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THE CAPACITATED TRANSSHIPMENT LOCATION PROBLEM UNDER UNCERTAINTY: A COMPUTATIONAL STUDY

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PRESENTATION OUTLINE

- Introduction
- The Stochastic Problem
- The Deterministic Approximation
- Instance Generation
- Computational Results
- Conclusion
INTRODUCTION

- Freight transportation is:
  - a fundamental issue in urban areas (economic, social reasons, etc.)
  - a disturbing factor in terms of traffic and environment pollution
- Bulky vehicles carrying goods stop at the so called City Distribution Centers (CDCs), where consolidation and coordination activities take place
- In a two-tiered organization, intermediate platforms, the satellites, among CDCs and final customers, are present
- The flows are consolidated and smaller vehicles carrying goods are used to make the final tour in the urban area where customers are reached
INTRODUCTION

- Origin to destination costs are deterministic and well measurable
- Uncertainty must be taken into account at the transshipment facilities (satellites)
- Stochastic terms may represent:
  - Throughput costs at the facility due to handling operations or consolidation activities
  - Time to wait to load the freight into smaller vehicles
  - A measure of the network congestion in the city, i.e. beyond the transshipment facility
- The Capacitated Transshipment Location Problem under Uncertainty helps to cope with these issues
A two-tiered City Logistics system

CDC

FIRST TIER

City center

SECOND TIER

SATELLITES
The Stochastic Problem

The goal is:

- Find an optimal location for the facilities
- Determine optimal freight flows

by

- Minimizing the total cost:
  - Total fixed locating cost
  - Total random generalized transportation cost

while

- Satisfying balancing and capacity constraints
NOTATION

Let be:

- $I$: set of origins (CDCs)
- $J$: set of destinations (customers)
- $K$: set of potential transshipment facility locations
- $L_k$: set of throughput operation scenarios at transshipment facility $k \in K$
- $n_k$: number of different throughput operation scenarios at the transshipment facility $k$, i.e. $n_k = |L_k|$  
- $P_i$: supply at origin $i \in I$
- $Q_j$: demand at destination $j \in J$
- $U_k$: throughput capacity of transshipment facility $k \in K$
- $f_k$: fixed cost of locating a transshipment facility $k \in K$
- $y_k$: binary variable which takes value 1 if transshipment facility $k \in K$ is located, 0 otherwise
- $c_{kij}$: unit transportation cost from origin $i \in I$ to destination $j \in J$ through transshipment facility $k \in K$
- $\theta_{kl}$: unit throughput cost of transshipment facility $k \in K$ in throughput operation scenario $l \in L_k$
- $s_{kij}$: flow from origin $i \in I$ to destination $j \in J$ through transshipment facility $k \in K$
ASSUMPTIONS

The following assumptions are made:

- the system is balanced (total demand = total supply)
- the unit throughput costs $\theta_{kl}$ are independent and identically distributed (i.i.d.) random variables with a common and unknown probability distribution

$$
Pr\{\theta_{kl} \geq x\} = F(x)
$$

TOWARDS THE MODEL

- The stochastic generalized unit transportation cost from origin $i$ to destination $j$ through transshipment facility $k$ in throughput scenario $l$ is given by

$$
r_{kl}^{ij}(\theta) = c_{ij}^k + \theta_{kl}, \quad i \in I, j \in J, k \in K, l \in L_k
$$

- $Pr\{r_{kl}^{ij}(\theta) \geq x\} = Pr\{c_{ij}^k + \theta_{kl} \geq x\} = Pr\{\theta_{kl} \geq x - c_{ij}^k\} = F(x - c_{ij}^k)$
TOWARDS THE MODEL

- We define
  \[ \bar{\theta}_k = \min_{l \in L_k} \theta_{kl}, \quad k \in K \]

- Under independence assumption of \( \theta_{kl} \)
  \[ H(x) = \Pr\{\bar{\theta}_k \geq x\} = \prod_{l \in L_k} \Pr\{\theta_{kl} \geq x\} = \prod_{l \in L_k} F(x) = [F(x)]^{n_k} \]

- The stochastic generalized unit transportation cost from origin \( i \) to destination \( j \) through transshipment facility \( k \) is the minimum among the different throughput operation scenario costs
  \[ \bar{r}_{ij}^k(\theta) = \min_{l \in L_k} r_{ij}^{kl}(\theta) = c_{ij}^k + \min_{l \in L_k} \theta_{kl} = c_{ij}^k + \bar{\theta}_k, \quad i \in I, \; j \in J, \; k \in K \]
The Stochastic Model

The Capacitated Transshipment Location Problem under Uncertainty (CTLPU) is as follows

\[
\min_y \sum_{k \in K} f_k y_k + E_\theta \left[ \min_s \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \bar{r}_{ij}^k (\theta) s_{ij}^k \right]
\]

s.t.

\[
\begin{align*}
\sum_{j \in J} \sum_{k \in K} s_{ij}^k &= P_i, & i \in I \\
\sum_{i \in I} \sum_{k \in K} s_{ij}^k &= Q_j, & j \in J \\
\sum_{i \in I} \sum_{j \in J} s_{ij}^k &\leq U_k y_k, & k \in K \\
\end{align*}
\]

\[
\begin{align*}
s_{ij}^k &\geq 0, & i \in I, & j \in J, & k \in K \\
y_k &\in \{0, 1\}, & k \in K 
\end{align*}
\]
It can be proven that, by using the asymptotic approximation method derived from the Extreme Value Theory, the deterministic approximation of the Capacitated Transshipment Location Problem under Uncertainty, named CTLPD, becomes

\[
\min_y \sum_{k \in K} f_k y_k + \max_s \left[ -\frac{1}{\beta} \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} s_{ij}^k \ln s_{ij}^k - \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} s_{ij}^k \left(c_{ij}^k - \frac{1}{\beta}\right) \right]
\]

s.t.

\[
\sum_{j \in J} \sum_{k \in K} s_{ij}^k = P_i, \quad i \in I
\]

\[
\sum_{i \in I} \sum_{k \in K} s_{ij}^k = Q_j, \quad j \in J
\]

\[
\sum_{i \in I} \sum_{j \in J} s_{ij}^k \leq U_k y_k, \quad k \in K
\]

\[
s_{ij}^k \geq 0, i \in I, j \in J, k \in K
\]

\[
y_k \in \{0, 1\}, k \in K
\]
 INSTANCE GENERATION

Since no instances for CTLPU are available in literature, ten new instances have been generated starting from a subset of those of Keskin and Uster (2007).

In particular
- number of depots $|I|$ is drawn from $U[2, 3]$
- number of customers $|J|$ is drawn from $U[30, 40]$
- number of potential locations for the transshipments $|K|$ is drawn from $U[10, 20]$
- supply $P_i$ is drawn from $U[900, 1000]$
- demand $Q_j$ is drawn from $U[1, \sum_i P_i/|J|].$
- capacity $U_k$ is drawn from $U[0.5 \text{av}U, 3 \text{av}U]$, where $\text{av}U = \sum_i P_i/|K|$
- unit transportation cost $c_{ij}^k$ is drawn from $U[1, 10]$
- fixed cost $f_k = \text{TC}_k/(|I||J|)$, where $\text{TC}$ is the total unit transportation cost over all the possible arcs
- random costs are generated by using three different cumulative probability distributions, Gumbel, Laplace, and Uniform, as follows
INSTANCE GENERATION

- Gumbel: \( \text{exp}(\text{exp}(-\beta x)) \) with \( \beta = 0.68 \), to have a mean (≈ 5.7) close to the mean of the distribution used to obtain the deterministic unit costs \( c^k_{ij} \). In this way, the random costs have the same order of magnitude of the deterministic unit costs.

- Laplace:

\[
\begin{align*}
0.5\exp\left(\frac{x - \mu}{b}\right) & \quad \text{if } x < \mu \\
1 - 0.5\exp\left(-\frac{x - \mu}{b}\right) & \quad \text{if } x \geq \mu
\end{align*}
\]

with mean equal to \( \mu \). The parameters of the distribution are set such that the mean of the Laplace distribution is the same of the Gumbel one.

- Uniform:

\[
\begin{align*}
0 & \quad \text{if } x < a \\
\frac{x - a}{b - a} & \quad \text{if } a \leq x < b \\
1 & \quad \text{if } x \geq b
\end{align*}
\]

The costs are generated in the range \([a, b] = [1, 10]\), such that the mean of the Uniform distribution is quite close to the Gumbel one.
COMPUTATIONAL RESULTS

- We compare CTLPU with its deterministic approximation CTLPD.
- The stochastic model CTLPU has been solved by using Xpress solver with 100 scenarios (set by the tuning procedure).
- The deterministic approximation CTLPD has been solved by using BonMin solver with $\beta = 0.68$.

<table>
<thead>
<tr>
<th>Instances</th>
<th>Det</th>
<th>Objective function Stoch</th>
<th>Gap</th>
</tr>
</thead>
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<tr>
<td></td>
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<td>137460</td>
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</tr>
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## Computational Results

<table>
<thead>
<tr>
<th>Instances</th>
<th>Number of open facilities</th>
<th>Common open facilities</th>
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</thead>
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<td></td>
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<td>Gumbel</td>
</tr>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Number</td>
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<td>8</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
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<td>12</td>
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<tr>
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<td>7</td>
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<td>8</td>
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<td>9</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Mean</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>
COMPUTATIONAL RESULTS

≈ 75% of open facilities in common.

When the open facilities are exactly the same, a gap between the two models is still present (≈0.4%), given by a different flow distribution in the two solutions.

<table>
<thead>
<tr>
<th>Instances</th>
<th>Gumbel</th>
<th>Laplace</th>
<th>Uniform</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00%</td>
<td>0.00%</td>
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<tr>
<td>2</td>
<td>0.22%</td>
<td>0.16%</td>
<td>0.36%</td>
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<tr>
<td>3</td>
<td>0.19%</td>
<td>0.47%</td>
<td>0.34%</td>
</tr>
<tr>
<td>4</td>
<td>0.73%</td>
<td>0.08%</td>
<td>0.29%</td>
</tr>
<tr>
<td>5</td>
<td>1.31%</td>
<td>0.40%</td>
<td>0.77%</td>
</tr>
<tr>
<td>6</td>
<td>0.57%</td>
<td>0.81%</td>
<td>0.03%</td>
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<tr>
<td>7</td>
<td>0.36%</td>
<td>0.45%</td>
<td>0.06%</td>
</tr>
<tr>
<td>8</td>
<td>0.27%</td>
<td>0.40%</td>
<td>0.79%</td>
</tr>
<tr>
<td>9</td>
<td>0.53%</td>
<td>0.26%</td>
<td>0.15%</td>
</tr>
<tr>
<td>10</td>
<td>0.38%</td>
<td>0.45%</td>
<td>0.84%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.49%</td>
<td>0.35%</td>
<td>0.38%</td>
</tr>
</tbody>
</table>
Computational Results

- When $\beta \rightarrow +\infty$ the coefficient of the Entropy term tends to 0 and CTLPD turns into the classical CTLP

<table>
<thead>
<tr>
<th>Instances</th>
<th>Gap</th>
<th>Common open facilities</th>
<th>Common open facilities (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.68%</td>
<td>5</td>
<td>71%</td>
</tr>
<tr>
<td>2</td>
<td>11.89%</td>
<td>6</td>
<td>75%</td>
</tr>
<tr>
<td>3</td>
<td>6.48%</td>
<td>8</td>
<td>73%</td>
</tr>
<tr>
<td>4</td>
<td>10.59%</td>
<td>5</td>
<td>71%</td>
</tr>
<tr>
<td>5</td>
<td>13.66%</td>
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<td>75%</td>
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<td>6</td>
<td>3.85%</td>
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<td>77%</td>
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<td>7</td>
<td>8.93%</td>
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<td>8</td>
<td>8.87%</td>
<td>6</td>
<td>75%</td>
</tr>
<tr>
<td>9</td>
<td>10.24%</td>
<td>5</td>
<td>71%</td>
</tr>
<tr>
<td>10</td>
<td>3.89%</td>
<td>11</td>
<td>92%</td>
</tr>
<tr>
<td>Mean</td>
<td>9.31%</td>
<td>7</td>
<td>78%</td>
</tr>
</tbody>
</table>
COMPUTATIONAL RESULTS
In order to use the model with actual data, it requires to tune the value of $\beta$ and the costs $c_{ij}^k$ of CTLPD.

$c_{ij}^k$ can be derived by considering historical data from databases by simple statistical computations.

Vice-versa tuning of $\beta$ requires to consider the full probability distribution of $q_{kl}$ (which is now a Gumbel one).

Let the costs be distributed in the interval $[m, M]=[1, 10]$ and consider the Gumbel distribution with mode $\zeta$.

$$G(x) = \exp(-\exp(-\beta(x - \zeta)))$$

If an approximation error of 0.01 is accepted then, after some manipulations, one gets

$$\beta = \frac{6.12}{(M - m)} = \frac{6.12}{(10-1)} = 0.68$$
CONCLUSION

- The Capacitated Transshipment Location Problem under Uncertainty, CTLPU, has been approximated by a non-linear deterministic model (CTLPD) belonging to the class of Entropy maximizing models.
- The results are very promising showing a mean gap between stochastic and deterministic around 2%.
- The facilities opened by the two models are almost the same.
- The role of the Entropy term is relevant but when $\beta \to +\infty$ the Entropy contribution disappears and the CTLPD turns into the classical Capacitated Transshipment Location Problem.
THANK YOU FOR YOUR ATTENTION!