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₁ **Comparison of Stochastic Methods for the**
₂ **Variability Assessment of Technology**
₃ **Parameters**

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4 This paper provides and compares two alternative solutions for the sim-
5 ulation of cables and interconnects with the inclusion of the effects of param-
6 eter uncertainties, namely the Polynomial Chaos (PC) method and the Re-
7 sponse Surface Modeling (RSM). The problem formulation applies to the tele-
8 graphers equations with stochastic coefficients. According to PC, the solu-
9 tion requires an expansion of the unknown parameters in terms of orthog-
10 onal polynomials of random variables. On the contrary, RSM is based on a
11 least-square polynomial fitting of the system response. The proposed meth-
12 ods offer accuracy and improved efficiency in computing the parameter vari-
13 ability effects on system responses with respect to the conventional Monte
14 Carlo approach. These approaches are validated by means of the application
15 to the stochastic analysis of a commercial multiconductor flat cable. This anal-
16 ysis allows us to highlight the respective advantages and disadvantages of
17 the presented methods.

1. Introduction

18 The constant and rapid pace of technological innovation today has produced in-
19 creasingly complex electronic devices to such an extent that they are often physically
20 separated into several sub-devices and then connected together. The integrity of
21 signals propagating on interconnections is then a fundamental point for the smooth
22 functioning of the overall system. Cable bundles represent one of the most common
23 means by which modern electronic systems and subsystems are interconnected. A
24 large variety of examples exists, ranging from transportation vehicles (cars, aircrafts,
25 ships) to Information Technology equipment and to industrial plants. The electro-
26 magnetic interaction among closely spaced individual wires induces disturbances in
27 all other adjacent circuits. This crosstalk can cause functional degradation of the
28 circuits at the ends of the cable. The magnitude of the electromagnetic interference
29 varies significantly as a function of a number of factors including the wires geometries.

30 The sensitivity of crosstalk to random wires position in the cable has led to several
31 probabilistic models for the crosstalk according to the frequency ranges. Instead of
32 using brute-force Monte Carlo (MC) method, some alternative solutions based on
33 the derivation of pseudo-analytical expressions for the statistical parameters of the
34 responses of distributed systems have been proposed so far [*Shiran et al.*, 1993; *Bellan*
35 *et al.*, 2003]. However, their principal limitation is related to their scarce flexibility
36 and restriction to the particular structures and output variables for which they have
37 been derived. Possible complementary methods based on the optimal selection of
38 the subset of model parameters in the whole design space [*Zhang et al.*, 2001] have

39 also been proposed. However, these methods turn out to be extremely inefficient,
40 since they require a large set of simulations with different samples of the random
41 parameters and prevent us from applying them to the analysis of complex realistic
42 structures.

43 Recently, an effective solution based on the so-called polynomial chaos (PC) has
44 been proposed to overcome the previous limitations. This methodology is based
45 on the representation of the stochastic solution of a dynamical circuit in terms of
46 orthogonal polynomials. For a comprehensive and formal discussion of PC theory,
47 the reader is referred to [*Ghanen and Spanos, 1991; Xiu and Karniadakis, 2002;*
48 *Debusschere et al., 2004*] and references therein; also, it should be pointed out that
49 the word *chaos* is used in the sense originally defined by Wiener [*Wiener, 1938*] as
50 an approximation of a Gaussian random process by means of Hermite polynomials.
51 This technique has been successfully applied to several problems in different domains,
52 including the extension of the classical circuit analysis tools, like the modified nodal
53 analysis (MNA), to the prediction of the stochastic behavior of circuits [*Strunz and*
54 *Su, 2008; Zout et al., 2007; Stievano and Canavero, 2010*]. However, so far, the
55 application has been mainly focused on the gaussian variability of model parameters
56 and limited to dynamical circuits consisting only of lumped elements. The authors
57 of this contribution have recently proposed an extension of PC theory to distributed
58 structures described by transmission-line equations [*Manfredi et al., 2010*], also in
59 presence of uniform random variables.

60 The main drawback of PC is related to the reduction of its efficiency when the
61 number of random variables increases. A possible solution consists in performing
62 preliminary tests to identify the most influential variables to be included in the
63 model [*Manfredi and Canavero, 2011*]. Yet, an approach based on the Response
64 Surface Modeling (RSM) is also possible and presented in this paper as an alterna-
65 tive to PC, followed by a comparison between the two methods. The RSM is based
66 on the fitting of a system response using polynomial terms, whose coefficients are
67 computed in a least-square sense starting from a reduced set of samples.

68 In order to be validated and compared, the advocated techniques are applied to
69 the stochastic analysis of a commercial multiconductor flex-cable used for the com-
70 munication between PCB cards.

2. Variability via Polynomial Chaos

71 This section outlines the PC method, focusing in particular on the application to
72 transmission lines described by telegraphers equations and validating it against a
73 traditional MC simulation on a commercial cable bundle. For further information,
74 readers are referred to [*Manfredi et al., 2010*] and references therein, where a more
75 comprehensive and detailed discussion is available.

2.1. PC Primer

76 The idea underlying the PC technique is the spectral expansion of a stochastic
77 function (intended as a given function of a random variable) in terms of a truncated
78 series of orthogonal polynomials. Within this framework, a function H , that in

our specific application will be the expression of the parameters and the resulting frequency-domain response of an interconnect described as a transmission line, can be approximated by means of the following truncated series

$$H(\xi) = \sum_{k=0}^P H_k \cdot \phi_k(\xi), \quad (1)$$

where $\{\phi_k\}$ are suitable orthogonal polynomials expressed in terms of the random variable ξ . The above expression is defined by the class of the orthogonal bases, by the number of terms P and by the expansion coefficients H_k . The choice of the orthogonal basis relies on the distribution of the random variables being considered. The tolerances given in product documentation and datasheets are usually expressed in terms of minimum, maximum and typical values. Since the actual distribution is generally unknown, a reasonable assumption is to consider the parameters as random variables with uniform distribution between the minimum and maximum values. Hence, the most appropriate orthogonal functions for the expansion (1) are the Legendre polynomials, the first three being $\phi_0 = 1$, $\phi_1 = \xi$ and $\phi_2 = (\frac{3}{2}\xi^2 - \frac{1}{2})$, where ξ is the normalized uniform random variable with support $[-1, 1]$. It is relevant to remark that any random parameter in the system, e.g., a dielectric permittivity ε_r , can be related to ξ as follows

$$\varepsilon_r = \frac{b+a}{2} + \frac{b-a}{2}\xi, \quad (2)$$

where a and b are the minimum and maximum values assumed by the parameter, respectively. The orthogonality property of Legendre polynomials is expressed by

$$\langle \phi_k, \phi_j \rangle = \langle \phi_k, \phi_k \rangle \delta_{kj}, \quad (3)$$

97 where δ_{kj} is the Kronecker delta and $\langle \cdot, \cdot \rangle$ denotes the inner product in the Hilbert
 98 space of the variable ξ with uniform weighting function, i.e.,

$$\langle \phi_k, \phi_j \rangle = \frac{1}{2} \int_{-1}^1 \phi_k(\xi) \phi_j(\xi) d\xi. \quad (4)$$

99 With the above definitions, the expansion coefficients H_k of (1) are computed via
 100 the projection of H onto the orthogonal components ϕ_k . It is worth noting that
 101 relation (1), which is a known nonlinear function of the random variable ξ , can
 102 be used to predict the probability density function (PDF) of $H(\xi)$ via numerical
 103 simulation or analytical formulae [Papoulis, 1991].

104 The basic results of PC theory outlined above can be extended to the case of
 105 multiple independent random variables. The application of orthogonality relations
 106 allows to build higher dimensional polynomials as the product combination of poly-
 107 nomials in one variable. As an example, Tab. 1 shows the first bivariate Legendre
 108 polynomials, up to the third order.

2.2. Application to Transmission-Line Equations

109 This section discusses the modification to the transmission-line equations, allowing
 110 to include the effects of the statistical variation of the per-unit-length (p.u.l.) pa-
 111 rameters via the PC theory. For the sake of simplicity, the discussion is based on the
 112 multiconductor transmission-line structure shown in Fig. 1, that represents the typ-
 113 ical problem of two wires whose heights above ground and separation are not known
 114 exactly, thus leading to a probabilistic definition of crosstalk between the wires.

115 In the structure of Fig. 1, the height h and the separation d are assumed to be
 116 defined by

$$\begin{cases} h = \bar{h} + (\Delta_h/2)\xi_1 \\ d = \bar{d} + (\Delta_d/2)\xi_2, \end{cases} \quad (5)$$

117 where ξ_1 and ξ_2 are independent normalized uniform random variables, with \bar{h}_1 and \bar{d}
 118 mean values and Δ_h and Δ_d supports. It should be noted that these variations define
 119 different possible configurations for the wire couple of Fig. 1, but each configuration
 120 is still uniform along the propagation direction.

121 The electrical behavior in frequency-domain of the line of Fig. 1 is described by
 122 the well-known telegraph equations:

$$\frac{d}{dz} \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix} = -s \begin{bmatrix} 0 & \mathbf{L} \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix}, \quad (6)$$

123 where s is the Laplace variable, $\mathbf{V} = [V_1(z, s), V_2(z, s)]^T$ and $\mathbf{I} = [I_1(z, s), I_2(z, s)]^T$ are
 124 vectors collecting the voltage and current variables along the multiconductor line
 125 (z coordinate) and \mathbf{C} and \mathbf{L} are the p.u.l. capacitance and inductance matrices
 126 depending on the geometrical and material properties of the structure [Paul, 1994].

127 In order to account for the uncertainties affecting the guiding structure, we must
 128 consider the p.u.l. matrices \mathbf{C} and \mathbf{L} as random quantities, with entries depending
 129 on the random vector $\boldsymbol{\xi} = [\xi_1, \xi_2]^T$. In turn, (6) becomes a stochastic differential
 130 equation, leading to randomly-varying voltages and currents along the line.

131 For the current application, the random p.u.l. matrices in (6) are represented
 132 through the Legendre expansion as follows:

$$\mathbf{C} = \sum_{k=0}^P \mathbf{C}_k \cdot \phi_k(\boldsymbol{\xi}), \quad \mathbf{L} = \sum_{k=0}^P \mathbf{L}_k \cdot \phi_k(\boldsymbol{\xi}), \quad (7)$$

133 where $\{\mathbf{C}_k\}$ and $\{\mathbf{L}_k\}$ are expansion coefficients matrices with respect to the orthog-
 134 onal components $\{\phi_k\}$ defined in Tab. 1. For a given number of random variables n
 135 and order p of the expansion – that corresponds to the highest order of the polyno-
 136 mials in (7) and generally lies within the range 2 to 5 for practical applications – the
 137 total number of terms is

$$(P + 1) = \frac{(n + p)!}{n!p!}, \quad (8)$$

138 that turns out to be equal to ten for the case $n = 2$ and $p = 3$.

139 The randomness of the p.u.l parameters reflects into stochastic values of the voltage
 140 and current unknowns and makes us decide to use expansions similar to (7) for the
 141 electrical variables. This yields a modified version of (6), whose second row is provided
 142 below in extended form for $P = 2$, as an exemplification

$$\begin{aligned} \frac{d}{dz} [\mathbf{I}_0(z, s)\phi_0(\boldsymbol{\xi}) + \mathbf{I}_1(z, s)\phi_1(\boldsymbol{\xi}) + \mathbf{I}_2(z, s)\phi_2(\boldsymbol{\xi})] = -s[\mathbf{C}_0\phi_0(\boldsymbol{\xi}) + \\ + \mathbf{C}_1\phi_1(\boldsymbol{\xi}) + \mathbf{C}_2\phi_2(\boldsymbol{\xi})][\mathbf{V}_0(z, s)\phi_0(\boldsymbol{\xi}) + \mathbf{V}_1(z, s)\phi_1(\boldsymbol{\xi}) + \mathbf{V}_2(z, s)\phi_2(\boldsymbol{\xi})], \end{aligned} \quad (9)$$

143 where the expansion coefficients of electrical variables are readily identifiable.

144 Projection of (9) on the first three Legendre polynomials leads to the following set
 145 of equations, where the explicit dependence on variables is dropped for notational
 146 convenience:

$$\begin{aligned} \frac{d}{dz} (\mathbf{I}_0\langle\phi_0, \phi_j\rangle + \mathbf{I}_1\langle\phi_1, \phi_j\rangle + \mathbf{I}_2\langle\phi_2, \phi_j\rangle) = -s(\mathbf{C}_0\langle\phi_0^2, \phi_j\rangle\mathbf{V}_0 + \\ + \mathbf{C}_0\langle\phi_0\phi_1, \phi_j\rangle\mathbf{V}_1 + \dots + \mathbf{C}_2\langle\phi_2^2, \phi_j\rangle\mathbf{V}_2), \quad j = 0, 1, 2 \end{aligned} \quad (10)$$

147 The above equation, along with the companion relation arising from the first row
 148 of (6), can be further simplified by using the orthogonality relations for the com-
 149 putation of the inner products $\langle\phi_k, \phi_j\rangle$ and $\langle\phi_k\phi_l, \phi_j\rangle$, leading to the following
 150 augmented system, where the random variables collected in vector $\boldsymbol{\xi}$ do not appear,

151 due to the integration process:

$$\frac{d}{dz} \begin{bmatrix} \tilde{\mathbf{V}}(z, s) \\ \tilde{\mathbf{I}}(z, s) \end{bmatrix} = -s \begin{bmatrix} 0 & \tilde{\mathbf{L}} \\ \tilde{\mathbf{C}} & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}(z, s) \\ \tilde{\mathbf{I}}(z, s) \end{bmatrix}. \quad (11)$$

152 In the above equation, the new vectors $\tilde{\mathbf{V}} = [\mathbf{V}_0, \mathbf{V}_1, \mathbf{V}_2]^T$ and $\tilde{\mathbf{I}} = [\mathbf{I}_0, \mathbf{I}_1, \mathbf{I}_2]^T$ collect
 153 the coefficients of the PC expansion of the unknown variables. The new p.u.l. matrix
 154 $\tilde{\mathbf{C}}$ turns out to be

$$\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_0 & \frac{1}{3}\mathbf{C}_1 & \frac{1}{5}\mathbf{C}_2 \\ \mathbf{C}_1 & \mathbf{C}_0 + \frac{2}{5}\mathbf{C}_2 & \frac{2}{5}\mathbf{C}_1 \\ \mathbf{C}_2 & \frac{2}{3}\mathbf{C}_1 & \mathbf{C}_0 + \frac{2}{7}\mathbf{C}_2 \end{bmatrix}, \quad (12)$$

155 and a similar relation holds for matrix $\tilde{\mathbf{L}}$.

156 It is worth noting that (11) is analogous to (6) and plays the role of the set of
 157 equations of a multiconductor transmission line with a number of conductors that is
 158 $(P + 1)$ times larger than those of the original line. It should be remarked that the
 159 increment of the equation number is not detrimental for the method, since for small
 160 values of P (as typically occurs in practice), the additional overhead in handling the
 161 augmented equations is much less than the time required to run a large number of
 162 MC simulations.

163 The extension of the proposed technique to different multiconductor structures pos-
 164 sibly including losses and to a larger number of random variables is straightforward.
 165 For instance, the procedure to include losses amounts to including the resistance and
 166 conductance matrices in (6) and the corresponding augmented matrices in (11).

167 The solution of a transmission-line equation requires the definition of boundary
 168 conditions, such as the Thevenin equivalent networks depicted in Fig. 2, defining
 169 sources and loads. For the deterministic case, the simulation amounts to combining

170 the port electrical relations of the two terminal elements with the transmission-line
 171 equation, and solving the system. This is a standard procedure as illustrated for
 172 example in [Paul, 1994] (see Ch.s 4 and 5). The port equations of the terminations
 173 of Fig. 2 in the Laplace domain become

$$\begin{cases} \mathbf{V}_a(s) = \mathbf{E}(s) - \mathbf{Z}_S(s)\mathbf{I}_a(s) \\ \mathbf{V}_b(s) = \mathbf{Z}_L(s)\mathbf{I}_b(s), \end{cases} \quad (13)$$

174 where $\mathbf{Z}_S = \text{diag}([Z_{S1}, Z_{S2}])$, $\mathbf{Z}_L = \text{diag}([Z_{L1}, Z_{L2}])$ and $\mathbf{E} = [E_1, 0]^T$. Also, in the
 175 above equation, the port voltages and currents need to match the solutions of the
 176 differential equation (6) at line ends (e.g., $\mathbf{V}_a(s) = \mathbf{V}(z=0, s)$, $\mathbf{V}_b(s) = \mathbf{V}(z=\mathcal{L}, s)$).

177 Similarly, when the problem becomes stochastic, the augmented transmission-line
 178 equation (11) is used in place of (6) together with the projection of the characteris-
 179 tics of the source and the load elements (13) on the first P Legendre polynomials.
 180 It is worth noticing that in this specific example, no variability is included in the
 181 terminations and thus the augmented characteristics of the source and load turn out
 182 to have a diagonal structure.

183 Once the unknown voltage and currents are computed, the quantitative informa-
 184 tion on the spreading of circuit responses can be readily obtained from the analyt-
 185 ical expression of the unknowns. As an example, the frequency-domain solution of
 186 the magnitude of voltage V_{a1} with $P = 2$, leads to $|V_{a1}(j\omega)| = |V_{a10}(j\omega)\phi_0(\boldsymbol{\xi}) +$
 187 $V_{a11}(j\omega)\phi_1(\boldsymbol{\xi}) + V_{a12}(j\omega)\phi_2(\boldsymbol{\xi})|$. As already outlined in the introduction, the above
 188 relation turns out to be a known nonlinear function of the random vector $\boldsymbol{\xi}$ that
 189 can be used to compute the PDF of $|V_{a1}(j\omega)|$ via standard techniques as numerical
 190 simulation or analytical formulae [Papoulis, 1991].

2.3. Validation

191 As a proof of the capabilities of the proposed technique, the analysis of the test
192 structure depicted in Fig. 3 is presented. The structure represents a 0.050" High
193 Flex Life Cable in a standard 9-wire configuration. Figure 3 collects both the key
194 parameters defining the geometry of the wires as well as the information on the two-
195 terminal circuit elements connected at the near-end of the cable. The cable length
196 is 80 cm and the far-end terminations are defined by identical RC parallel elements
197 ($R = 10\text{ k}\Omega$, $C = 10\text{ pF}$) connecting the wires #1, ..., #8 to the reference wire #0.

198 In this example, the goal is to estimate the response variability of the near-end
199 crosstalk between two adjacent wires in a bundle of many wires. As highlighted in
200 Fig. 3, line #4 is energized by the voltage source E_S and the other lines are quiet and
201 kept in the low state via the R_S resistances. From the official datasheet of the cable,
202 tolerance limits regarding the separation between wires ($d_{ij} \in [48, 52]$ mils) and the
203 overall radius of each wire including the dielectric coating ($r_{c,i} \in [16, 19]$ mils) are
204 available. There is no information about the permittivity value ε_r of the PVC dielec-
205 tric coating. Nevertheless, this value typically represents a primary source of uncer-
206 tainty and therefore cannot be neglected; a possible realistic range is $\varepsilon_r \in [2.9, 4.1]$.
207 In order to reduce the number of random variables included in the PC model, a rea-
208 sonable choice is to assume that only the separations between the generator and the
209 two adjacent wires are effective on crosstalk, as well as the permittivity. Therefore,
210 the variability is considered to be provided by the relative permittivity ε_r of the coat-
211 ing and the separations d_{34} and d_{45} between the active and its immediately adjacent

212 lines. These quantities are assumed to behave as independent uniform random vari-
213 ables lying in the aforementioned ranges. All the other parameters are considered to
214 be equal to their nominal values. For this comparison a third order PC expansion
215 of the p.u.l. parameters is computed via numerical integration based on the method
216 described in [Paul, 1994] (see Sec. 3.2.4 and cylindrical structures).

217 Figure 4 shows a comparison of the Bode plot (magnitude) of the transfer function
218 $H(j\omega) = V_3(j\omega)/E_S$ defining the near-end crosstalk computed via the advocated PC
219 method and determined by means of the MC procedure. The solid black thin curves
220 of Fig. 4 represent the $\pm 3\sigma$ interval of the transfer function, where σ indicates the
221 standard deviation, determined from the results of the proposed technique. For com-
222 parison, the deterministic response with nominal values of all parameters is reported
223 in Fig. 4 as a solid black thick line; also, a limited set of MC simulations (100, out of
224 the 40,000 runs, in order not to clutter the figure) are plotted as gray lines. Clearly,
225 the thin curves of Fig. 4 provide a qualitative information of the spread of responses
226 due to parameters uncertainty. A better quantitative prediction can be appreciated
227 in Fig. 5, comparing the PDF of $|H(j\omega)|$ computed for different frequencies (circles)
228 with the distribution obtained via the analytical PC expansion (squares). The fre-
229 quencies selected for this comparison correspond to the dashed lines shown in Fig. 4.
230 The good agreement between the actual and the predicted PDFs and, in particu-
231 lar, the accuracy in reproducing the tails and the large variability of non-uniform
232 shapes of the reference distributions, confirm the potential of the proposed method.
233 Moreover, it should be noted that the reference MC distribution is computed by con-

234 sidering 9 random variables, i.e., the permittivity and all the wire-to-wire separations.
235 The good agreement between the curves allows us to conclude that the limited set of
236 variables included in the PC model represented a smart choice. For this example, it
237 is also clear that a PC expansion with $P = 3$ is already accurate enough to capture
238 the dominant statistical information of the system response.

3. Variability via Response Surface Modeling

239 Although PC provides an accurate stochastic model, even at high frequencies,
240 the amount of time taken by the overhead and by the solution of the augmented
241 system rapidly grows with the number of polynomial terms. Hence, the indiscriminate
242 inclusion of any possible random variable in the PC model may be critical for this
243 method and should be avoided. The variables should be carefully chosen among the
244 most influential instead. Nonetheless, an alternative and effective method for the
245 inclusion of a higher number of random variables exists and it is provided by the
246 RSM. This section introduces this alternative method and compares it against MC
247 and PC solutions of the same cable configuration shown in Fig. 3.

3.1. RSM Primer

248 The Response Surface Model [*Myers and Montgomery, 2002*] is a polynomial func-
249 tion which approximates the input/output behaviour of a complex system; the model
250 is a non-linear equation constructed by fitting observed responses and inputs via a
251 least-square fitting technique and it is used to predict the system output in response
252 to arbitrary combinations of input variables.

253 A second-order RSM has the following general form:

$$y = \beta_0 + \sum_{i=1}^n \beta_i x_i + \sum_{i=1}^n \beta_{ii} x_i^2 + \sum_{i=1}^{j-1} \sum_{j=2}^n \beta_{ij} x_i x_j, \quad (14)$$

254 where y is the system response, β_i and β_{ij} are the model fit coefficients, x_i are
 255 the system inputs and n is the number of independent input variables. Even if
 256 quadratic and interaction terms are introduced to model weak non-linearities, RSM
 257 is still a linear function of fit coefficients β , whose amount is equal to $k = 1 +$
 258 $2n + n(n - 1)/2$. Therefore they can be evaluated through a least-square fitting
 259 technique, which calculates the coefficients from the system response and inputs by
 260 minimizing the sum of the square errors.

261 A second-order RSM is chosen noticing that it is flexible enough to model the
 262 observed stochastic behavior. Although in the field of parametric modeling there
 263 are more complex and powerful approaches, e.g., surrogate modeling [*Gorissen et*
 264 *al.*, 2010], which are more capable of extracting information from a lower amount
 265 of computationally-expensive data samples, in our application the most important
 266 requirement is the inclusion of a higher number of random variables. Despite its
 267 relative structural simplicity, RSM turns out to be suitable for our purpose, featuring
 268 a good model accuracy compared to standard MC approach.

269 The set of samples used for model fitting is determined in order to obtain accu-
 270 rate response surfaces over a wide range; a Latin Hypercube Sampling (LHS) plan
 271 yields a randomized space-filling sample set, whose projections on each design space
 272 dimension are uniformly spread, modeling appropriately all experimental corners of
 273 the design space. Sample size r is increased, starting from $r = k$, until standard

274 model evaluation criteria, e.g. RMSE and coefficients of multiple determination R^2
 275 and R_{adj}^2 [Morris and Mitchell, 1995; devore, 2000], are satisfied on a separate sample
 276 subset.

3.2. Application to Stochastic Frequency-Domain Response

277 This section discusses the application of RSM to interconnects, like the one depicted
 278 in Figs. 1 and 2, with the inclusion of the effects of the statistical variation of
 279 geometrical and material parameters. The goal is to model the response variability
 280 of some output, for instance the transfer function $H(j\omega) = V_{a2}(j\omega)/E_1$, defining the
 281 near-end crosstalk, with a polynomial RSM using normalized random variables as
 282 inputs.

283 For the sake of simplicity, we start considering the influence of two parameters,
 284 described by uniform random variables ξ_1 and ξ_2 . The second-order RSM of $|H(j\omega)|$
 285 in dB scale is composed of $k = 6$ terms and takes the following form, according
 286 to (14):

$$\begin{aligned}
 |H(j\omega)|_{\text{dB}} &= |H_0(j\omega)|_{\text{dB}} + \beta_1(\omega)\xi_1 + \beta_2(\omega)\xi_2 + \beta_{11}(\omega)\xi_1^2 + \\
 &+ \beta_{22}(\omega)\xi_2^2 + \beta_{12}(\omega)\xi_1\xi_2,
 \end{aligned}
 \tag{15}$$

287 where β_0 is set equal to the nominal transfer function $|H_0(j\omega)|_{\text{dB}}$ without any effect
 288 of parameter variability. The remaining five terms have to be estimated through a
 289 least square fitting technique; it is relevant to remark that the system response and
 290 therefore model coefficients are frequency-dependent, hence a least square problem
 291 has to be solved for each frequency point. The choice of normalized random variables
 292 with support $[-1, 1]$ as inputs and of the magnitude of the transfer function in dB

293 as output reduces the variation of the fit coefficients, thus avoiding numerical insta-
294 bilities in the model. However, a Response Surface Model for the estimation of the
295 linear magnitude or phase may be created as well. It has been experimentally proven
296 that a LHS-based design for $n = 2$ input variables requires a total of $r = 10$ samples,
297 which are obtained from the solutions of line equation (6) computed for the values
298 of the input variables specified by the sampling plan.

299 Once the fit coefficients are determined, the RSM represents an analytical function
300 of the random variables (similarly to the case discussed earlier for the PC expansion),
301 and it can be used to compute the PDF of $|H(j\omega)|_{\text{dB}}$ through standard techniques.
302 It is worth noting that the time required to evaluate the function output is much
303 smaller than a single MC simulation, and this motivates the use of the proposed
304 technique for a significantly large number of random variables.

3.3. Validation

305 This section refers to the stochastic analysis of the test structure already presented
306 in Section 2.3, extending the considered variability to other parameters. A first
307 RSM of $|H(j\omega)|_{\text{dB}}$ is built considering $n = 9$ random variables as inputs, in order to
308 include the variability of each wire-to-wire separation d_{ij} , as well as of the relative
309 dielectric constant. The resulting polynomial function needs $k = 55$ terms, whose fit
310 coefficients are estimated evaluating deterministic responses for a LHS composed of
311 $r = 250$ samples.

312 Fig. 5 additionally shows the PDF obtained from the RSM. Again, the good
313 correspondance demonstrates that RSM is indeed capable of handling a larger number
314 of variables, assuring a good accuracy.

315 Moreover, a second RSM is created to perform a complete stochastic analysis of
316 the structure, including also the thickness of wire insulators. Hence, the new model
317 contains a total number of $n = 18$ variables, i.e., 8 wire-to-wire separations, 9 coating
318 radii and the dielectric permittivity. To estimate the $k = 190$ fit coefficients of the
319 polynomial function, a LHS composed of $r = 600$ samples is used. Fig. 6 shows the
320 PDF of $|H(j\omega)|$ computed via MC simulations and by means of RSM polynomial
321 function. The good agreement confirms that second-order Response Surface Models
322 are sufficient to capture the non-uniform distribution of the statistical responses of
323 this class of structures, when affected by a large number of random parameters.

4. Conclusions

324 This paper presents two alternative methods enabling to compute quantitative
325 information on the sensitivity to parameters uncertainties of complex distributed
326 interconnects described by multiconductor transmission-line equations.

327 PC is based on the expansion of the voltage and current variables into a sum of
328 a limited number of orthogonal polynomials. It is shown that it provides very high
329 accuracy when compared to conventional solutions like Monte Carlo in the evaluation
330 of statistical parameters, even at high frequencies. Besides, PC allows to build a
331 stand-alone (augmented) model describing an interconnect affected by parameters

332 variability. This model can be reused when simulating different test conditions, such
333 as different loads and line lengths, as well as it can be integrated into more complex
334 systems. However, it suffers from a loss of computational efficiency when the number
335 of included random variables is raised.

336 RSM represents an alternative solution to overcome the previous limitation and
337 it is based on a polynomial fitting of the desired output variables in a least-square
338 sense. Yet, the model is limited to the specific conditions for which it is computed,
339 and it needs to be re-built whenever the loads or the line length change. Typically,
340 it is less accurate since some interaction terms are neglected to limit the amount of
341 samples required.

342 Both methods have been applied to the stochastic analysis of a commercial mul-
343 ticonductor flex cable with uncertain parameters described by independent uniform
344 random variables. Table 2 collects the main figures on the efficiency of the proposed
345 methods vs. the conventional MC for a 300-point frequency sweep. It is worth noting
346 that the setup time refers to the computation of the expansion and the augmented
347 matrices for the PC case, while it refers to the computation of the solutions at sam-
348 pling plan points for the RSM model. Table 2 data indicate that the PC and RSM
349 computation of curves like those in Figs. 5 and 6 on the whole frequency range is
350 faster by a factor ranging between 50 and 150 with respect to MC computation.
351 This holds even if for fairness we consider the computational overhead required by
352 the generation of the proposed models. Additionally, thanks to the analytical model
353 provided by either PC or RSM, designers might achieve superior insight into the

354 influence of each system parameter, compared to the relatively blind MC approach.
355 This comparison confirms the strength of the proposed methods, that allow to gen-
356 erate accurate predictions of the statistical behavior of a realistic interconnect with
357 a great efficiency improvement.

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Figure captions

408 **Figure 1.** Cross-section of two coupled wires, whose height above ground and
409 separation are uncertain parameters.

410 **Figure 2.** Definition of Thevenin equivalent boundary conditions at source and
411 load terminations.

412 **Figure 3.** Application test structure: 80-cm long commercial flex cable (.050”
413 High Flex Life Cable, 28 AWG Standard, PVC, 9-wire configuration). $R_S = 50 \Omega$,

414 $d_w = 15$ mils, $d_c = 35$ mils. The nominal value of the distance between adjacent wires
415 (e.g., d_{34} and d_{45}) is 50 mils.

416 **Figure 4.** Bode plots (magnitude) of the near-end crosstalk transfer function
417 $H(j\omega)$ of the example test case (see text for details). Solid black thick line: de-
418 terministic response; solid black thin lines: 3σ tolerance limit of the third order
419 polynomial chaos expansion; gray lines: a sample of responses obtained by means of
420 the MC method (limited to 100 curves, for graph readability).

421 **Figure 5.** Probability density function of $|H(j\omega)|$ for the example of this study,
422 computed at different frequencies. Of the three distributions, the one marked PC
423 (3) refers to the response obtained via a third-order polynomial chaos expansion
424 with 3 random variables, the one marked RSM (9) is generated from a second-order
425 Response Surface Model including 9 random variables, while the one marked MC (9)
426 refers to 40,000 MC simulations, involving the same nine variables of the RSM.

427 **Figure 6.** Probability density function of $|H(j\omega)|$ resulting from the variability of
428 18 independent parameters. Of the two distributions, the one marked RSM refers to
429 the response obtained via second-order Response Surface Model, and the one marked
430 MC refers to 40,000 MC simulations.

Table 1. Legendre Polynomials for the case of two independent random variables ($\xi = [\xi_1, \xi_2]^T$) and a third-order expansion ($p = 3$).

index k	order p	k -th basis ϕ_k	$\langle \phi_k, \phi_k \rangle$
0	0	1	1
1	1	ξ_1	$\frac{1}{3}$
2	1	ξ_2	$\frac{1}{3}$
3	2	$\frac{3}{2}\xi_1^2 - \frac{1}{2}$	$\frac{1}{5}$
4	2	$\xi_1\xi_2$	$\frac{1}{9}$
5	2	$\frac{3}{2}\xi_2^2 - \frac{1}{2}$	$\frac{1}{5}$
6	3	$\frac{5}{2}\xi_1^3 - \frac{3}{2}\xi_1$	$\frac{1}{7}$
7	3	$\frac{3}{2}\xi_1^2\xi_2 - \frac{1}{2}\xi_2$	$\frac{1}{15}$
8	3	$\frac{3}{2}\xi_1\xi_2^2 - \frac{1}{2}\xi_1$	$\frac{1}{15}$
9	3	$\frac{5}{2}\xi_2^3 - \frac{3}{2}\xi_2$	$\frac{1}{7}$

Table 2. CPU time required for the simulation of the setup of Fig. 3 by the standard MC method and the advocated PC and RSM techniques. See text for explanation of columns.

Method	# of random variables	Order	Setup	Simulation time	Speed-up
MC	—	—	—	3 h 53 min	—
PC	3	2	4.1 sec	1 min 50 sec	116×
PC	3	3	5 sec	4 min 20 sec	52×
RSM	9	2	1 min 23 sec	2.6 sec	163×
RSM	18	2	3 min 20 sec	21.7 sec	63×

Figure 1. Cross-section of two coupled wires, whose height above ground and separation are uncertain parameters.

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Figure 6. Probability density function of $|H(j\omega)|$ resulting from the variability of 18 independent parameters. Of the two distributions, the one marked RSM refers to the response obtained via second-order Response Surface Model, and the one marked MC refers to 40,000 MC simulations.











