The Capacitated Transshipment Location Problem under Uncertainty: a computational study

M.M. Baldi, M. Ghirardi, G. Perboli, and R. Tadei
Department of Control and Computer Engineering
Politecnico di Torino, Turin, Italy

Presentation Outline

- Introduction
- The Stochastic Problem
- The Deterministic Approximation
- Instance Generation
- Computational Results
- Conclusion

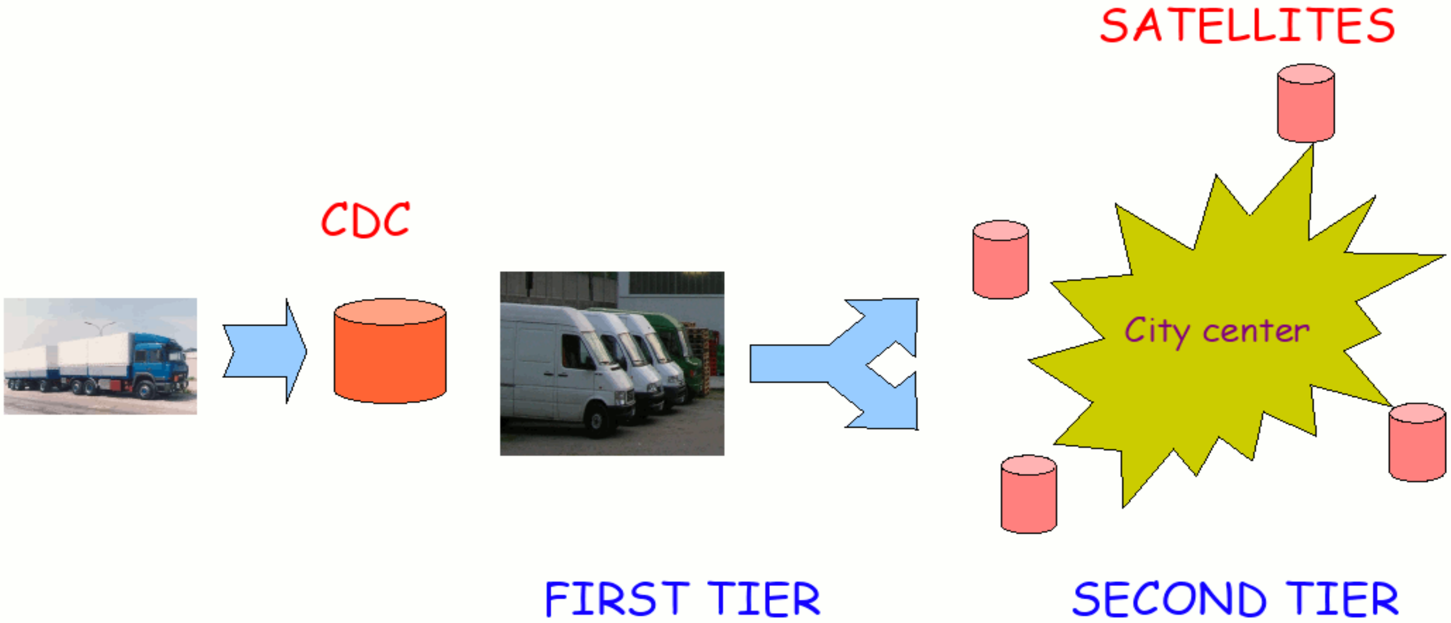
Introduction

- Freight transportation is:
  - a fundamental issue in urban areas (economic, social reasons, etc.)
  - a disturbing factor in terms of traffic and environment pollution
- Bulky vehicles carrying goods stop at the so called *City Distribution Centers* (CDCs), where consolidation and coordination activities take place
- In a two-tiered organization, intermediate platforms, the satellites, among CDCs and final customers, are present
- The flows are consolidated and smaller vehicles carrying goods are used to make the final tour in the urban area where customers are reached

INTRODUCTION

- Origin to destination costs are deterministic and well measurable
- Uncertainty must be taken into account at the transshipment facilities (satellites)
- Stochastic terms may represent:
  - Throughput costs at the facility due to handling operations or consolidation activities
  - Time to wait to load the freight into smaller vehicles
  - A measure of the network congestion in the city, i.e. beyond the transshipment facility
- The Capacitated Transshipment Location Problem under Uncertainty helps to cope with these issues

A two-tiered City Logistics system



The Stochastic Problem

The goal is:

- Find an optimal location for the facilities
- Determine optimal freight flows

by

- Minimizing the total cost:
  - Total fixed locating cost

  plus

  - Total random generalized transportation cost

while

- Satisfying balancing and capacity constraints

NOTATION

Let be:

- I : set of origins (CDCs)
- J : set of destinations (customers)
- K : set of potential transshipment facility locations
- L_k : set of throughput operation scenarios at transshipment facility $k \in K$
- n_k : number of different throughput operation scenarios at the transshipment facility k, i.e. $n_k = |L_k|$
- P_i : supply at origin $i \in I$
- Q_j : demand at destination $j \in J$
- U_k : throughput capacity of transshipment facility $k \in K$
- f_k : fixed cost of locating a transshipment facility $k \in K$
- y_k : binary variable which takes value 1 if transshipment facility $k \in K$ is located, 0 otherwise
- c^k_{ij} : unit transportation cost from origin $i \in I$ to destination $j \in J$ through transshipment facility $k \in K$
- θ_{kl} : unit throughput cost of transshipment facility $k \in K$ in throughput operation scenario $l \in L_k$
- s^k_{ij} : flow from origin $i \in I$ to destination $j \in J$ through transshipment facility $k \in K$

ASSUMPTIONS

The following assumptions are made:

- the system is balanced (total demand = total supply)
- the unit throughput costs θ_{kl} are independent and identically distributed (i.i.d.) random variables with a common and unknown probability distribution

$$Pr\{\theta_{kl} \geq x\} = F(x)$$

TOWARDS THE MODEL

- The stochastic generalized unit transportation cost from origin i to destination j through transshipment facility k in throughput scenario l is given by

$$r^{kl}_{ij}(\theta) = c^k_{ij} + \theta_{kl}, \qquad i \in I, j \in J, k \in K, l \in L_k$$

- $Pr\{r^{kl}_{ij}(\theta) \geq x\} = Pr\{c^k_{ij} + \theta_{kl} \geq x\} = Pr\{\theta_{kl} \geq x - c^k_{ij}\} = F(x - c^k_{ij})$

TOWARDS THE MODEL

- We define

$$\bar{\theta}_k = \min_{l \in L_k} \theta_{kl}, \quad k \in K$$

- Under independence assumption of θ_{kl}

$$H(x) = \Pr\{\bar{\theta}_k \geq x\} = \prod_{l \in L_k} \Pr\{\theta_{kl} \geq x\} = \prod_{l \in L_k} F(x) = [F(x)]^{n_k}$$

- The stochastic generalized unit transportation cost from origin i to destination j through transshipment facility k is the minimum among the different throughput operation scenario costs

$$\bar{r}_{ij}^{k}(\theta) = \min_{l \in L_k} r_{ij}^{kl}(\theta) = c_{ij}^{k} + \min_{l \in L_k} \theta_{kl} = c_{ij}^{k} + \bar{\theta}_k, \quad i \in I, j \in J, k \in K$$

The Stochastic Model

- The Capacitated Transshipment Location Problem under Uncertainty (CTLPU) is as follows

$$\min_{y} \sum_{k \in K} f_k y_k + E_\theta \left[\min_{s} \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \bar{r}_{ij}^k (\theta) s_{ij}^k \right]$$

s.t.

$$\sum_{j \in J} \sum_{k \in K} s_{ij}^k = P_i, \quad i \in I$$

$$\sum_{i \in I} \sum_{k \in K} s_{ij}^k = Q_j, \quad j \in J$$

$$\sum_{i \in I} \sum_{j \in J} s_{ij}^k \leq U_k y_k, \quad k \in K$$

$$s_{ij}^k \geq 0, i \in I, j \in J, k \in K$$

$$y_k \in \{0,\ 1\}, k \in K$$

The Deterministic Approximation

- It can be proven that, by using the asymptotic approximation method derived from the Extreme Value Theory, the deterministic approximation of the Capacitated Transshipment Location Problem under Uncertainty, named CTLPD, becomes

$$\min_{y} \sum_{k\in K} f_k y_k + \max_{s}\left[-\frac{1}{\beta}\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} s_{ij}^k \ln s_{ij}^k - \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} s_{ij}^k\left(c_{ij}^k - \frac{1}{\beta}\right)\right]$$

s.t.

$$\sum_{j\in J}\sum_{k\in K} s_{ij}^k = P_i, \quad i \in I$$

$$\sum_{i\in I}\sum_{k\in K} s_{ij}^k = Q_j, \quad j \in J$$

$$\sum_{i\in I}\sum_{j\in J} s_{ij}^k \le U_k y_k, \quad k \in K$$

$$s_{ij}^k \ge 0, i \in I, j \in J, k \in K$$

$$y_k \in \{0,\ 1\}, k \in K$$

Instance Generation

Since no instances for CTLPU are available in literature, ten new instances have been generated starting from a subset of those of Keskin and Uster (2007).

In particular

- number of depots $|I|$ is drawn from *U[2, 3]*
- number of customers $|J|$ is drawn from *U[30, 40]*
- number of potential locations for the transshipments $|K|$ is drawn from *U[10, 20]*
- supply P_i is drawn from *U[900, 1000]*
- demand Q_j is drawn from $U[1, \sum_i Pi/|J|]$.
- capacity U_k is drawn from $U[0.5\ avU, 3\ avU]$, where $avU = \sum_i Pi/|K|$
- unit transportation cost c^k_{ij} is drawn from *U[1, 10]*
- fixed cost $f_k = TC\ U_k/(|I|\,|J|)$, where TC is the total unit transportation cost over all the possible arcs
- random costs are generated by using three different cumulative probability distributions, Gumbel, Laplace, and Uniform, as follows

Instance Generation

- Gumbel: *exp(-exp(-βx))* with $\beta = 0.68$, to have a mean (≈ 5.7) close to the mean of the distribution used to obtain the deterministic unit costs c^k_{ij}. In this way, the random costs have the same order of magnitude of the deterministic unit costs
- Laplace:

$$\begin{cases} 0.5\exp\left(\dfrac{x-\mu}{b}\right) & \text{if } x < \mu \\ 1-0.5\exp\left(-\dfrac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

  with mean equal to μ. The parameters of the distribution are set such that the mean of the Laplace distribution is the same of the Gumbel one
- Uniform:

$$\begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x \geq b \end{cases}$$

  The costs are generated in the range *[a, b] = [1, 10],* such that the mean of the Uniform distribution is quite close to the Gumbel one.

Computational Results

- We compare CTLPU with its deterministic approximation CTLPD
- The stochastic model CTLPU has been solved by using Xpress solver with 100 scenarios (set by the tuning procedure),
- The deterministic approximation CTLPD has been solved by using BonMin solver with $\beta = 0.68$

Instances	Objective function				Gap		
	Det	Stoch					
		Gumbel	Laplace	Uniform	Gumbel	Laplace	Uniform
1	142713	137460	139664	134570	3.82%	2.18%	6.05%
2	209429	207013	209238	202495	1.17%	0.09%	3.42%
3	150860	144510	145031	147152	4.39%	4.02%	2.52%
4	167359	164393	165939	161654	1.80%	0.86%	3.53%
5	157160	151061	152683	148561	4.04%	2.93%	5.79%
6	211108	210291	210567	213969	0.39%	0.26%	-1.34%
7	244105	243214	245251	239280	0.37%	-0.47%	2.02%
8	248086	243645	245213	249019	1.82%	1.17%	-0.37%
9	247005	243887	246621	239930	1.28%	0.16%	2.95%
10	188291	181987	184353	185853	3.46%	2.14%	1.31%
Mean	**196612**	**192746**	**194456**	**192248**	**2.25%**	**1.43%**	**2.93%**

Computational Results

Instances	Number of open facilities				Common open facilities					
	Det	Stoch								
		Gumbel	Laplace	Uniform	Gumbel		Laplace		Uniform	
					Number	%	Number	%	Number	%
1	8	6	8	8	4	67%	8	100%	8	100%
2	9	7	7	8	5	71%	5	71%	6	75%
3	12	10	10	10	8	80%	8	80%	7	70%
4	9	6	7	8	4	67%	3	43%	6	75%
5	9	8	8	8	5	63%	6	75%	6	75%
6	13	12	12	12	9	75%	10	83%	10	83%
7	9	7	7	7	5	71%	3	43%	5	71%
8	9	8	9	8	7	88%	9	100%	6	75%
9	9	8	9	8	6	75%	6	67%	5	63%
10	13	12	11	11	11	92%	9	82%	9	82%
Mean	**10**	**8**	**9**	**9**	**6**	**75%**	**7**	**74%**	**7**	**77%**

Computational Results

≈ 75% of open facilities in common.

When the open facilities are ***exactly*** the same, a gap between the two models is still present (≈0,4%), given by a different flow distribution in the two solutions.

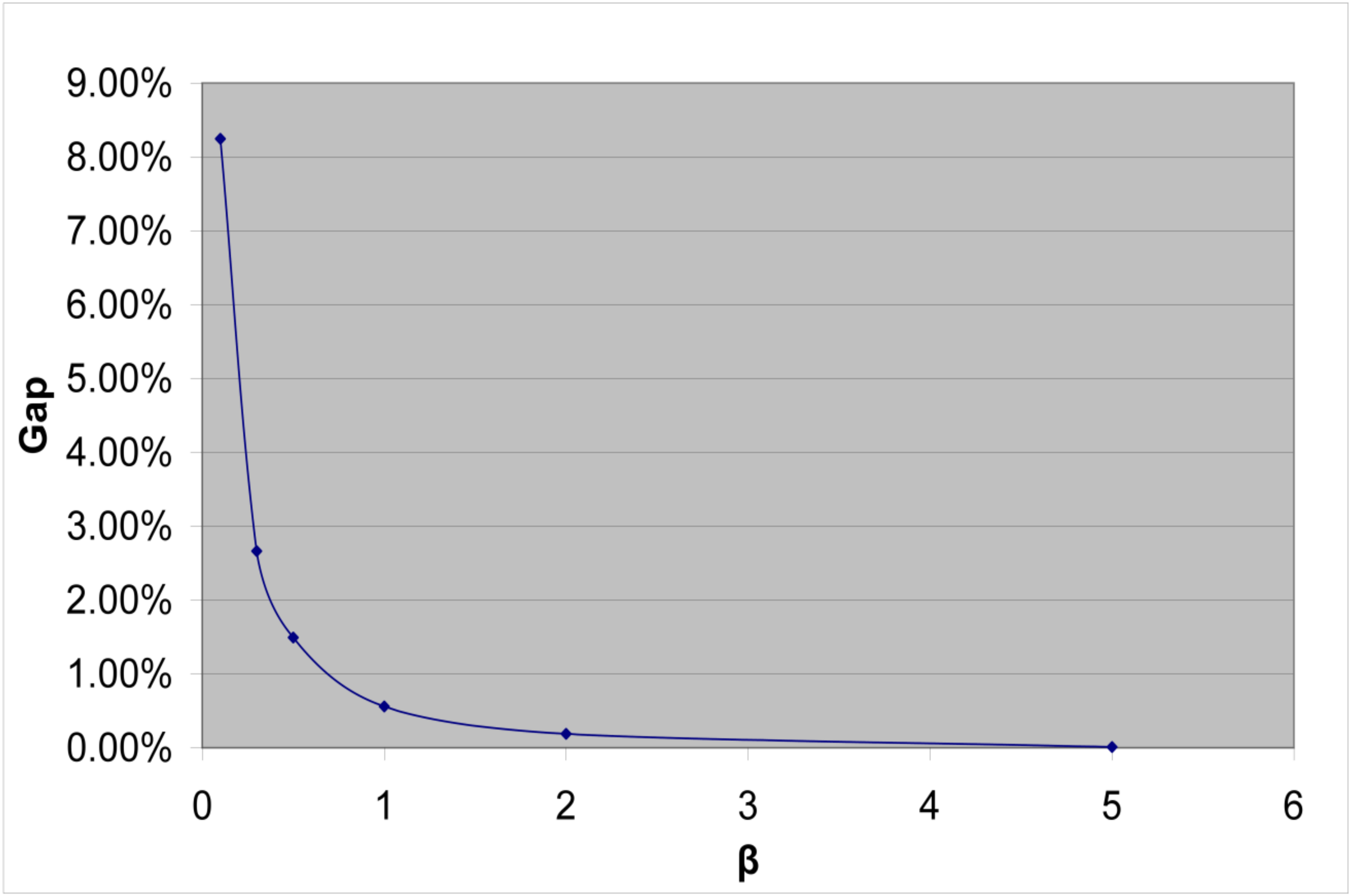
Instances	Gumbel	Laplace	Uniform
1	0.30%	0.00%	0.00%
2	0.22%	0.16%	0.36%
3	0.19%	0.47%	0.34%
4	0.73%	0.08%	0.29%
5	1.31%	0.40%	0.77%
6	0.57%	0.81%	0.03%
7	0.36%	0.45%	0.06%
8	0.27%	0.40%	0.79%
9	0.53%	0.26%	0.15%
10	0.38%	0.45%	0.84%
Mean	**0.49%**	**0.35%**	**0.38%**

Computational Results

- When $\beta \rightarrow +\infty$ the coefficient of the Entropy term tends to 0 and CTLPD turns into the classical CTLP

Instances	Comparison		
	Gap	Common open facilities	Common open facilities (%)
1	14.68%	5	71%
2	11.89%	6	75%
3	6.48%	8	73%
4	10.59%	5	71%
5	13.66%	6	75%
6	3.85%	10	77%
7	8.93%	9	100%
8	8.87%	6	75%
9	10.24%	5	71%
10	3.89%	11	92%
Mean	**9.31%**	**7**	**78%**

COMPUTATIONAL RESULTS



Tuning of the model in real situations

- In order to use the model with actual data, it requires to tune the value of β and the costs c^k_{ij} of CTLPD
- c^k_{ij} can be derived by considering historical data from databases by simple statistical computations
- Vice-versa tuning of β requires to consider the full probability distribution of θ_{kl} (which is now a Gumbel one)
- Let the costs be distributed in the interval *[m, M]= [1, 10]* and consider the Gumbel distribution with mode ζ

$$G(x) = exp(-exp(-\beta(x - \zeta)))$$

- If an approximation error of 0.01 is accepted then, after some manipulations, one gets

$$\beta = 6.12/(M - m)=6.12/(10-1)=0.68$$

Conclusion

- The Capacitated Transshipment Location Problem under Uncertainty, CTLPU, has been approximated by a non-linear deterministic model (CTLPD) belonging to the class of Entropy maximizing models
- The results are very promising showing a mean gap between stochastic and deterministic around 2%
- The facilities opened by the two models are almost the same
- The role of the Entropy term is relevant but when $\beta \to +\infty$ the Entropy contribution disappears and the CTLPD turns into the classical Capacitated Transshipment Location Problem

THANK YOU FOR YOUR ATTENTION !