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Achievable Positioning Accuracies in a Network of GNSS Reference Stations

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1. Introduction

The Network Real Time Kinematic (NRTK) positioning is nowadays a very common practice not only in academia but also in the professional world. Since its appearance, over 10 years ago, a growing number of people use this type of positioning not only for topographic applications, but also for the control of vehicles fleets, precision agriculture, land monitoring, etc.. To support these users several networks of Continuous Operating Reference Stations (CORSs) were born. These networks offer real-time services for NRTK positioning, providing a centimetric positioning accuracy with an average distance of 25-35 kms between the reference stations.

What is the effective distance between reference stations that allows to achieve the precision required for real-time positioning, using both geodetic and GIS receivers? How the positional accuracy changes with increasing distances between CORS? Can a service of geostationary satellites, such as the European EGNOS, be an alternative to the network positioning for medium-low cost receivers? These are only some of the questions that this chapter try to answer.

First, the GNSS network positioning will be discussed, with particular attention to the differential GNSS corrections such as the Master Auxiliary Concept (MAC), Virtual Reference Station (VRS) and Flächen Korrektur Parameter (FKP).

After this short review, the results obtained during a national experiment designed to verify both the quality and the potential of existing real-time and post-processing positioning services will be presented, with particular attention to the variability of the same depending on the network geometry, the type of rover receiver and the duration of his survey, as well as the use of the different GNSS constellations currently available for our area.

This experiment was conducted using already existing CORSs. Three real-time networks, characterized by different distances between the stations (50, 100 and 150 kms), were designed. The real-time products were tested, for each network, by sessions during 24-hour on a centroid point, using both geodetic and GIS receivers provided by different companies. A so large time session is made to avoid, on final results, the constellation geometry based influence, making results fully comparable.

In addition to the real-time network corrections, a post-processing analysis will be conducted, using the raw data acquired from geodetic and GIS receivers and combining

them to the RINEX from the nearest network station and to the RINEX of virtual stations, generated by the network software close to the measurement site.

The ultimate goal of this chapter is to quantify the accuracy achievable nowadays with geodetic and GIS receivers when they are used into a network of reference stations, as well as to verify (or deny) the possibility that, thanks to the continuous GNSS modernization program, the improvement of new satellite constellations and new algorithms for computing and positioning, networks that are characterized by large distances between reference stations can be used for high accuracy real-time positioning.

2. The network positioning concept

Between 1990 and 1995, the carrier-phase differential positioning has known an enormous evolution due to phase ambiguity fixing method named "On The Fly" Ambiguity Resolution (Landau & Euler, 1992). Using this technique, a cycle slip recovery, also for moving items, was not more problematic, but positioning problems when distances between master and rover exceed 10-15 kms were not solved. For this reasons, at the end of the 90's, the Network Real Time Kinematic (NRTK) or, more generally, Ground Based Augmentation Model (GBAS) was realized. (Vollath et al, 2000, Raquet & Lachapelle, 2001, Rizos, 2002).

First, to understand the network positioning concept it is necessary keep in mind some concepts about differential positioning. To do this, it is possible to write the carrier-phase equation in a metric form:

$$\phi_k^p(i) = \rho_k^p - cdT_k + cdt^p - \alpha_i I_k^p + T_k^p + Mi_k^p + E_k^p + \lambda_i Ni_k^p + \varepsilon_k^p \tag{1}$$

In this equation, the $\phi_k^p(i)$ term represents the carrier-phase measurement on the *i*-th frequency. On the right-hand side of the equation, in addition to the geometric range ρ_k^p between the satellite *p* and the receiver *k*, it is possible to find the biases related to receiver and satellite clocks multiply by the speed light (cdT_k and cdt^p), the ionospheric propagation delay $\alpha_i I_k^p$ (with a known coefficient $\alpha_i = f_1^2 / f_i^2$ that depend by the *i*-th frequency), the tropospheric propagation delay T_k^p, the multipath error Mi_k^p, the ephemeris error E_k^p, the carrier-phase ambiguity multiply by the frequency length $\lambda_i Ni_k^p$ and finally the random errors ε_k^p.

Single differences can be written considering two receivers (*h* and *k*). Neglecting multipath error, that depends only by the rover site and therefore can not be modelled, it is possible to write:

$$\phi_{hk}^p(i) = \phi_h^p(i) - \phi_k^p(i) = \rho_{hk}^p + \lambda_i Ni_{hk}^p - cdT_{hk} - \alpha_i I_{hk}^p + T_{hk}^p + E_{hk}^p + \varepsilon_{hk}^p \tag{2}$$

After that, double differences equations can be written considering two receivers (*h* and *k*) and two satellites (*p* and *q*). Subtracting the single difference calculated for the satellite *q* from those one calculated for the satellite *p*, it is possible to obtain the double differences equation, neglecting random errors contribution:

$$\phi_{hk}^{pq}(i) = \phi_{hk}^p(i) - \phi_{hk}^q(i) = \rho_{hk}^{pq} + \lambda_i Ni_{hk}^{pq} - \alpha_i I_{hk}^{pq} + T_{hk}^{pq} + E_{hk}^{pq} + \cancel{\varepsilon_{hk}^{pq}} \tag{3}$$

When the distance between the two receivers is lower than 10 kms, the atmospheric propagation delays and the ephemeris errors can be irrelevant, allowing to achieve a centimetrical accuracy. Over this distance, these errors grow up and can not be neglected. Otherwise, these errors are very spatially correlated and can be spatially modelled (Wübbena et al., 1996). However, to be able to predict and use in real-time these biases, three conditions must be satisfied: the knowledge with a centimetric accuracy of the masters positions, a control centre able to process in real-time data of all the stations, the continuous carrier-phase ambiguity fixing also when inter-station distances reach 80-100 kms. This concept is equal to bring to the left-hand side of (3), among the known terms, the first two terms on the right-hand side, i.e.:

$$\phi_{hk}^{pq}(i) - \rho_{hk}^{pq} - \lambda_i N i_{hk}^{pq} = \alpha_i I_{hk}^{pq} + T_{hk}^{pq} + E_{hk}^{pq} \tag{4}$$

In this way, it is possible to model, not only between stations *h* and *k*, but also among all the reference stations of the network, the residual ionospheric and tropospheric biases and the ephemeris error. When these errors are modelled, they can be broadcasted to any rover receiver.

3. From the concept to the implementation

First, it is possible to note that the (4) was written using two satellites. Otherwise, if a network of GNSS reference stations is considered, the same satellites that are visible and usable for the two or more master stations can not be necessarily visible from the rover receiver. Therefore, it is better to move from ionospheric and tropospheric delays, which depend by a couple of satellites, to something which depends only by a single satellite.

The network biases can be calculated in real-time using double differences, single differences or non-differential equations. The achievement of a common value of ambiguities is required. A theoretical proof of the equivalence between the non-differential and differential methods, in particular, can be found in Schaffrin & Grafarend (1986). The use of a differential method has pros and cons. The best advantage of the differential method is that the unknown parameters are fewer. Otherwise, the main disadvantage is that there is a correlation problem.

Although the approaches are identical, in recent years the trend is to use a non-differential approach. The network state parameters are evaluated by the use of a Kalman filter.

This methodology is obviously more complex, since both dispersive and non-dispersive components of each station are considered as unknowns in the Kalman filter state vector. This increased complexity is balanced by many advantages. The use of a Kalman filter allows to increase the number of equations available at each epoch, including for example measurements related to satellites that are not tracked by all the stations, in order to make the network estimation more robust also when one or more permanent stations are not available (e.g. transmission problems).

3.1 The non-differential model

Starting from pseudorange and carrier-phase equations written for the L1 ($\overline{P}_k^p$ and $\overline{\phi}_k^p$) and L2 ($\overline{\overline{P}}_k^p$ and $\overline{\overline{\phi}}_k^p$) GPS frequencies considering only one receiver (*k*) and one satellite (*p*), it is possible to separate the unknowns that depend on the receiver and the satellite:

$$\begin{bmatrix} \bar{P}_k^p \\ \bar{\bar{P}}_k^p \\ \bar{\phi}_k^p \\ \bar{\bar{\phi}}_k^p \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & \alpha & 0 & 0 \\ 1 & -1 & \lambda_1 & 0 \\ 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \rho_k^p + cdt^p - cdT_k + T_k^p \\ I_k^p \\ \bar{N}_k^p \\ \bar{\bar{N}}_k^p \end{bmatrix} \tag{5}$$

Although it stands to reason that the pseudorange measurements have a beneficial contribution to the unknowns estimation, now only the phase equations are considered. In addition, the geometric range ρ_k^p can be moved on the left-hand side of the equation.

$$\begin{bmatrix} \bar{\phi}_k^p - \rho_k^p \\ \bar{\bar{\phi}}_k^p - \rho_k^p \end{bmatrix} = \begin{bmatrix} 1 & -1 & \lambda_1 & 0 \\ 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} cdt^p - cdT_k + T_k^p \\ I_k^p \\ \bar{N}_k^p \\ \bar{\bar{N}}_k^p \end{bmatrix} \tag{6}$$

After that, in the right-hand side, it is possible to separate the tropospheric bias from the clock errors:

$$\begin{bmatrix} \bar{\phi}_k^p - \rho_k^p \\ \bar{\bar{\phi}}_k^p - \rho_k^p \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & \lambda_1 & 0 \\ 1 & 1 & -\alpha & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} cdt^p - cdT_k \\ T_k^p \\ I_k^p \\ \bar{N}_k^p \\ \bar{\bar{N}}_k^p \end{bmatrix} \tag{7}$$

Through (7), and after few mathematical processes that are not shown, it is possible to separate the tropospheric propagation delay and the clock errors, solving the network positioning in a non-differential way.

4. The biases interpolation

After the dispersive and not-dispersive biases estimation, three solutions can be followed:

- to consider data from the reference stations of the network and to interpolate these data on the rover position, generating a virtual reference station close to the rover (VRS positioning);
- to model with a plane the biases and to broadcast the model parameters to the rover (FKP positioning);
- to broadcast to the rover the estimated biases together with data from a master reference station of the network (MAC positioning).

4.1 The VRS positioning

When the previous biases are estimated, the easiest and oldest way to broadcast differential corrections is the VRS (Virtual Reference Stations).

As mentioned before, the idea is to create a synthetic correction generated as if the reference station is close to the rover. For this reason, the rover communicates its approximate position (e.g. through an NMEA GGA message). Using this position, it is possible to interpolate data following different strategies, e.g. using:

- plane triangles (Vollath et al., 2000, Landau et al., 2002);
- an Inverse Distance Weighting (IDW) estimation;
- least squares method for estimating polynomial coefficients;
- collocation (Raquet et al., 2001).

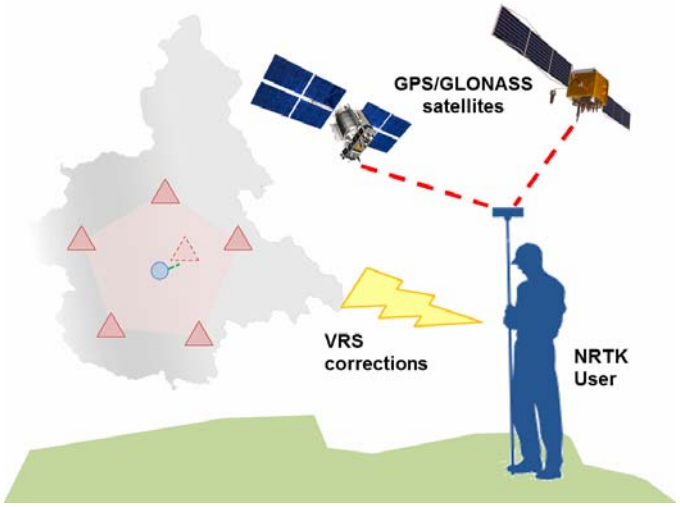

Fig. 1. The VRS positioning concept

This strategy can also be applied for post-processing positioning, by mean of a virtual data file (usually, in RINEX format). This file contains the observations that a virtual reference station may acquire in a well-known position selected by the network user.

As said above, the VRS positioning is the oldest network positioning strategy. Even if it has some advantages and is widely applied, there are also some disadvantages. The VRS method not allows, for example, a multi-base positioning (such as other methods) and it is not always well-regulated and repeatable.

4.2 The FKP positioning

Another differential network strategy is to calculate the interpolative area parameters and to broadcast them together with data from one reference station. This allows to have a one-way communication system and to maintain a relatively low transmission load.

The idea was first used by Wübbena et al. (1996, 2002), who uses a flat to interpolate the network biases in a given area. This positioning strategy was called FKP, which is the acronym of the German sentence “Flächen Korrektur Parameter” (flat correction parameters, in English).

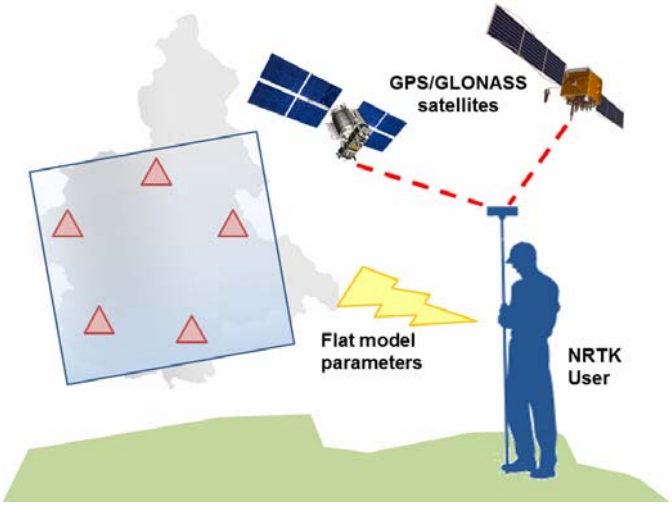

Fig. 2. The FKP positioning concept

The application of the positioning strategy is very simple. Four parameters, named E_0, N_0, E_1, N_1 can be computed considering the estimated values of geometric and ionospheric delays, using a given reference position (φ_R, λ_R). After that, it is possible to calculate the terms:

$$\begin{aligned} \delta r_0 &= 6.37\left(N_0\left(\varphi-\varphi_R\right)+E_0\left(\lambda-\lambda_R\right)\cos\varphi_R\right) \\ \delta r_I &= 6.37\,H\left(N_1\left(\varphi-\varphi_R\right)+E_1\left(\lambda-\lambda_R\right)\cos\varphi_R\right) \end{aligned} \tag{8}$$

where:

$$H = 1+16\left(0.53-E/\pi\right)^3 \tag{9}$$

where E is the satellite elevation (in radians). Finally, the two carrier-phase corrections (in meters) are:

$$\begin{aligned} \delta r_{f1} &= \delta r_0 + \left(60/77\right)\delta r_I \\ \delta r_{f2} &= \delta r_0 + \left(77/60\right)\delta r_I \end{aligned} \tag{10}$$

4.3 The MAC positioning

In 2001, Euler et al. (2001) had proposed a new approach to the use and transmission of network corrections called Master Auxiliary Concept (MAC). The concept is the same as above: a common level of network ambiguity fixing is estimated and the corrections are transmitted to the rover separating dispersive and non-dispersive components.

In the MAC positioning, the coordinates and the biases of a single reference station (master station) are broadcasted to the rover in addition to the single differences (both corrections and coordinates) of the other stations in the network (auxiliary stations).

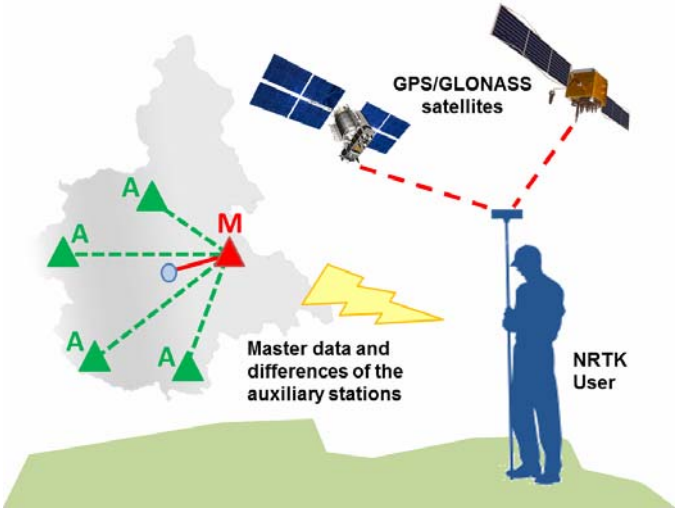

Fig. 3. The MAC positioning concept

These single differences are numerically small and have not a relevant transmission size, considering for example that the tropospheric corrections can be transmitted with a lower rate than the ionospheric ones (e.g. 2 - 5 seconds).

This new strategy implies that the network software should not perform any interpolation of the estimated biases. This interpolation, however, is only shifted to the rover, which has the possibility to choose different interpolative models or to apply a multi-base positioning.

Therefore, the rover receiver must have more computing power, so this positioning mode does not fit well to older receivers.

For very large networks, it is possible to transmit data from a subset of the network stations (sub-network or cell). Even in this case the positioning performed by the rover is accurate and fast. Even in this case, the result of the rover positioning is independent of the used cell.

5. NRTK developments and problems

It wonders if, due to the GPS and GLONASS modernization and the development of the Compass and Galileo constellations, the NRTK positioning will become obsolete. Over the last ten years, several authors (e.g. Chen et al., 2004) ask this question. In summary:

- After a large number of simulations, it is possible to conclude that, in the master-rover differential real-time positioning, the phase ambiguity solution will be almost instantaneous, making it unnecessary the use of a network of GNSS reference stations. However, in high ionospheric activity scenarios, the ambiguity fixing probability in the master-rover positioning will be very low. With a reference station network the ionosphere bias will be reduced (Stankov & Jakowskia, 2007).
- In differential positioning, the maximum distance between the master and the rover will be increased from 10 kms to 20 kms with the same reliability. Even the network

spacing will be increase with more frequencies (up to 80-100 kms), but the rover receiver will improve the reliability of fixing.
- The tropospheric biases will not be removed by using three or more frequencies. With a higher number of satellites tracked by a network of reference stations, instead, these errors will be estimated with a greater accuracy (Zhang & Lachapelle, 2001).
- Regardless of the GNSS future improvements, the multipath error in the rover will still be present. However, this error can be modelled on reference stations, giving the benefit of a more reliable estimation of the other biases.

To achieve a high real time positioning accuracy, in conclusion, a network solution will be required. The new GNSS constellations will decrease the time-to-fix the ambiguities, for both the network reference stations and the rover receiver.

A major technical problem of the network positioning is the correction signal broadcasting. A GPRS/GSM coverage in the survey site is not always available: in these cases, a radio link between the master and the rover receiver is a common solution. Otherwise, this solution does not involve the network positioning. A possible solution, especially for large networks, would be the integration between the NRTK correction and the SBAS architecture. This integration, in fact, does not require the use of additional antennas but only the payment of an access fee to the satellite band. Another solution might be to use digital subcarriers of TV channels. This solution fits very well, for example, with the MAC technique in the RTCM3 format.

On the other hand, the two-way internet communication could allow the network manager to offer additional services that are not usually provided. It may be possible, for example, to broadcast the number of satellite with a fixed network ambiguity, the maps on the survey site, the geoid undulation, etc… At the same time, the network user could transmit measurement data, updating in real time its survey, in addition to the quality of the data and of the fixed ambiguities, and to other parameters that can provide useful information for increase the reliability of GNSS positioning.

6. The experiments

In the previous sections, the network positioning concept and the different correction strategies were presented. But what is the effective distance between reference stations that allows to achieve the precision required for real-time positioning, using both geodetic and GIS receivers? And how the positional accuracy changes with increasing distances between Continuously Operating Reference Stations (CORSs)? These are only some of the questions that the experiments reported in the following try to answer.

The experiments were based on three different networks, with different inter-station distances: the first one (in the following, "red network" or "small network"), with distances of about 50 kms, is comparable with the existing GNSS networks in Italy. The second network ("green network" or "medium network") is characterized by distances of about 100 kms, which is the average spacing of the national geodetic network which materializes the Italian reference system (*Rete Dinamica Nazionale* - RDN). The last one ("blue network" or "large network") has inter-station distances of about 150 kms and it is used to verify the possibility of use not too thick networks.

The experiments were conducted using, as rover site, the reference one located on the roof of the headquarters of the Politecnico di Torino at Vercelli. For this reason, other reference stations have been chosen in the north-west side of Italy, so that the rover can be in a centroid point with respect to the three different GNSS networks (Fig. 4). The reference stations that are involved in this test (see the Table 1) belong to networks operated by public entities (such as administrative regions) and by private organizations (for example Surveyor Colleges or private companies).

	Station ID	*Station Name*	*Receiver type (IGS name)*	*Antenna type (IGS name)*
Red Network (50kms)	ALES	Alessandria	LEICA GRX1200+GNSS	LEIAR25.R3
	CRES	Crescentino	LEICA GRX1200+GNSS	LEIAR25.R3
	BIEL	Biella	LEICA GRX1200+GNSS	LEIAR25.R3
	LENT	Lenta	TPS NETG3	TPSCR.G3
	VIGE	Vigevano	TPS ODYSSEY_E	TPSCR3_GGD
Green Network (100 kms)	TORI	Torino	LEICA GRX1200+GNSS	LEIAR25.R3
	CHAT	Chatillon	TPS NETG3	TPSCR.G3
	LUIN	Luino	TPS NETG3	TPSG3_A1
	CREA	Crema	TPS ODYSSEY_E	TPSCR3_GGD
	SESC	Serravalle Scrivia	TPS NETG3	TPSCR.G3
Blue Network (150 kms)	CARP	Carpenedolo	LEICA GRX1200+GNSS	LEIAS10
	BUSL	Bussoleno	LEICA GRX1200+GNSS	LEIAR25.R3
	LOAN	Loano	TPS NETG3	TPSCR.G3
	TARO	Borgo val di Taro	TPS ODYSSEY_E	TPSCR3_GGD
	DOMO	Domodossola	TPS NETG3	TPSCR.G3

Table 1. Characteristics of permanent stations

The reference coordinates of all the stations were computed by processing 15 days of data with the Bernese GPS scientific software (version 5.0), linking the networks reference system with the ETRF2000 (2008.0), that is the Italian reference system materialized by the RDN. The different antennas used for the rover receivers were mounted on a pillar, as mentioned above, located on the roof of the Politecnico di Torino at Vercelli (Fig. 2).

The network software that was used is GNSMART (GNSS State Monitoring and Representation Technique), distributed by Geo++®. This software allows to quantify and to estimate tropospheric and ionospheric errors in addition to allows the modelling of the satellites ephemeris errors, of the multipath estimation and of the satellite and receiver clock errors. Even if it has no theoretical limitations to the minimum number of permanent stations, for a correct functioning, at least 5 stations are suggested.

In the experiments, double frequency, geodetic GNSS receivers of the main companies operating in Italy were used. The characteristics of the instruments are shown in the Table 2.

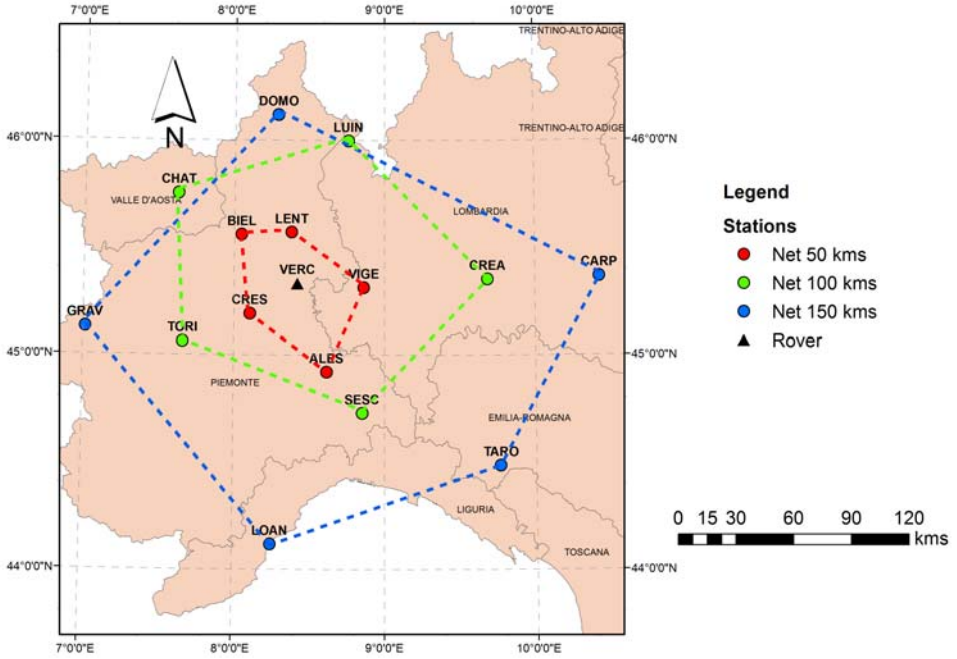

Fig. 4. Types of networks

	GX1230+GNSS (Leica Geosystems)	*GRS-1 (Topcon)*	*S9 GNSS (Stonex)*
Image			
Antenna	LEIAX1203+GNSS	TPSPG_A1	TRM55970.00
Nr. of channels	120	72	220
Constellations	GPS+GLONASS	GPS+GLONASS	GPS+GLONASS
Position update rate	20 Hz	N/A	1 Hz
Type of protocols	RTCM 2.x RTCM 3.0 CMR / CMR+	RTCM 2.x RTCM 3.0 CMR / CMR+	RTCM 2.x RTCM 3.0 CMR / CMR+
Internal modem	Yes	Yes	Yes
Type of connection	GSM	GSM	GSM/GPRS

Table 2. Geodetic receivers

Fig. 5. Rover installation

In addition to the geodetic receivers, GIS L1 receivers with their external antennas were used in the experiments. As previously, the characteristics of these instruments are summarized in the Table 3.

	Zeno 10 (Leica Geosystems)	*GRS-1 (Topcon)*	*GeoXH (Trimble)*
Image			
Antenna	LEIAT502	TPSPG_A5	TRM53406.00
Nr. of channels	14	72	220
Constellations	GPS+GLONASS	GPS+GLONASS	GPS only
Position update rate	5 Hz	N/A	1 Hz
External antenna	Yes	Yes	Yes
Time to first position	35 ÷ 120 s	N/A	45 s
Type of protocols	RTCM 2.x RTCM 3.0 CMR / CMR+	RTCM 2.x RTCM 3.0 CMR / CMR+	RTCM 2.x RTCM 3.0 CMR / CMR+
Phase corrections	Yes	Yes	No
Internal modem	Yes	Yes	No
Type of connection	GSM/UMTS 3.5G	GSM	No

Table 3. GIS instruments

7. Real time positioning accuracies

The results reported below are average values, considered significant for the two sets of instruments used in the experiments. All data collected during the experiment were analysed using two types of charts:

- Cumulative Distribution Function (CDF), i.e. a curve that describes the probability that a variable X with a given probability distribution will be found at a value less than or equal to x%.
- Cumulative moving average, i.e. the arithmetic mean of a series of values over a period that increase with respect to time. Assuming equidistant measuring or sampling times, it can be computed as the sum of the values over a period divided by the number of values.

In the following, the planimetric and elevation positioning errors for both the instrument categories, for the different network size and products are analysed.

7.1 Geodetic receivers

The tests carried out using geodetic receivers have involved the use of the three types of NRTK corrections analysed in the previous paragraphs: VRS, MAC and FKP. For each receiver and each NRTK correction, 24 hours of real time positioning results have been stored. For this analysis, only the positions with both fixed ambiguities and a HDOP (Horizontal Diluition Of Precision) index lower than 4 have been considered.

Analysing the stored positions, it was possible to highlight the behaviour that each receiver has depending on the type of differential correction: the Fig. 6 shows the quality of the planimetric and height positioning that one of the receivers used has into the "red" network.

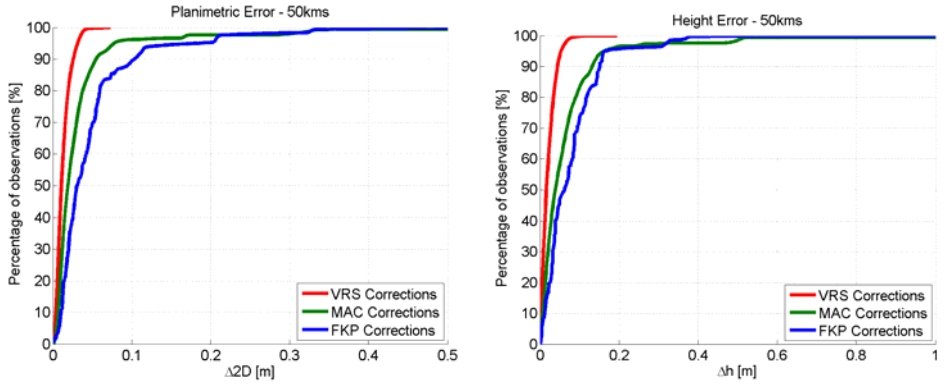

Fig. 6. CDF of planimetric (left) and elevation (right) errors of a geodetic receiver

In addition to the different behaviour that a receiver has with respect to differential correction, it is possible to consider also the position quality variation of different receivers. The Fig. 7 shows the planimetric and elevation error distributions using the three different geodetic receivers presented in Table 2, with a VRS correction broadcasted by the "green" network.

The analysis of the curves above allows to highlight a homogeneous behaviour among the receivers, which are separated only at the end of the distribution (around the 85% of probability). The planimetric accuracy, for example, changes from about 2 cms (95% of probability) for the "Receiver 2" to about 7 cms for "Receiver 1" and "Receiver 3".

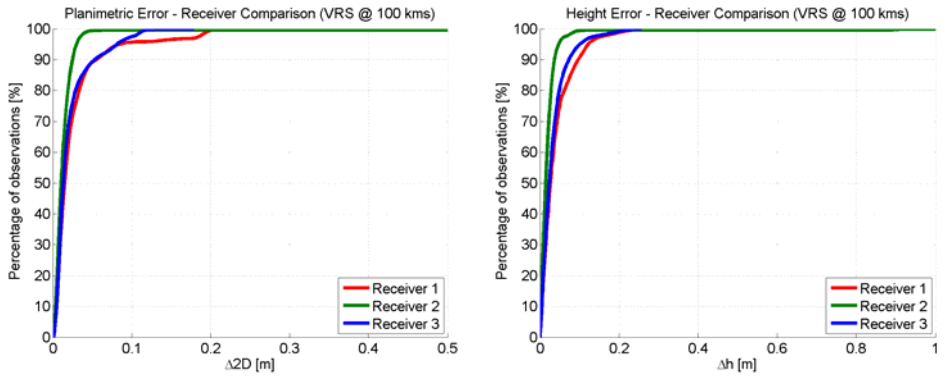

Fig. 7. CDF of planimetric (left) and elevation (right) errors of the three geodetic receivers

For this reason, it is acceptable to consider the behaviour of an "average" receiver, focusing on the variation in the positioning quality with respect to the size of the network.

The following sections show the results of these tests with the different NRTK correction used.

7.1.1 VRS positioning

The VRS is without doubt the most used differential correction in real-time positioning, as well as the easiest to manage for each receiver. As seen above, in fact, this type of differential correction is based on generating, starting from the data of network CORSs, a virtual reference station close to the measurement site.

It is therefore expected a deteriorating positioning quality with the increasing of the inter-station distances. If this variable is increasing, in fact, there are numerous inaccuracies that can be made by the interpolation step during the generation of the VRS correction.

The Fig. 8 shows the average behaviour of a geodetic receiver in the case of a VRS correction broadcasted by networks of different sizes.

The CDF analysis brings out an effective increase of the errors (both planimetric and altimetric) when the size of the GNSS network grows up. The planimetric error, for example, changes from values below 5 cms (95% of reliability) considering the "red" network up to 10 and 15 cm with the "green" and the "blue" one, respectively. A similar behaviour can be observed for the elevation error, with values from 6 cm ("red" network) to 10 cm ("green" network) and to about 25 cm ("blue" network).

Even with regard to the cumulative moving average, VRS positioning with a "red" network achieves a centimetre accuracy after few minutes, with a trend that remains constant over the time.

It is also interesting to analyse the trend of the cumulative moving average when "green" and "blue" networks are used: in both cases, there was a significant improvement of the position quality when the measurement period increases. This trend allows to reach a centimetre accuracy after a few hours-length measurement. This behaviour is evident for

example when the "blue" network is used, where the effect of few outliers positions, not detected in real-time by the receiver, disappears only after 5 hours of measurements.

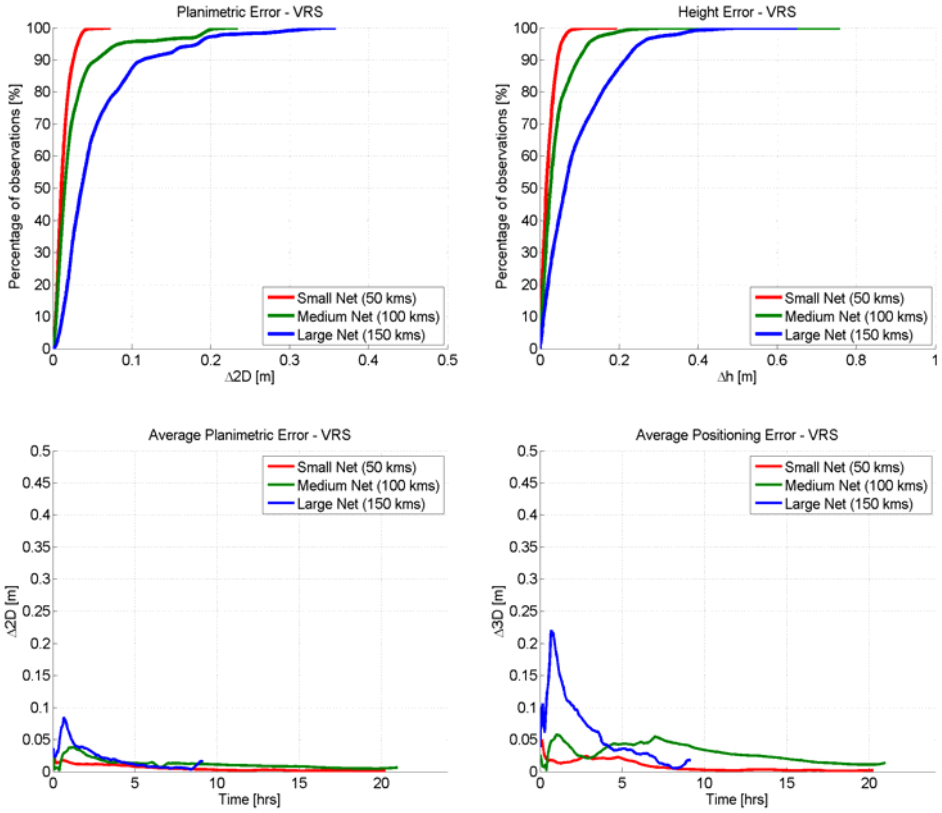

Fig. 8. Positioning quality of a geodetic receiver when a VRS correction is used: CDF of the planimetric error (top left) and of the elevation error (top right), cumulative moving average of the planimetric error (bottom left) and of the three-dimensional positioning error (bottom right)

7.1.2 MAC positioning

As said previously, the MAC correction is realized using observations from a single reference station (master) with additional information of other CORSs within a well-defined cell of the network (auxiliary stations). For this reason, this correction should be less sensitive to the variation of GNSS network sizes. As long as the distance between the master station and the rover is maintained below the permissible values for differential real-time positioning, the variation of the network size represents only a minor contribution to the differential correction (i.e., the contribution due to the auxiliary stations). The tests allow to extract an "average" behaviour of a geodetic dual frequency receiver when a MAC correction is used. This behaviour is summarized in the Fig. 9.

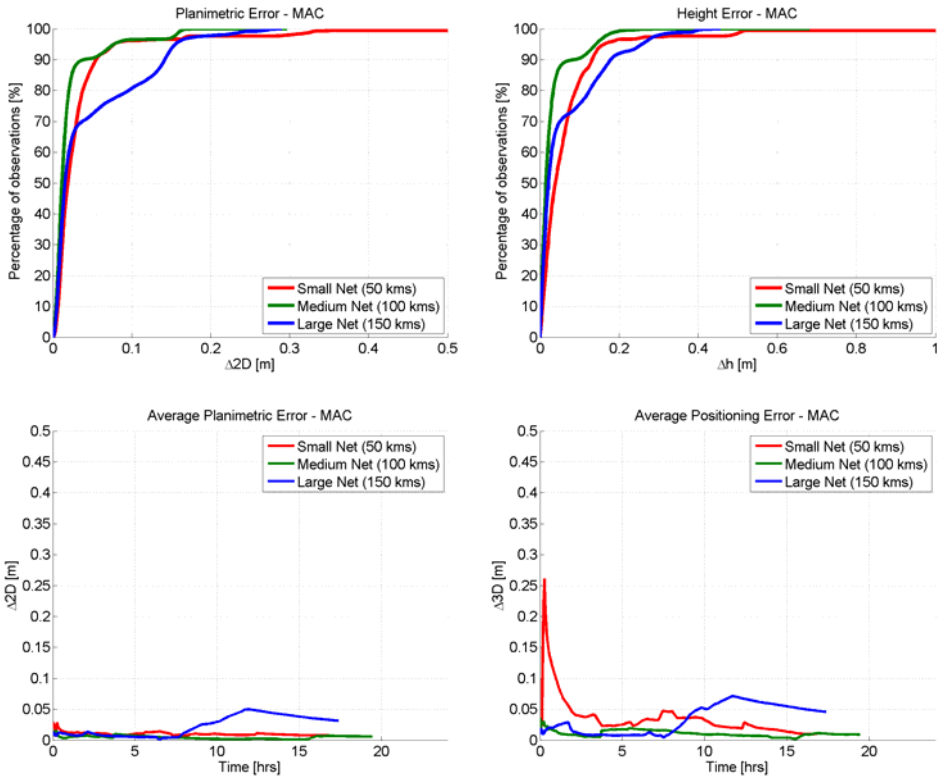

Fig. 9. Positioning quality of a geodetic receiver when a MAC correction is used: CDF of the planimetric error (top left) and of the elevation error (top right), cumulative moving average of the planimetric error (bottom left) and of the three-dimensional positioning error (bottom right)

The curves above confirms what was expected. Analysing, for example, the CDF curves of planimetric and elevation error, it is possible to see how positioning errors do not increase excessively when switching between the "red" and the "green" network. In these cases, the positioning quality is similar, and reaches about 5 cms (95% of observations) in planimetry and about 10 cms in altitude. A significant positioning deterioration occurs when differential MAC corrections broadcasted by the "blue" network are used. In this case, the master station is very far from the rover, causing problems on the quality of the positioning (15 cms in planimetry and 25 cms in elevation).

The trend of cumulative moving averages allows to highlight once again the similar behaviour of the MAC positioning performed with a "red" and a "green" network, as seen in bottom right in the Fig. 9. The cumulative moving average also shows how the MAC positioning with the "blue" network is not perfectly consistent over the time: as it is possible to see, after about 8 hours of measurement there is a worsening of the three-dimensional positioning quality, due to measurement error variations that are not well modelled by a so wide network.

7.1.3 FKP positioning

The FKP positioning, as seen in previous sections, consists of broadcasting to the rover the bias flat model parameters estimated by the GNSS network software. The hypothesis that spatial delay variations can be arranged along a plane is certainly reliable for small networks, but become trivial when the inter-station distances become too high. In that case, in fact, local atmospheric phenomena, which can cause considerable disturbances in the GNSS observations, are not taken into account. The positioning results obtained by the use of a geodetic receiver corrected by a FKP model are shown in the Fig. 10.

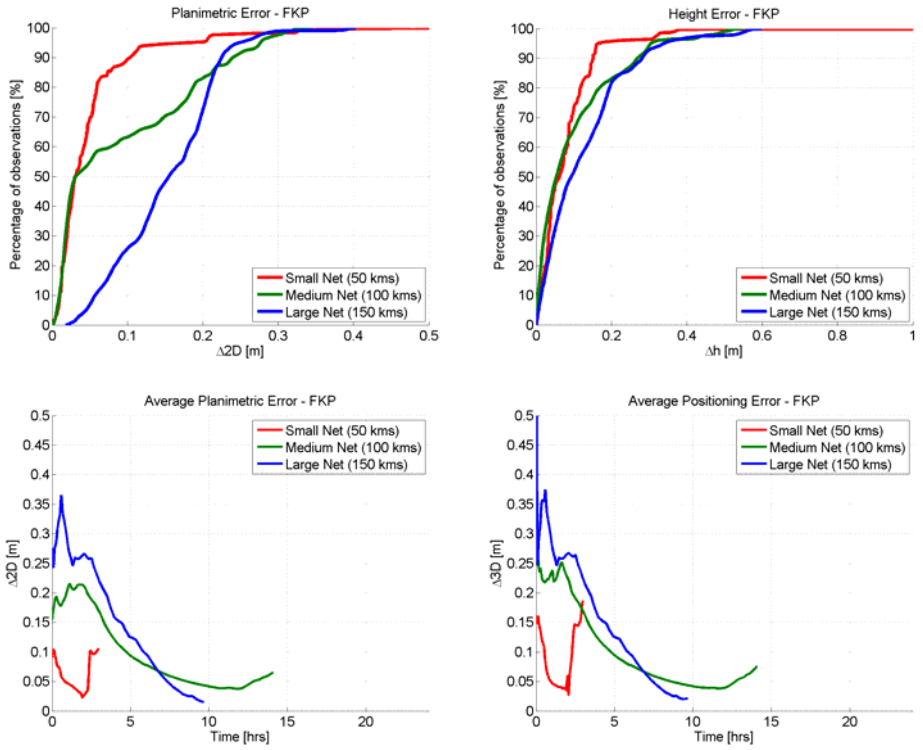

Fig. 10. Positioning quality of a geodetic receiver when a FKP correction is used: CDF of the planimetric error (top left) and of the elevation error (top right), cumulative moving average of the planimetric error (bottom left) and of the three-dimensional positioning error (bottom right)

As it is possible to see in the previous figures, a flat interpolation model allows to achieve a positioning error equal to or slightly greater than 10 cms (95% of reliability) only when small networks (e.g. the "red" one) are used. When medium-sized and large-sized networks (the "green" and "blue" ones, respectively) are used, the planimetric average error exceeds 20 cms. A very similar trend is found also for the elevation error, which increases from 15 cms ("red" network) to over 30 cms ("green" and "blue" networks). There are, in this case, no significant differences between the two wider networks.

With regard to the performance of the cumulative moving average, it is possible to see that a positioning error always lower than 5 cm can be achieved only by averaging several hours of data. The analysis of the average length of the lines shows the small number of epochs with a fix ambiguity values (almost always less than 50% of the total measured times). This is not due to NRTK corrections transmission problems, but to the use of FKP corrections by the receiver (the flat model does not fit well with the rover measurement errors).

7.2 GIS receivers

The tests carried out on the three GIS receivers shown in Table 3 were designed to study their accuracy within GNSS networks with different inter-station distances. The corrections from a VRS (used by all receivers in this class) and from the nearest reference station (NRT) were tested. Given the receiver category and the metric accuracies expectations, the EGNOS[1] corrections were also used, in order to assess whether could be, for GIS receivers, the benefits of a network of GNSS reference stations compared with the area corrections broadcasted by a geostationary satellites constellation.

In the following, the results obtained using VRS corrections are discussed. After that, the comparison between these results and those obtained using corrections from the NRT station and from the EGNOS satellites are presented.

7.2.1 VRS positioning

First, planimetric and elevation accuracies achievable with a GIS receiver into networks with different inter-station distances are analysed. As said above, 24 hours of measurements (to be independent of satellites geometry) and only positions with a HDOP index lower than or equal to 4 (to exclude outliers) were considered.

The Fig. 11, in the next page, shows the results obtained considering an "average" receiver. From the pictures analysis, it may notice that the positioning accuracy changes when the inter-station distance increases. However, it is possible to see that, unlike the geodetic receivers, in this case there is not a significant positioning deterioration with the increasing network size (from the "red" network to the "blue" one). The planimetric error at the 95% of reliability, for example, goes from 80 cms ("red" network) to 60 cms ("green" network) and to about 1 m ("blue" network). The improvement obtained by considering a medium-sized ("green") network is not surprising, but it must be analysed considering the quality of GIS receivers. This behaviour shows a substantial stability of the positioning accuracy, which remains always around metric values. This trend is more evident when the elevation error is analysed. Cumulative moving average lines achieve a sub-decimetric accuracy only after about 5 hours, showing no particular differences between the three different networks.

7.2.2 NRT and EGNOS positioning

The analysis carried out considering the positioning quality with VRS corrections were compared with these obtained using the corrections from both the nearest reference station and the European geostationary satellites constellation EGNOS. In order to highlight the benefits of differential corrections, the stand-alone positioning results are also reported in the figures.

[1] http://www.esa.int/esaNA/egnos.html

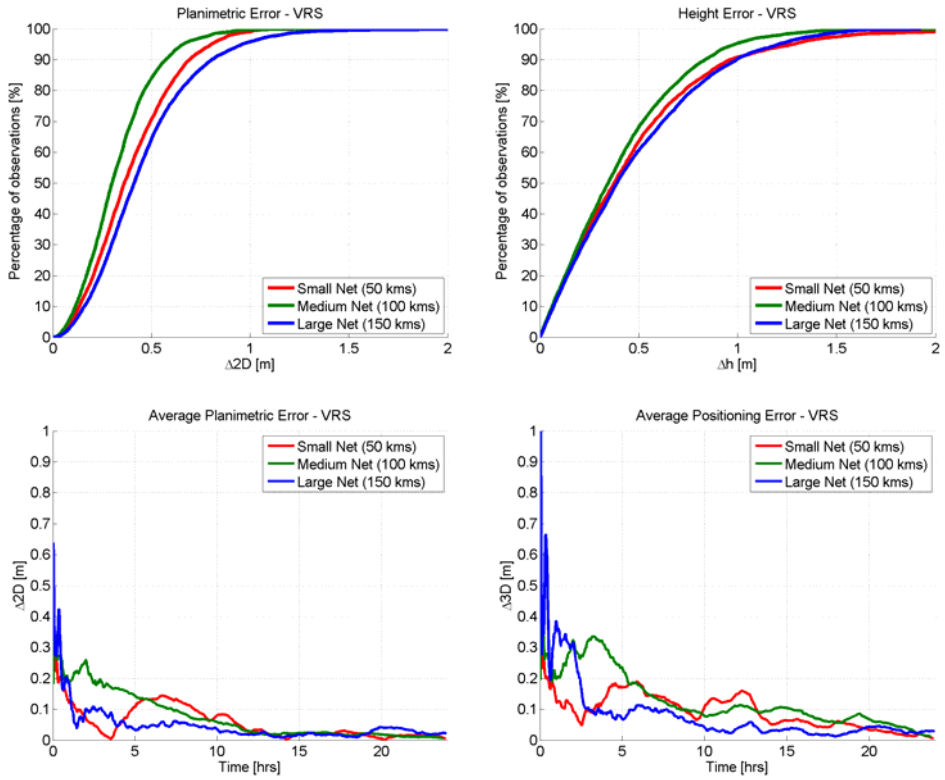

Fig. 11. Positioning quality of a GIS receiver when a VRS correction is used: CDF of the planimetric error (top left) and of the elevation error (top right), cumulative moving average of the planimetric error (bottom left) and of the three-dimensional positioning error (bottom right)

The Fig. 12 shows the comparison between the stand-alone positioning error and this one obtained using the two corrections said above. The figure shows the results obtained considering the network with inter-station distances of about 100 kms ("green" network), which in previous tests gave better results. As shown, both the NRT and EGNOS corrections allow to obtain a positioning quality that is fully comparable to that one achievable using VRS corrections. This result, although it may seem in contrast with the virtual stations and with the GNSS network positioning concepts, must not surprise. Common GIS receivers, in fact, are not able to well use carrier-phase corrections that difficultly can be modelled when the reference stations are too far from the measurement site.

The analysis of figures above allows also to highlight benefits due to the use of differential corrections with respect to the stand-alone positioning. The planimetric error (at the 95% of reliability), for example, decreases from values close to 1.7 m for stand-alone positioning up to about 70 cms when differential corrections are used. This improvement is even more evident observing the height accuracy trend (which decreases from about 4.5 ms to 1 m) and when cumulative moving average is considered (Fig. 13).

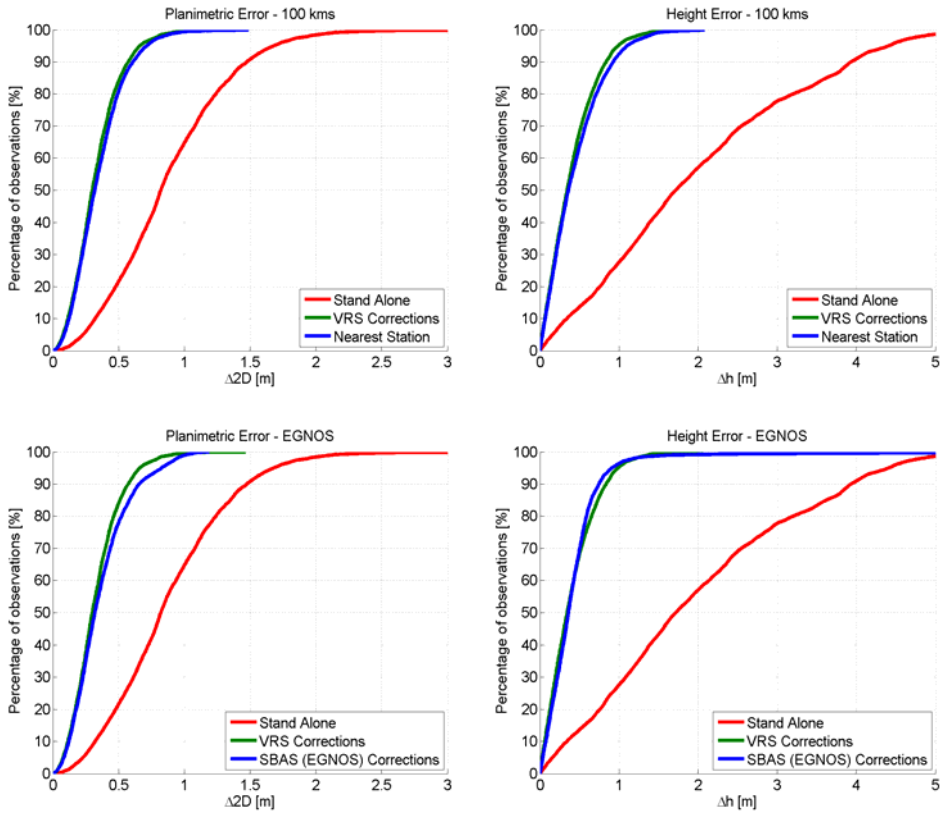

Fig. 12. Positioning quality comparison of a GIS receiver: CDF of planimetric error (left) and of elevation error (right) when a NRT correction is used (top) and when an EGNOS area correction is involved (bottom)

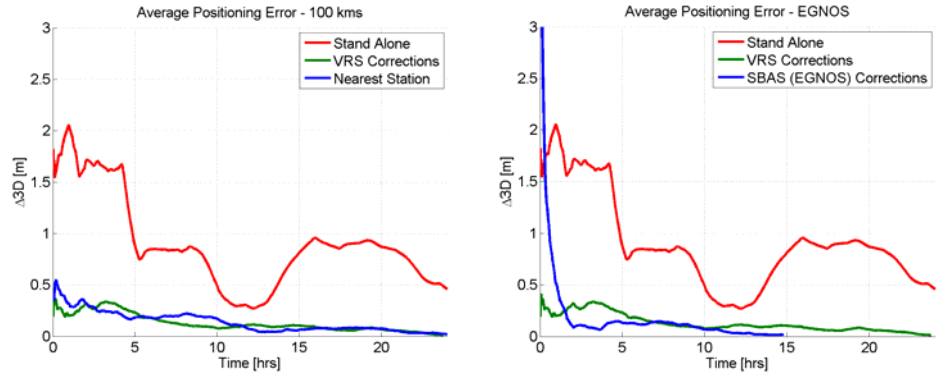

Fig. 13. Positioning quality comparison of a GIS receiver: cumulative moving average of the positioning error when NRT (left) and EGNOS (right) corrections are used

8. Post-processing positioning accuracies

Raw data files in a RINEX format were stored in order to estimate the accuracies achievable in post-processing and the performance when the average inter-station distance increases.

These files, with a length of 24 hours, were split in many shorter files with different duration, in order to statistically evaluate the planimetric and altimetric accuracy.

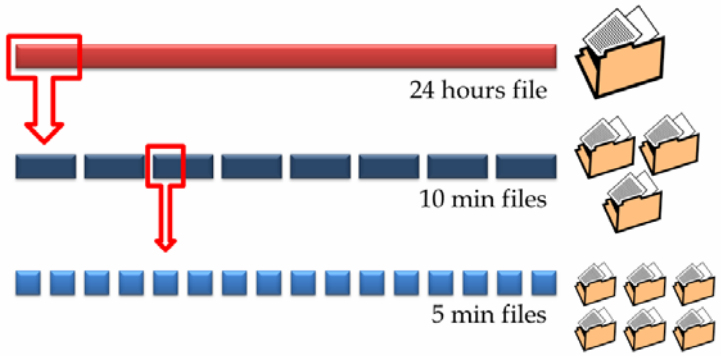

Fig. 14. Data files split schema

The data files were processed by a commercial software (Leica Geomatics Office™ v.8.0) based on the double differences approach, using as master station the nearest permanent station to each considered network and a VRS generated by the network software close to the measurement site.

The post-processing results show a no significant difference among the three geodetic receivers, due to the goodness raw data quality. The same behaviour was observed also when the three GIS receivers were used. For this reason, an "average" instrument for each class of receivers is considered in the following analysis.

8.1 Geodetic receivers

Raw data files of a geodetic receiver were split in many files of 5 and 10 minutes long, and they were post-processed as said above. The CDF of the planimetric and altimetric error (calculated using the "true position" evaluated from the network adjustment previously described) were computed for each time session.

The Fig. 15, in the next page, shows the results obtained using the nearest station for the three considered networks. A low deterioration of the positioning accuracy can be observed when different reference stations (at different distances from the rover) were used as master.

This can be seen, for instance, considering the planimetric accuracy obtained by the post-processing of the 5 minutes long data. In this case, for the "green" and the "blue" networks, a significant degradation can be observed only in the last 10% of the distribution.

Considering the 10 minutes long files, no significant improvements are observed, as expected, in the "red" network, while a better accuracy can be seen when "green" and "blue" networks are used. Also the percentage of epochs with fixed ambiguities are similar

between the "red" and the "green" networks (98-99% of epochs using 5 minutes files and 99-100% using 10 minutes files). For the "blue" one, this percentage decreases to 92% and 97% respectively.

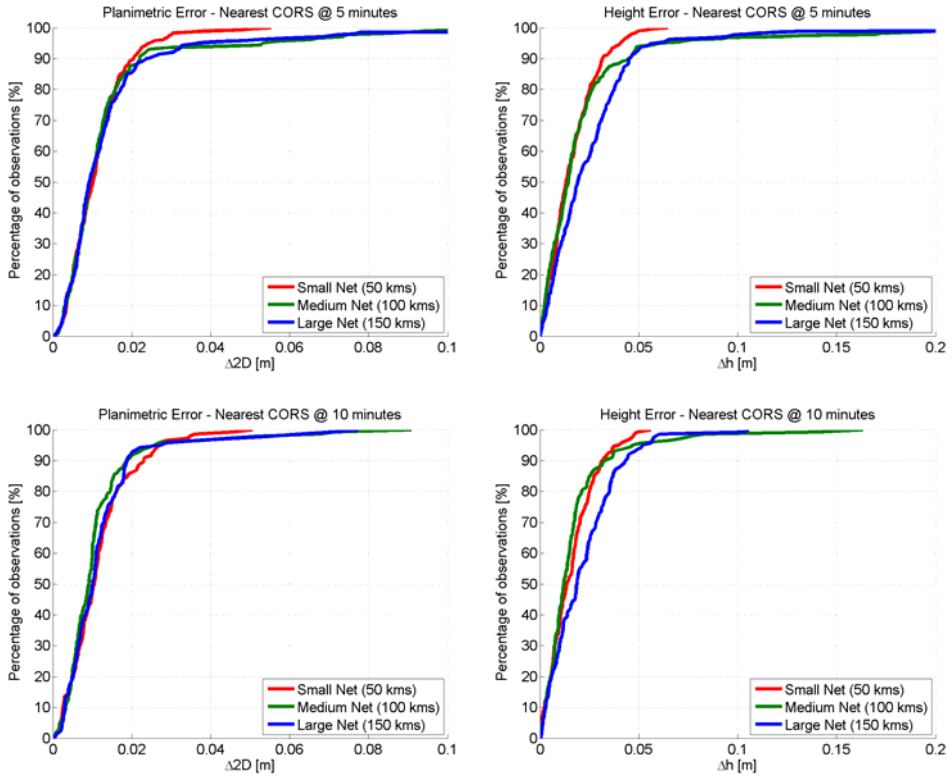

Fig. 15. Positioning quality of a geodetic receiver after the post-processing with the nearest reference station. CDF of the planimetric (left) and altimetric (right) error using static time sessions of 5 (top) and 10 (bottom) minutes

These results clearly show what is the limit (about 2 cms) of the post-processing approach when a master station is located farther than 25-30 kms from the rover. This distance is until today comparable with the actual inter-station distances of GNSS networks. To improve the 2 cms limit with a reasonable reliability, two strategies can be adopted:

- increase the static measurement length, resulting in a lack of the productivity;
- use a post-processing network product, i.e. a virtual RINEX file generated from the error models estimated by the GNSS network.

In the last case, the main advantage for the user consists in having a raw data file located close to the rover.

In this way, the rover has a higher probability to fix the phase ambiguities. Otherwise, this product shows the problems already discussed for the real-time VRS positioning. The VRS

RINEX files, in fact, are generated interpolating the error model estimated by the network software. When the inter-station distances grows up, a positioning quality deterioration is expected, due to the approximations made in the interpolation process of a wider area. The Fig. 16, that shows the results obtained using a VRS RINEX file as master, confirms what was expected.

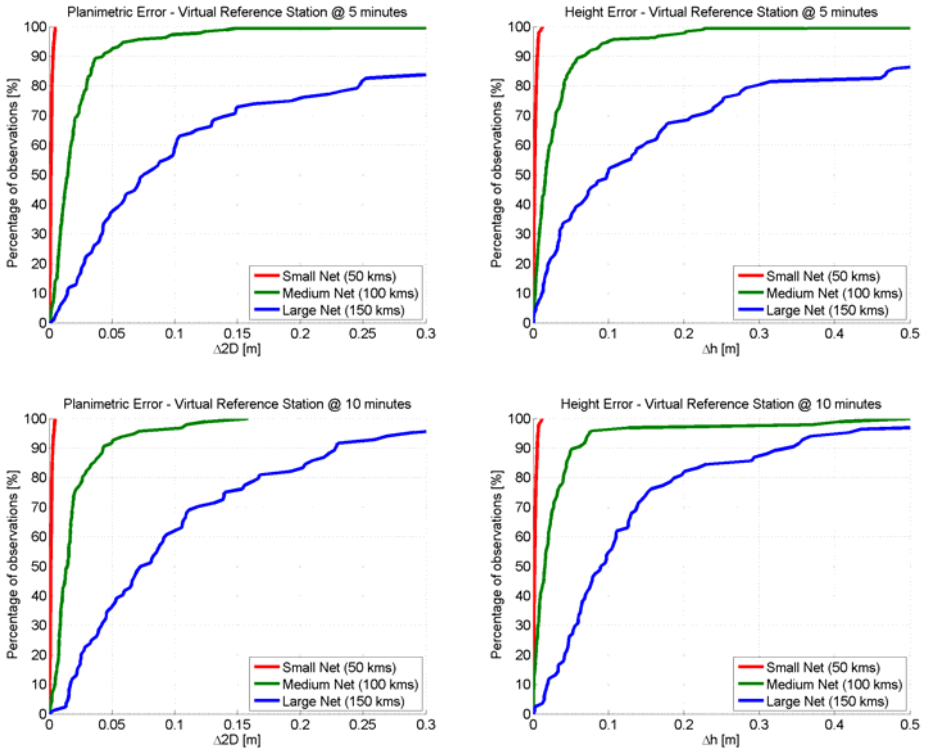

Fig. 16. Positioning quality of a geodetic receiver after the post-processing with the VRS. CDF of the planimetric (left) and altimetric (right) error using static time sessions of 5 (top) and 10 (bottom) minutes

It is easy to note that the VRS post-processing positioning improves the planimetric and altimetric accuracy only in the case of small- ("red") and medium-sized ("green") networks (about 1 cm and 4 cms respectively, considering the planimetric error).

These results confirm the goodness of this product when GNSS permanent stations, far each other about 50-100 kms, are used. When these distances exceed 100 kms, the positioning quality is comparable, or even worse, with the one obtained using the nearest reference station as master.

The percentage of fixed ambiguities is about 100% of the epochs considering 5 minutes or 10 minutes long files in "red" network, while it is very low for "green" (50%) and "blue" (12%) networks.

8.2 GIS receivers

As in the previous section, an "average" GIS receiver was considered, and the same processing methods were adopted. However, it was a priori decided to increase the static processing length session, splitting the raw data in 10 or 20 minutes long files. This is the average time that an operator could wait to achieve a sub-decimeter positioning accuracy using a low-cost receiver.

The Fig. 17 shows the post-processing results of raw data files, obtained using the nearest reference station.

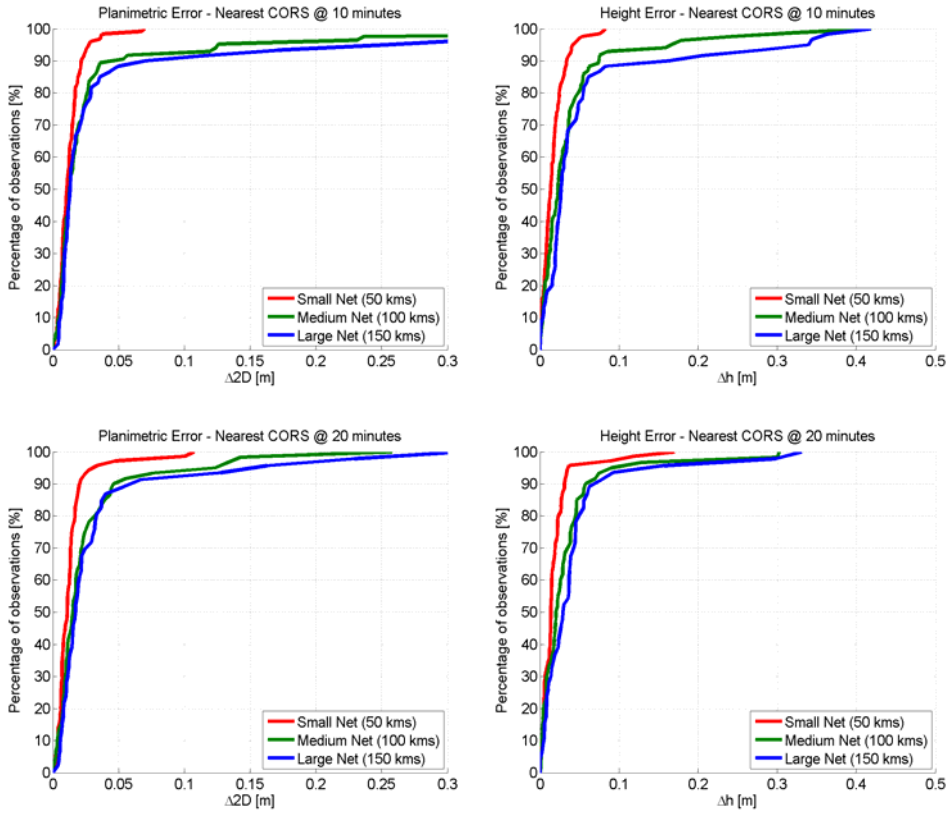

Fig. 17. Positioning quality of a GIS receiver after the post-processing with the nearest reference station. CDF of the planimetric (left) and altimetric (right) error using static time sessions of 10 (top) and 20 (bottom) minutes

A low deterioration both in planimetric and in altimetric accuracy can be observed, when the master is farther than 30 kms from the rover. This deterioration is not due to the low quality of raw data, but, instead, because GIS receivers are not able to track the L2 frequency. This frequency, in fact, allows to linearly combine measurements to reduce some of the biases (e.g. iono-free combination).

It should also be noted that increasing the measurement time from 10 to 20 minutes does not entail a real improvement in the positioning accuracy, that reaches values from about 2-3 cms ("red" network) to 7 cms ("green" network) at the 90% of reliability.

As before, a better accuracy can be obtained using VRS RINEX files generated by the network software. The analysis of the positioning accuracy, shown in Fig. 18, confirms the expected behaviour, already seen for geodetic receivers.

A reduction of the maximum planimetric and altimetric error for "red" and "green" networks is observed (few centimetres at 90% of reliability).

The percentage of measurement sessions with fixed ambiguities, goes from 68% ("red" network) to 48% ("green" network) and only to 31% ("blue" network). These percentages do not appreciably changes when 20 minutes long files are considered.

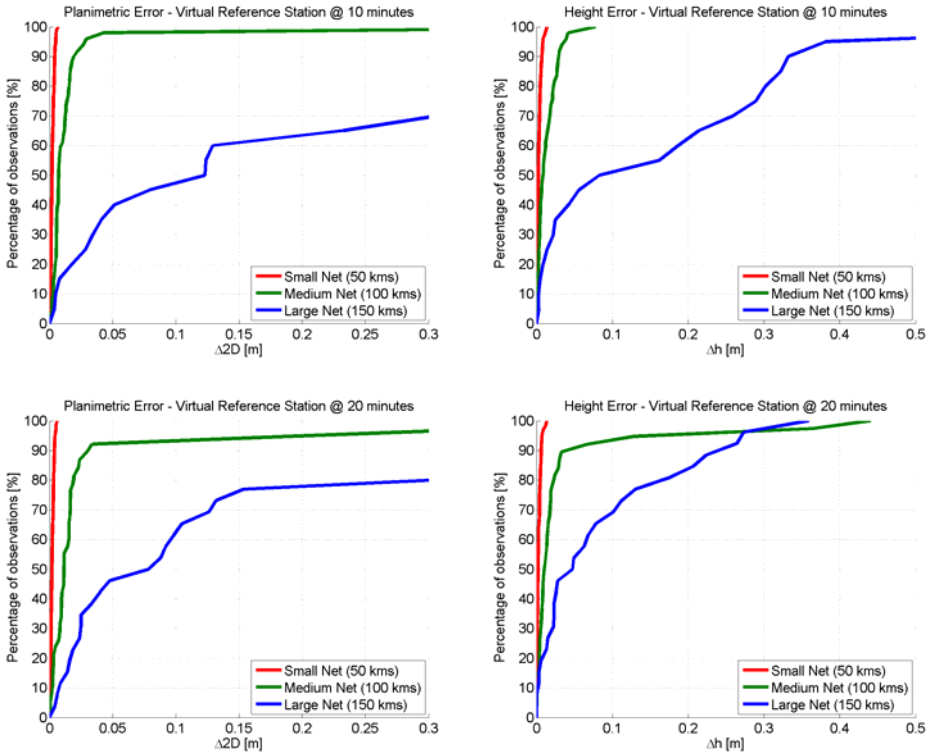

Fig. 18. Positioning quality of a GIS receiver after the post-processing with the VRS file. CDF of the planimetric (left) and altimetric (right) error using static time sessions of 10 (top) and 20 (bottom) minutes

The combined use of single frequency instruments and virtual data is very useful only for the "red" network, while the virtual data generated by the "green" and the "blue" networks significantly increase the errors, as clearly shown in Fig. 18.

It is representative the planimetric positioning achieved with the VRS generated by the wider ("blue") network, which has a percentage of about 20-30% of the data that lies outside the maximum axis value (30 cms). In this case, the percentage of measurement sessions with fixed ambiguities is 100% ("red" network) and collapse to 40% ("green" network) and only to 10% ("blue" network).

9. Conclusions

In this chapter, the accuracy of geodetic and GIS receivers in small-, medium- and large-sized networks of GNSS reference stations were analysed, comparing the results obtained with different network products. The accuracies achieved with a 95% of reliability, referring to a well-known rover position and using 24 hours of measurement, were considered.

Geodetic receivers can benefit from the VRS corrections transmitted by networks with inter-station distances up to 100 kms, allowing it to achieve planimetric accuracies from 2 to 8 cms and from 5 to 12 cms in elevation. A similar behaviour can be found when MAC corrections are used. This network product, in fact, provides comparable results for the small- and medium-sized networks (about 5 cm in planimetry and 10 cm in elevation).

If large networks are considered, the NRTK positioning is often inefficient and unreliable. Due to their lower accuracy to model biases of large areas, FKP corrections are not suitable for positioning even in medium-sized networks.

The performance of GIS receivers in real-time is poorly influenced by the size of the network. Planimetric error achieves accuracies from 65 to 85 cms in the three considered networks, and elevation error is always about 1 m. This improvement is noticeable when it is compared to the stand-alone position, with planimetric accuracies of 1.7 m and 4.5 m in altitude. Even with the EGNOS corrections it is possible to reach the same altitude accuracy (1 m at 95%) and a planimetric accuracy of about 75 cms. Using the network differential corrections, a planimetric accuracy of 50 cm can be achieved by averaging few minutes of real-time positions.

Regarding the post-processing positioning, no substantial differences were noted in the accuracy considering static session of 5 and 10 minutes long for geodetic receivers, and of 10 and 20 minutes long for GIS receivers.

For geodetic instruments, it is found that the positioning using a VRS RINEX file allows an improvement only when small-sized networks are involved. For wider networks, the best accuracies are always obtained using the RINEX file from the nearest reference station, although the number of ambiguity fixes may drop up to about 30% of the epochs.

Considering GIS receivers, the best performance is obtained when the nearest station data are used in a small-sized network (inter-station distances of about 50 kms), with a planimetric error of 2 cms and an elevation error of 3 cms. A VRS RINEX file generated by a large network does not improve the position accuracy with respect to the results obtained from the nearest station, while some advantages can be found when a medium-sized network is involved. The planimetric accuracy, in fact, goes from 10 cms, when data from the nearest station are used, to about 4 cms considering virtual data generated by a GNSS network. A similar behaviour can be also found when elevation accuracy is considered (from 15 cms to 8 cms).

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