Fast Iterative Simulation of High-Speed Channels via Frequency-Dependent Over-Relaxation

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***Abstract*—This paper presents an optimized Waveform Relaxation solver for electrically-long high-speed channels terminated by nonlinear networks. The time-domain scattering operators of channel and terminations are cast as recursive convolutions and nonlinear discrete-time filters, respectively. A transverse and longitudinal decoupling is then applied to the channel operator, with the introduction of suitable relaxation sources, and solved iteratively until convergence. A frequency-dependent over-relaxation parameter is introduced in order to optimize the convergence rate. Numerical results show significantly reduced runtime and iteration count for critical benchmarks with respect to previous Waveform Relaxation formulations.**

I. Introduction

The qualification of high-speed electrical links requires extensive transient simulations. The usual objective of these simulations is the computation of eye diagrams for the assessment of intersymbol interference, jitter, and any other effects that may prevent safe transmission of digital signals from driver to receiver [1], [2]. A reliable and statistically meaningful eye diagram must be generated with a large number of test pseudo-random bit patterns. Such constraints make the direct transient analysis a very challenging task.

State-of-the-art methods for eye diagram generation can be subdivided in two main classes. Some techniques are based on linear convolution analysis and superposition, combined with statistical post-processing [1]–[3]. Execution speed is not critical for such methods, which are very efficient. However, some approximations are intrinsicallly performed due to the linear nature of the overall methodology, which does not allow accurate analysis of strongly nonlinear drivers and receivers. More precise techniques are based on a direct transient simulation of the terminated channel, performed with a time-domain solver. Methods based on SPICE subcircuits or macromodels for channel, drivers, and receivers fall in this class [4]. It is however well-known that SPICE is not optimal for this class of problems, and despite the superior accuracy that may be achieved, a very long runtime is usually observed even with moderate-length bit streams.

In [5], a new approach for fast channel simulation based on Waveform Relaxation (WR) was presented. This method is based on macromodels for drivers/receivers [6], and channel [7]. Each of these macromodels is cast as a linear delayed recursive convolution (for the channel part) or a nonlinear digital filter (for drivers and receivers). A two-level partitioning is used for decoupling each channel from its weakly-coupled neighbors, as well as from its termination network, through suitable relaxation sources. A fixed-point iteration is then established for solving the transient analysis within a Waveform Relaxation framework [8]–[13]. This method combines the advantage of SPICE-level accuracy with a fast runtime, as discussed in [5].

One major issue affects the latter methodology. Although convergence of the WR iteration can be assessed a priori based on the characteristics of channel and terminations, it may be possible that the convergence rate is too slow. In extreme cases, convergence may not hold, and the scheme will diverge. This problem motivated the extensions in [14], where an over-relaxation parameter was introduced in order to optimize convergence. Although some improvement was observed, there is still margin for several optimizations.

This work builds on previous formulations [5], [14] and constructs a new iterative scheme based on a frequency-dependent over-relaxation. The main novel contribution of this paper is the possibility of optimize separately and independently each spectral component of the solution, by maximizing the corresponding frequency-dependent convergence rate. The time-domain application of the corresponding frequency-dependent over-relaxation is achieved by an ad hoc frequency-weighted rational approximation, which allows to compute the over-relaxation sources through fast recursive convolutions. We demonstrate significantly reduced runtime and iteration count with respect to previous formulations on critical benchmarks.

II. Problem statement and background

We consider a P-port (P even) coupled high-speed channel $\mathcal{H}$, with each port terminated into individual drivers and/or receivers $\mathcal{F}_i$, $i = 1, \ldots, P$. The channel, which is known via tabulated scattering frequency samples available in a Touchstone file, is processed by a delay-rational approximation algorithm [7] and cast as a passive Delay-Rational Macromodel (DRM), with transfer matrix elements

$$H^{i,j}(s) = \sum_{m=0}^{M^{i,j}} \sum_{n=1}^{N_m^{i,j}} \frac{R_{mn}^{i,j}}{s - p_{mn}^{i,j}} e^{-s\tau_m^{i,j}} + D^{i,j}\,. \quad (1)$$

The corresponding impulse response matrix $\mathbf{h}(t)$ is a combination of delayed exponential terms, which can be cast as fast recursive convolutions in time-domain

$$\boldsymbol{b}(t) = \mathbf{h}(t) * \boldsymbol{a}(t)\,, \tag{2}$$

where $\boldsymbol{a}(t), \boldsymbol{b}(t)$ denote respectively incident and reflected transient scattering waves from the channel at its interface ports. The possibly nonlinear terminations are cast as nonlinear discrete-time macromodels using the Mπlog approach [6]. Using the same transient scattering waves as port variables, we can express driver and receiver characteristics in explicit form, as detailed in [5]. The resulting transient simulation problem requires the solution of system

$$\begin{cases} \boldsymbol{b} = \mathcal{H}\,\boldsymbol{a}\,, \\ \boldsymbol{a} = \mathcal{F}\left(\boldsymbol{b}\right)\,, \end{cases} \tag{3}$$

where the linear convolution operator $\mathcal{H}$ represents the channel, and the explicit nonlinear operator $\mathcal{F}$ is diagonal. The formulation of (3) assumes a discretized time axis $t_k = k\delta$, and represents with $\boldsymbol{a}, \boldsymbol{b}$ the complete time evolution of the scattering waves over the entire time interval of interest.

The direct solution of (3) would require the inversion of nonlinear operator $\mathcal{I} - \mathcal{H}\mathcal{F}(\cdot)$. The proposed Waveform Relaxation scheme is one possible method for performing this inversion. As discussed in [5], [14], we perform a two-level decoupling. First, the channel operator $\mathcal{H}$ is split into its (block-) diagonal part $\mathcal{D}$, including direct transmission and reflection coefficients of each independent channel, and remainder $\mathcal{C}$, including all coupling terms between adjacent channels. Practical structures are in fact designed in order to have small couplings, and operator $\mathcal{C}$ is thus expected to be small with respect to the dominant part $\mathcal{D}$. Second, we decouple each individual channel from its termination by keeping the two equations in (3) separate.

System (3) can be rewritten in a fully equivalent form

$$\begin{cases} \boldsymbol{b} &= \mathcal{D}\boldsymbol{a} + \boldsymbol{\theta} \\ \boldsymbol{a} &= \mathcal{F}\left(\boldsymbol{b}\right) + \boldsymbol{\varphi}\,, \end{cases} \tag{4}$$

where $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ are suitable decoupling sources, defined as

$$\begin{cases} \boldsymbol{\theta} &= (1-\eta)\left(\boldsymbol{b} - \mathcal{D}\boldsymbol{a}\right) + \eta\mathcal{C}\boldsymbol{a}\,, \\ \boldsymbol{\varphi} &= (1-\eta)\left(\boldsymbol{a} - \mathcal{F}\left(\boldsymbol{b}\right)\right)\,, \end{cases} \tag{5}$$

and η is an unknown parameter that will be determined later. It is straightforward to prove that (3) and (4)-(5) have the same solution.

A two-level iterative scheme with outer index μ and inner index ν is applied to solve (4)-(5). Denoting as $\boldsymbol{a}_{\mu,\nu}, \boldsymbol{b}_{\mu,\nu}$ the estimates of the solution at iteration μ, ν, we have

$$\begin{cases} \boldsymbol{b}_{\mu,\nu} &= \mathcal{D}\boldsymbol{a}_{\mu,\nu-1} + \boldsymbol{\theta}_{\mu-1} \\ \boldsymbol{a}_{\mu,\nu} &= \mathcal{F}\left(\boldsymbol{b}_{\mu,\nu}\right) + \boldsymbol{\varphi}_{\mu-1}\,, \end{cases} \tag{6}$$

which are evaluated alternatively for $\nu = 1, \ldots, \nu_{\max}$ (the inner loop). Upon stabilization of all waveforms $\boldsymbol{a}_\mu = \boldsymbol{a}_{\mu,\nu_{\max}}$, $\boldsymbol{b}_\mu = \boldsymbol{b}_{\mu,\nu_{\max}}$, new estimates for the relaxation sources are computed as

$$\begin{cases} \boldsymbol{\theta}_\mu &= (1-\eta)\left(\boldsymbol{b}_\mu - \mathcal{D}\boldsymbol{a}_\mu\right) + \eta\mathcal{C}\boldsymbol{a}_\mu\,, \\ \boldsymbol{\varphi}_\mu &= (1-\eta)\left(\boldsymbol{a}_\mu - \mathcal{F}\left(\boldsymbol{b}_\mu\right)\right)\,. \end{cases} \tag{7}$$

and the process is iterated for $\mu = 1, \ldots, \mu_{\max}$ (the outer loop). At startup, all waveforms are initialized with zeros.

The convergence of the above iterative scheme can be assessed in the frequency domain by assuming linear terminations with scattering matrix $\mathbf{\Gamma}(s)$. It can be shown [14] that the error of the solution estimate $\boldsymbol{A}_{\mathcal{K},\mathcal{I}}$ at outer iteration $\mathcal{K}$ when a constant number of inner iterations $\nu_{\max} = \mathcal{I}$ are performed can be characterized as

$$\boldsymbol{\mathcal{E}}_{\mathcal{K},\mathcal{I}} = \boldsymbol{A}_{\mathcal{K},\mathcal{I}} - \boldsymbol{A}_{\text{exact}} = -\eta \mathbf{P}^{\mathcal{K}}_{\mathcal{I},\eta}(s)\boldsymbol{A}_{\text{exact}}\,, \tag{8}$$

where $\boldsymbol{A}_{\text{exact}}$ is the exact solution in the frequency-domain and where

$$\mathbf{P}_{\mathcal{I},\eta}(s) = \mathbf{I} - \eta\left[\mathbf{I} - \left(\mathbf{\Gamma}(s)\mathbf{D}(s)\right)^{\mathcal{I}}\right]\left(\mathbf{I} - \mathbf{P}(s)\right)\,. \tag{9}$$

is a frequency-dependent iteration operator with $\mathbf{P}(s) = \left(\mathbf{I} - \mathbf{\Gamma}(s)\mathbf{D}(s)\right)^{-1}\left(\mathbf{\Gamma}(s)\mathbf{C}(s)\right)$. Convergence is guaranteed if the spectral radius (the maximum eigenvalue magnitude) of this operator is unitary bounded, $\rho_{\max}\{\mathbf{P}_{\mathcal{I},\eta}(s)\} < 1$ at any frequency $s = j\omega$.

Operator $\mathbf{P}_{\mathcal{I},\eta}$ is parameterized by η. It is thus possible to find an optimal constant value η_0 such that the spectral radius $\rho_{\max}$ is minimized and the convergence rate is optimal. It was shown in [14] that the optimal constant η_0 can be found as the solution of

$$\eta_0 = \arg\min_{\eta,\omega}\max_{q\in\{1,\ldots,P\}}\left|1 - \eta\lambda_q(\omega)\right|\,, \tag{10}$$

where $\lambda_q(\omega)$ denotes the q-th eigenvalue of matrix

$$\mathbf{\Lambda}(j\omega) = \left\{\left[\mathbf{I} - \left(\mathbf{\Gamma}(j\omega)\mathbf{D}(j\omega)\right)^{\mathcal{I}}\right]\left(\mathbf{I} - \mathbf{P}(j\omega)\right)\right\}\,. \tag{11}$$

The results in [14] show that the constant-η_0 over-relaxation scheme (denoted as WR-SOR, "Successive Over-Relaxation") is able to achieve or speedup convergence for critical industrial channels.

III. Frequency-Dependent Over-Relaxation

In this section, we outline our proposed strategy to further accelerate the convergence of the Waveform Relaxation scheme discussed in Sec. II. The key observation is that the determination of the optimal over-relaxation parameter η_0 in (10) considers all frequencies at the same time and with equal weight. Solution of (10) provides a single real number η_0 that minimizes the maximum of the frequency-dependent spectral radius $\rho_{\max}(s)$ among all frequencies. However, as Fig. 1 shows, the typical behavior of this parameter, henceforth denoted as $\rho_0(j\omega)$, as a function of frequency is characterized by several maxima and minima. A single constant η_0 can be tuned to reduce the value of a single local maximum, but we need to reduce as many peaks as possible, in order to achieve a uniformly small spectral radius. In other words, we have many independent constraints in (10), but only one degree of freedom, and the problem is severely overconstrained.

This motivates the following strategy of increasing the degrees of freedom by allowing the over-relaxation parameter to be frequency-dependent $\eta(s)$. In order to be able to use this frequency-dependent parameter within the proposed Waveform

Relaxation scheme, which operates on discrete-time waveforms, we need to perform a frequency-to-time conversion. Following the same approach that we used for the channel operator, we will assume a rational function form $\eta_R(s)$ as

$$\eta_R(s) = \eta_\infty + \sum_{n=1}^{N} \frac{r_n}{s - q_n}\,, \tag{12}$$

so that time-domain application results in a convolution with the corresponding impulse response

$$\xi(t) = \eta_\infty \delta(t) + \sum_{n=1}^{N} r_n e^{q_n t} u(t)\,. \tag{13}$$

The discretization of the convolution between $\xi(t)$ and any waveform $w(t)$ results in a recursive convolution or, equivalently, in a digital IIR filter, which will be described formally as operator $\mathcal{N}$. The proposed frequency-dependent over-relaxation scheme can thus be formulated as (6), which is unchanged, together with the modified relaxation source updates

$$\begin{cases} \boldsymbol{\theta}_\mu &= (1-\mathcal{N})\left(\boldsymbol{b}_\mu - \mathcal{D}\boldsymbol{a}_\mu\right) + \mathcal{N}\mathcal{C}\boldsymbol{a}_\mu\,, \\ \boldsymbol{\varphi}_\mu &= (1-\mathcal{N})\left(\boldsymbol{a}_\mu - \mathcal{F}\left(\boldsymbol{b}_\mu\right)\right)\,. \end{cases} \tag{14}$$

A. Optimal over-relaxation

We first consider the determination of the optimal frequency-dependent over-relaxation parameter $\eta_{\text{opt}}(j\omega)$ that minimizes the spectral radius $\rho_{\max}$ at each independent frequency. Therefore, we reformulate the optimization problem (10) in a simpler form

$$\eta_{\text{opt}}(j\omega) = \arg\min_{\eta} \max_{q\in\{1,\dots,P\}} \left|1 - \eta\lambda_q(\omega)\right|\,, \tag{15}$$

which needs to be solved repeatedly over a sufficiently fine grid of frequencies ω_k, $k = 1,\dots,K$. It follows that for a single frequency point ω_k, the problem (15) is equivalent to the epigraph form [16],

$$\begin{aligned} &\min_{t,\eta} t, \\ &\text{subject to } |1 - \eta\lambda_{q,k}|^2 < t, \quad q = 1,\dots,P\,. \end{aligned} \tag{16}$$

where $\lambda_{q,k} = \lambda_q(\omega_k)$. Hence, the original problem (15) can be transformed in a standard quadratically constrained linear problem (QCLP)

$$\begin{aligned} &\min_{\mathbf{x}} t, \\ &\text{subject to } \frac{1}{2}\mathbf{x}^T\mathbf{U}_q\mathbf{x} + \mathbf{v}_q^T\mathbf{x} + \frac{1}{2}, \quad q = 1,\dots,P\,. \end{aligned} \tag{17}$$

where

$$\mathbf{U}_q = \begin{bmatrix} |\lambda_{q,k}|^2 & 0 & 0 \\ 0 & |\lambda_{q,k}|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{v}_q = \begin{bmatrix} -\Re\{\lambda_{q,k}\} \\ \Im\{\lambda_{q,k}\} \\ -1/2 \end{bmatrix} \tag{18}$$

and where $\mathbf{x}$ collects all the optimization variables,

$$\mathbf{x}^T = \begin{bmatrix} \Re\{\eta\} & \Im\{\eta\} & t \end{bmatrix}. \tag{19}$$

Since both the objective function and the constraints are convex, the problem (17) is convex. For this reason its solution can be efficiently computed using the interior point method [17].

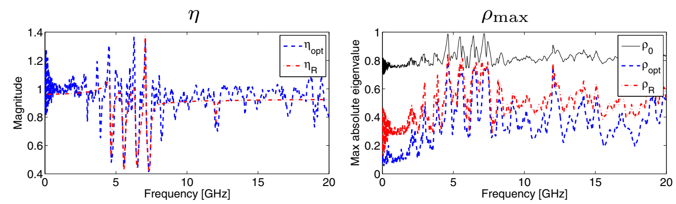

Fig. 1. Left: optimal over-relaxation parameter $\eta_{\text{opt}}(j\omega)$ and its rational approximation $\eta_R(j\omega)$. Right: frequency-dependent spectral radius for constant (ρ_0), optimal (ρ_{opt}), and rational (ρ_R) over-relaxation.

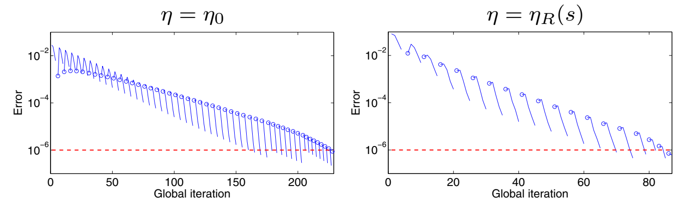

Fig. 2. Evolution of the inner (continuous line) and outer (dots) approximation errors through iterations for WR with constant over relaxation η_0 (left panel) and frequency-dependent over-relaxation $\eta_R(s)$ (right panel).

B. Suboptimal rational over-relaxation

Once the samples $\eta_{\text{opt}}(j\omega_k)$ have been determined, our aim is to fit a rational function $\eta_R(s)$ as in (10) such that

$$\sum_{k=1}^{K} \left\|\eta_R(j\omega_k) - \eta_{\text{opt}}(j\omega_k)\right\|_{\mathcal{W}}^2 \tag{20}$$

is minimized. At the same time, we would like to keep the number of poles of $\eta_R(s)$ as low as possible, in order to preserve efficiency in the application of the recursive convolution updates of the relaxation sources in (14). The specific choice of the norm $\|\cdot\|_{\mathcal{W}}$ is crucial to meet this objective.

We define a frequency-dependent weighting function

$$F(j\omega_k) = \left[\rho_{\text{opt}}(j\omega_k)\right]^{\alpha}\,, \tag{21}$$

where $\alpha \geq 0$ is a free parameter for tuning the fitting process. A Vector Fitting scheme is then applied using $F(j\omega_k)$ as a frequency-dependent weight, see [15] for details. This results in minimization

$$\sum_{k=1}^{K} F(j\omega_k)\left|\eta_R(j\omega_k) - \eta_{\text{opt}}(j\omega_k)\right|_2^2 \tag{22}$$

over the set of poles q_n, residues r_n, and direct coupling constant η_∞ in (12). We remark that if $\alpha = 0$ a standard Vector Fitting scheme is obtained. If α increases, a larger weight is applied to those frequencies corresponding to larger values of the spectral radius $\rho_{\text{opt}}(j\omega)$. Fitting accuracy will be larger at those frequencies, whereas a less aggressive accuracy will be achieved at less critical frequencies. In this work, we used $\alpha = 10$.

IV. Results

We report some numerical results for one critical benchmark. This case was already analyzed in [14], where it is

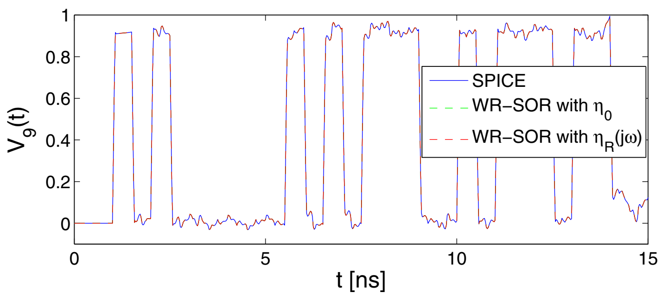

Fig. 3. Transient simulation: comparison between SPICE, constant and frequency-dependent WR-SOR schemes.

labeled as "case B". The physical structure is a 18-port channel connecting two boards through a LGA connector. As discussed in [14], a standard Waveform Relaxation scheme without over-relaxation (corresponding to $\eta = 1$) does not converge, since the associated iteration operator has a spectral radius that exceeds one at some frequencies.

Applying a constant over-relaxation η_0 is able to fix convergence. The right panel in Fig. 1 (black solid line) shows that the achieved spectral radius $\rho_0(j\omega)$ is very close to one, but the number of required WR iterations is exceedingly large (Fig. 2, left panel). The optimal frequency-dependent over-relaxation parameter $\eta_{\rm opt}(j\omega)$, depicted in Fig. 1 (left panel, blue solid line) leads to an optimal spectral radius $\rho_{\rm opt}(j\omega)$, depicted in the right panel of Fig. 1 (blue dashed line). We clearly see that $\rho_{\rm opt}(j\omega)$ is considerably less than $\rho_0(j\omega)$.

Application of the frequency-dependent rational fitting to $\eta_{\rm opt}(j\omega)$ leads to a rational function $\eta_R(s)$ with 30 poles, depicted with a red dashed line in Fig. 1 (left panel). Accuracy is only important where $\rho_{\rm opt}(j\omega)$ is largest, which occurs in this case at the local minima of $\eta_{\rm opt}(j\omega)$. The resulting spectral radius $\rho_R(j\omega)$ is only suboptimal (red line in right panel of Fig. 1), but significantly better than $\rho_0(j\omega)$. This is further demonstrated by the evolution of the error through iterations for the corresponding frequency-dependent WR-SOR scheme (Fig. 2, right panel), which converges below the prescribed threshold 10^{-6} in much less iterations. Figure 3 provides a validation of the constant and frequency-dependent WR-SOR schemes with a direct SPICE run. The results are consistent, as expected. SPICE required 1630 seconds to run a pseudo-random bit sequence of 1000 bits; the constant over-relaxed scheme took 673 seconds; the proposed frequency-dependent over-relaxed scheme took 159 seconds, with a speedup factor of more than $10\times$ with respect to SPICE.

In conclusion, this paper presented a systematic methodology for improving convergence of Waveform Relaxation schemes for the transient analysis of high-speed channels with linear or nonlinear terminations. The main novel contribution is the introduction of a frequency-dependent over-relaxation parameter, which is determined adaptively through an ad hoc weighted rational fitting process. The results show significant improvement in runtime with respect to previous implementations.

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