General consensus towards S-parameter characterization
- Two modeling approaches were discussed
  - Direct convolution via FFT-derived impulse responses
  - Passive macromodeling (SPICE synthesis or recursive convolution)
  - Geometry discretization + extraction + MOR not discussed here
- A lot of care needs to be taken for both approaches
  - Still very active research activities
  - See part II of this tutorial…