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# DEVELOPMENT OF A SOFTWARE TOOL FOR THE EVALUATION OF THE SHADING FACTOR UNDER COMPLEX BOUNDARY CONDITIONS

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## ABSTRACT

With the aim of a detailed analysis of shading, a software tool has been developed, which calculates the shading factor under generic boundary conditions. In this paper, the implemented algorithms are described and a sensitivity analysis is performed. It is shown that, during the summer months, correct shading strategies can provide for a high reduction of the energy entering a building through fenestration. Thus, a proper evaluation of shading is especially important for computing the energy need for cooling and for sizing the air-conditioning systems.

## INTRODUCTION

A proper evaluation of shading is of great importance for predicting solar gains in buildings. A correct design of shading systems can lead to sensitive energy savings in winter and possibly even higher in summer, including an improvement of the internal comfort. According to the latitude of the site, different shading strategies should be applied, but general design guidelines are not always enough to model the uniqueness of each building in the urban context. Therefore, a detailed analysis would often be required. For an accurate calculation of solar heat gains in buildings, technical standards introduce a reduction coefficient of the incident solar radiation, called shading factor. Its value is provided for simple geometries, which are usually not suitable to properly describe a real environment. In addition, the relations adopted to evaluate the shading factor are based on simplified hypotheses, which cause non-negligible errors when compared to more complex algorithms.

In literature, many approaches to evaluate the shape of shadows cast on a window can be found. Finite element analyses and radiosity methods tend to be time-consuming, while trigonometric procedures are limited to a few simple geometries (McCluney, 1986, 1990). Other interesting approaches have been developed, which graphically evaluate the number of pixels or points that are projected onto the receiver area from the sun's point of view (Yezioro and Shaviv, 1994; Niewienda and Heidt, 1996). Software packages based on this approach give a quick response in the evaluation of the geometric shading coefficient, but they do not take the energy related aspects into account.

Algorithms for the evaluation of the shading factor can also be found within building energy simulation software. For example, EnergyPlus (2010) performs its calculation applying the homogeneous coordinate procedure proposed by Walton (1978) and coupling the sunlit area with the irradiation data returned by the ASHRAE clear sky model (2005) or with the Zhang-Huang all-sky model (Zhang et al., 2002). Diffuse radiation is taken into account through the application of the anisotropic model proposed by Perez et al. (1990).Even though Perez model has been proven to compute the diffuse radiation on a tilted surface effectively (Loutzenhiser et al., 2007), models that evaluate the radiance of the sky dome are more suitable for the application in an obstructed context. The majority of the software, anyway, considers an isotropic distribution of the sky radiance, as in the case of the TRNSHD module (Hiller et al., 2000) which was developed in order to improve the TRNSYS internal model. The shading calculation within TRNSYS (2006) can account for overhangs and wingwalls while other obstructions are computed through a shading mask, which simply works as a switch on direct and diffuse solar radiation. The decomposition into beam and diffuse radiation from global irradiation data is performed through the Reindl et al. model (1990a) and the computation of the solar radiation on a tilted surface can be performed with the assumption of isotropic sky or with the anisotropic models developed by Hay and Davies (1980), Reindl et al. (1990b) or Perez at al. (1990).

In order to overcome many of the limits in the existing models, a calculation procedure of the shading factor under complex boundary conditions has been developed, and its algorithms have been implemented in a software tool written in Matlab (2011) language.

## SHADING ALGORITHMS

### Definition of the shading factor

The shading factor is defined as the ratio between the global solar radiation received on a surface in presence of shading obstacles and in their absence. Its instantaneous value can be expressed as:

$$Fs = \frac{Fs_b \cdot I_b + Fs_d \cdot I_d + I_r}{I_b + I_d + I_r} \quad (1)$$

Similarly its average value, with respect to a reference period, is given by:

$$Fs_m = \frac{Fs_{b,m} \cdot H_b + Fs_{d,m} \cdot H_d + H_r}{H_b + H_d + H_r} \quad (2)$$

where $Fs_{b,m}$ and $Fs_{d,m}$ are, respectively, the average geometric shading coefficients for direct and diffuse radiation, given by equations (3).

$$Fs_{b,m} = \frac{\int_o^T Fs_b \cdot I_b d\tau}{H_b} \quad \text{and} \quad Fs_{d,m} = \frac{\int_o^T Fs_d \cdot I_d d\tau}{H_d} \quad (3)$$

The reference periods will be the day and the month. The monthly shading factor is calculated by assuming its value coincident with the daily shading factor of the average day of the month (Hiller et al., 2000). For each month, the average day is considered the one with average sun path, corresponding also to the one with average duration of the day. For the daily average calculation, the time step is set to an hour.

With regards to a correct evaluation of the reflected irradiance incident on a tilted surface, the knowledge of the reflection coefficients of each surface seen by the window and the mutual ones between each surface would be needed. Since the availability of such data is usually impossible, and due to the long calculation time that would be required for such computation, only reflection from the ground is considered. The reflected component is assumed as totally incident on the window. Thus, it will be:

$$I_r = \frac{1-\cos\Sigma}{2} \cdot \rho \cdot \left(I_{bh} + I_{dh}\right) \quad (4)$$

The effect of obstructions on reflected radiation will be analyzed in a future paper.

**Geometric shading factor for direct radiation**

The geometric shading coefficient for direct radiation is the ratio between the sunlit area of the window and its total area:

$$Fs_b = A_{w,s} / A_w \quad (5)$$

The calculation of the sunlit fraction, which is given by the total area of the window minus its shaded area, requires the evaluation of the shape of the shadows cast on the window by external obstructions. They vary with the geometry of the window-obstructions system and with the sun's position. Due to the application of the developed software mainly on fenestration and building energy simulation, the simplified algorithms proposed by the ASHRAE standards (2009) have been considered suitable for the calculation of the solar position, although the hour angle is assumed positive if counter-clockwise. It will thus be positive in the morning and negative in the afternoon.

**Geometric shading factor for diffuse radiation**

The amount of diffuse radiation that reaches a surface is given by the integration of the radiance distribution of the sky dome seen by it. It varies according to the orientation and tilt of the surface, to the cloud cover and to the presence and position of possible obstructions. Discretizing the sky dome in *n*m* angular zones, the diffuse irradiance on a tilted plane in absence of obstructions can be numerically computed:

$$I_d = \sum_{i=1}^{n}\sum_{j=1}^{m} R_{ij} \cos\theta_{ij} \cdot \Omega_{ij} \quad (6)$$

where:

$$\cos\theta_{ij} = \sin\beta_{ij} \cos\Sigma + \cos\beta_{ij} \sin\Sigma \cos\left(\psi_{ij} - \gamma\right) \quad (7)$$

$$\Omega_{ij} = \left[\sin\left(\beta_j + \frac{\pi}{4m}\right) - \sin\left(\beta_j - \frac{\pi}{4m}\right)\right] \cdot \frac{2\pi}{n} \quad (8)$$

Placing an observer on each point of the sky dome, the portion of window that can be seen by the observer is also the portion of window that sees that point on the sky dome. Hypothesizing the sun is the observer, the portion of sky seen by the window is equal to the geometric shading coefficient for direct radiation. Thus, the geometric shading coefficient for diffuse radiation is given by:

$$Fs_d = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} Fs_{b,ij} \cdot R_{ij} \cdot \cos\theta_{ij} \cdot \Delta\Omega_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{m} R_{ij} \cdot \cos\theta_{ij} \cdot \Delta\Omega_{ij}} \quad (9)$$

where the radiance is calculated according to the anisotropic all-sky model proposed by Brunger and Hooper (1993), and the sky dome is discretized each 5° in height and 10° in azimuth.

**Shadows' calculation procedure**

The evaluation of the shaded area is performed through a vector approach.

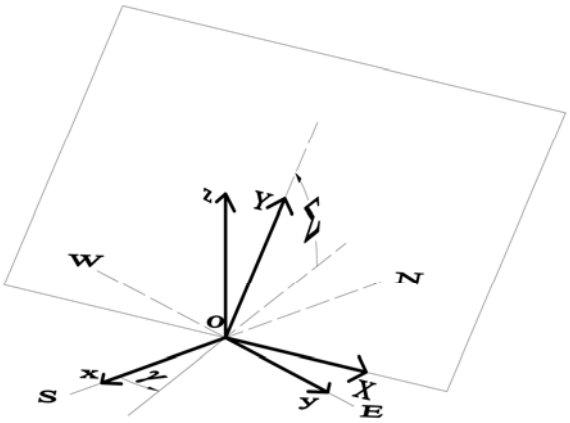


*Figure 1 Coordinate systems*

A global coordinate system *xyz* with the x-axis along the south direction, the y-axis along the east direction and the z-axis along the zenith direction is adopted,

as shown in Figure 1. Angles are considered positive when counter-clockwise.

The shadow *P'*, projected by a point *P* onto a generic plane, is given by the intersection with the plane of the straight line having the sun rays direction and that passes through the shadow-casting point. Repeating the procedure for all points in space, the shadows cast by whole objects can be evaluated.

To simplify the calculation of the shaded areas, since the coordinates of *P'* belong to the three-dimensional coordinate system *xyz*, a coordinate transformation of *P'* into the two-dimensional coordinate system *XY* of the shaded plane is useful. This transformation involves a first rotation around the z-axis, and a second rotation around the x-axis.

Once the vertex coordinates of both window and shadows are known in the *XY* coordinate system, the area of each shadow overlapping with the window and all the multiple overlaps between the shadows can be evaluated. This calculation is performed through the application of the homogeneous coordinates, according to the procedure explained by Walton (1978).

Modeling partially opaque shading surfaces is made possible by assigning them an opacity factor. For *N* shadows on the window, the total sunlit area is given by:

$$A_{w,s} = A_w - \sum_{k=1}^{N}\left[(-1)^{k+1}\sum_{1\le i_1<\ldots<i_k\le N}\left(\prod_{j=1}^{k}\zeta_{i_j}\bigcap_{j=1}^{k}A_{i_j}\right)\right] \quad (10)$$

To allow for a higher speed of the whole process, whether each object is two-dimensional or three-dimensional, only its vertex coordinates are memorized. This can be done if only convex objects are modeled, because they surely project convex shaped shadows. Concave polygons can be defined as sum of convex polygons. The vertices of the shadows are obtained by applying a convex hull procedure to the projected points. This will keep and sort all the points when a surface is being projected, and will remove the internal points when the shadow is cast by a solid.

Only when semi-transparent objects are considered, modeling a solid as a whole object or as the sum of its faces makes a difference. The shadow of a semi-transparent solid gets the transparency value given to the solid, while for a sum of faces, each face is given its own transparency, and the overlap of the single faces' transparencies is taken into account.

Mathematically, the intersection of a plane and a straight line is regardless of the position of the projected point. This means that objects behind the window surface can cast a false shadow. To prevent this from happening, before applying the shadow casting procedure, the position of each object needs to be checked. When an intersecting object is found, its face information is retrieved through a three-dimensional convex hull procedure. Every face placed behind the plane is removed, and the intersecting faces are sliced.

**Environment modeling**

The environment is characterized by external obstructions and by a horizon profile. The objects modeled as obstructions are shadow casting entities or elements that hide the sight of the sky dome. They can include other buildings as well as portions of the building itself. The horizon is a set of azimuth-altitude coordinates which draw a line on the sky dome. The radiance emitted by the portion of dome beneath the horizon is blocked, and the direct radiation is available only if the sun's elevation is greater than the horizon profile.

Among all the obstructions that can be found in an environment, vegetation is the most challenging to model, due not only to the wide variety of shapes, but also to the visual density of the foliage that changes during the year.

A tree trunk can be defined as a cylinder with given radius and height. The opacity of the trunk is set to 100%.

The foliage can be described either with conical frustum-shapes or with ellipsoidal ones. Combined foliage geometries can also be considered by modeling multiple trees in the same position.

The conical-frustum configuration can describe frusta of a cone, cones and cylinders. The required input data are the inferior and superior radiuses of the frustum of the cone and the height of the tree.

The ellipsoidal configuration can describe ellipsoids symmetrical to their vertical axes, spheres, portions of ellipsoids and portions of spheres. The required input data are the length of the semi-axes and the position of a secant plane which allows for the removal of the bottom part of the ellipsoid.

To model the deciduousness of the foliage, an opacity value is assigned. For deciduous plants, according to the species, the visual density of the foliage varies roughly between 70% and 90% in summer and 35% and 45% in winter (Heisler, 1986). For evergreen plants, a constant opacity value of 80% can be advised. The opacity values adopted in the software, assigned monthly, are shown in Table 1.

*Table 1*
*Monthly foliage opacity values for deciduous plants*

| MONTH | OPACITY |
|---|---|
| January, February, December | 40% |
| March, November | 50% |
| April, October | 60% |
| May, September | 70% |
| June, July, August | 80% |

**Sky conditions**

The shading factor is a function of the solar irradiance, whose value changes according to the sky conditions. For an instantaneous calculation, the sky can be modeled as clear, average or generic. For a monthly evaluation, only the average sky conditions can be considered.

The information required to describe the average sky are the daily average direct and diffuse irradiation on the horizontal plane. Starting from the daily irradiation values, the hourly global irradiation is estimated through the Collares-Pereira and Rabl correlation as modified by Gueymard (Collares-Pereira and Rabl, 1979; Gueymard, 1986). Then, with the Liu-Jordan relation (1960), the hourly values of the diffuse horizontal irradiation are estimated. This approach can be adopted also to evaluate the shading factor for general sky conditions, when direct and diffuse irradiation are known separately.

The information required to describe a generic sky is the global daily irradiation on the horizontal plane. From this datum, decomposition models provide for direct and diffuse irradiation values separately. For this purpose, among all three models have been chosen for the implementation in the software tool. These models are those developed by Erbs et al. (1982), Skartveit et al. (1998), and Ruiz-Arias et al. (2010). The reason for a multiple choice of the decomposition model is given by the different accuracies they show according to the site and to the sky conditions (Batlles et al., 2000; Gueymard, 2009). All decomposition methods generally underestimate direct radiation and overestimate the diffuse component under clear skies, while they invert their behaviour under all-sky conditions. The Erbs et al. model, which is a simple univariate method, is a better performer compared to many multivariate methods, especially for mid-range solar zenith angles. The Skartveit et al. one, tuned on data from Bergen, is more accurate for low solar elevations, and it also considers the ground albedo as a variable. Especially under cloudy skies, regional albedo affects diffuse radiation significantly. Finally, the model proposed by Ruiz-Arias et al., tuned on data from twenty-one stations in the USA and Europe, is based on a sigmoid function, whose variables are the clearness index and the air mass. This model does not need to consider different equations according to the clearness index interval and is statistically more reliable for extreme values of the clearness index.

For clear skies, the calculation of direct and diffuse solar irradiance is performed according to the REST2 model (Gueymard, 2008). REST2 represents a state-of-the-art in solar radiation modeling, since its accuracy is comparable with the instrumental error of the best radiometers, without the need for spectral models. The quantity and the quality of the input data, if known, allow for a precise modeling of the solar radiation. If unknown, standard values can be adopted for many parameters.

## DESCRIPTION OF THE SOFTWARE

**Inputs and outputs**

The first information required by the program is the definition of the time period of the simulation. The available choices are instantaneous, daily average or monthly average calculations.

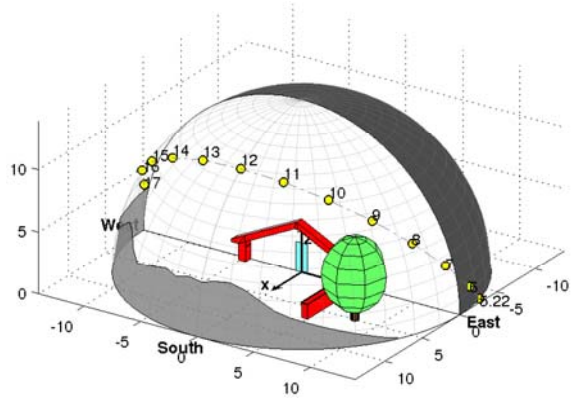


*Figure 2 Example of a modeled environment*

Afterwards, the following inputs will be required:

1. *Time definition of the simulation*: according to the chosen time period, month, day, hour and daylight saving information can be asked;
2. *Site's geographical position*: latitude, longitude, altitude and time zone are required;
3. *Radiative characteristics of the nearby environment*: the albedo value is requested;
4. *Geometry of the shaded surface*: after setting orientation and tilt angle of the window, its shape has to be defined. For a rectangular shaped window, height, width and position are required. For a curve shaped window, circles or ellipses can be modeled. They can be given a rotation and portions of the whole shape can be considered. Last, a generic shaped window can be defined by inserting its vertices' coordinates;
5. *Horizon profile*: its angular coordinates can be retrieved from a DXF file, can be manually inserted, or no horizon can be chosen. In case of DXF import, only *Line* entities are read by the program;
6. *Geometry of the external obstructions*: the objects that cast shadows on the window or cover the sky dome's sight can be imported from a DXF file, can be manually inserted, or no obstruction can be considered. In case of DXF import, to be read by the software, every object has to be modeled as one of the following entities: *3D Face*, *Polygon Mesh*, *Polyface Mesh*. An opacity factor can be given by assigning its value to the object layer's name;

*Table 2*
*Inputs and outputs of the software tool*

| **INPUTS** |
|---|
| Time period of the simulation |
| Time definition of the simulation |
| Site's geographical position |
| Radiative characteristics of the nearby environment |
| Geometry of the shaded surface |
| Horizon profile |
| Geometry of the external obstructions |
| Overhangs and vertical fins |
| Vegetation |
| Sky conditions |
| **OUTPUTS** |
| Hourly irradiances (for daily average calculation) |
| Hourly shading factors (for daily average calculation) |
| Global irradiance on the surface in absence of shading |
| Duration of the day (for average calculations) |
| Area of the shaded surface |
| Shading factor |

7. *Overhangs and vertical fins*: a detailed description of overhangs and fins is possible. For what concerns overhangs, their distance, depth, lateral projection, tilt angle and opacity are requested. A vertical projection of the overhang can also be added, and modeled with a different opacity value. Side fins can also be added. For what concerns vertical fins, it is possible to choose between left one, right one or both. Their distance, depth, vertical projection, tilt angle and opacity are requested. If both fins are modeled, different tilt angles and opacities can be chosen;
8. *Vegetation*: for each inserted plant, its geometry can be retrieved from a database or can be manually described. A total of seven tree shapes can be imported, scaled and placed in the modeled environment. If the manual insertion is chosen, the information required are those previously described. Each tree can be modeled as deciduous or evergreen. In the first case, the foliage opacity values are set according to Table 1, otherwise the user is asked to choose its value for each tree;
9. *Sky conditions*: sky conditions can be clear, average or generic. To describe the average sky, the information required are the daily average direct and diffuse irradiation on the horizontal plane. To describe the generic sky, the global daily irradiation on the horizontal plane is requested. The decomposition into direct and diffuse irradiation can be performed with the models developed by Erbs et al. (1982), Skartveit et al. (1998) or Ruiz-Arias et al. (2010). Finally, for clear skies, the calculation of direct and diffuse solar irradiance is performed according to the REST2 model (Gueymard, 2008). The required input data are: the ozone and the total nitrogen dioxide amounts, the precipitable water, the Ångström turbidity coefficient and the wavelength exponents.

All the input and output information for each simulation is written on a text file. In order for the shading factor to be meaningful, it needs to be combined with the corresponding irradiation data. Thus, the total solar irradiance on the tilted surface in absence of obstructions and, in case of average calculation, the duration of the day, are also provided. Coupled together, these data allow for the evaluation of the solar heat gains through the modeled window. For a daily average simulation, the hourly values of shading factor and irradiance are also provided.

A graphic output of the shadows cast on the window is available: image or video files can be created for instantaneous or average simulations, respectively.

**Software run time**

The calculation time is a function of the complexity of the geometry: the more the sky dome is obstructed by overlapping objects, the more time is needed. To allow for a quicker run time, it is possible to save user created environments. The results from calculation steps that are only dependant on the geometry, will also be stored. If the user chooses to generate the video output, this will contribute to slightly slow down the process.

As an example, a shaded surface placed horizontally (longest calculation: the entire sky dome can be seen), with an environment consisting of forty obstructions, has been launched with the program. The hardware information of the running machine is the following: processor: 2.20 GHz, cash size: 1024 kb, RAM: 2050 Mb. The calculation run time is reported in Table 3.

*Table 3*
*Run time of the software tool*

| **CASE** | **TIME** |
|---|---|
| First run | 148. 84 s |
| Second run | 3.10 s |
| First run with generation of video output | 159.99 s |
| Second run with generation of video output | 15.38 s |

**Further development**

Further improvement of the developed software is possible. In particular, a better modeling of the radiation reflected by external obstructions is advisable. Although the command line interface is good for giving the intructions step by step, work is already in progress for replacing it with a graphical one. This will allow for a higher flexibility in the software usage, for example by letting the user choose for a set of parameters which are currently set by default.

## SENSITIVITY ANALYSIS

A sensitivity analysis has been performed, which evaluates the incidence of overhangs and fins on the average irradiance impinging on a window, both in winter (December) and summer (July) conditions. The analyzed window is a square, whose sides have length 1.5 m. The shading environment consists in an overhang (first case), and in two vertical fins (second case), which are seen by the center of the window with a sight angle of 15°, 30° and 45°. Their length is equal to the side of the window. The location chosen for the simulation is Turin (45° N), and a ground reflection coefficient of 0.2 was assumed. The analyzed orientations are south and east. The time step of the calculation has been set to 5 minutes.

The results of the simulations are shown in Figure 3 and 4. In winter it can be observed that the presence of overhangs and fins does not significantly affect the amount of energy impinging on the window. The only exception is given by the vertical fins facing to the east, but the irradiance reaching the surface is low in any case. In summer, it can be observed that vertical fins almost do not contribute in shading the window. Instead, a high energy reduction can be achieved through overhangs, especially on south exposition.

From these results it can be summarized that for mid-ranged latitudes in winter, when the solar gains are low, a little variation on the shading factor is not very significant. In summer, instead, correct shading strategies can provide for a high reduction of the energy entering a building through fenestration.

Since the shading factor is just one of the several input values required to run a building energy simulation, it would be interesting to know how much it influences the energy need of a building. A case study has shown that a ±5% difference on the shading factor values can affect the energy need for heating of only ±2%, while it affects the energy need for cooling of -13%/+8% (Ballarini and Corrado, 2008). Thus, a proper evaluation of shading is especially important for computing the energy need for cooling and for sizing the air-conditioning systems.

## CONCLUSION

A software tool for the evaluation of the shading factor in presence of complex boundary conditions has been developed. It is characterized by a good versatility in the environment description, and it can provide for the irradiation data to couple with the shading factor. After setting the site and the time for the simulation, generic-shaped windows can be modeled. The external environment, which can be either imported from DXF files or manually described, can include a horizon profile, generic-shaped obstructions and vegetation. The calculation can be performed for every sky condition: clear, average or generic. In addition, the simulation can be run to obtain instantaneous, daily average or monthly average shading factor values.

The accuracy of the proposed model is mainly dependant on the accuracy of the selected algorithms and on how a real environment compares to a modeled one. At the current state of the software, the simplified evaluation of reflected radiation is probably the highest source of error. In order to verify the accuracy of the proposed procedure, a validation would be required. Since the shading factor cannot be directly measured, the knowledge of all the variables needed for its calculation would be necessary. The comparison between measured and calculated shading factor values goes beyond the scope of this paper, but it surely is an interesting task for the future.

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## NOMENCLATURE

| | | |
|---|---|---|
| $A$ | Area | [$m^2$] |
| $Fs$ | Shading factor | [-] |
| $H$ | Irradiation | [$J/m^2$] |
| $I$ | Irradiance | [$W/m^2$] |
| $R$ | Radiance | [$W/m^2sr$] |
| $h$ | Height | [m] |
| $\Sigma$ | Tilt angle | [rad] |
| $\Omega$ | Solid angle | [sr] |
| $\beta$ | Altitude angle | [rad] |
| $\gamma$ | Orientation | [rad] |
| $\theta$ | Incidence angle | [rad] |
| $\rho$ | Albedo | [-] |
| $\psi$ | Azimuth angle | [rad] |
| $\zeta$ | Opacity factor | [-] |

**Subscripts**

| | |
|---|---|
| $b$ | Direct |
| $d$ | Diffuse |
| $h$ | Horizontal |
| $m$ | Average |
| $r$ | Reflected |
| $s$ | Shaded |
| $w$ | Window |

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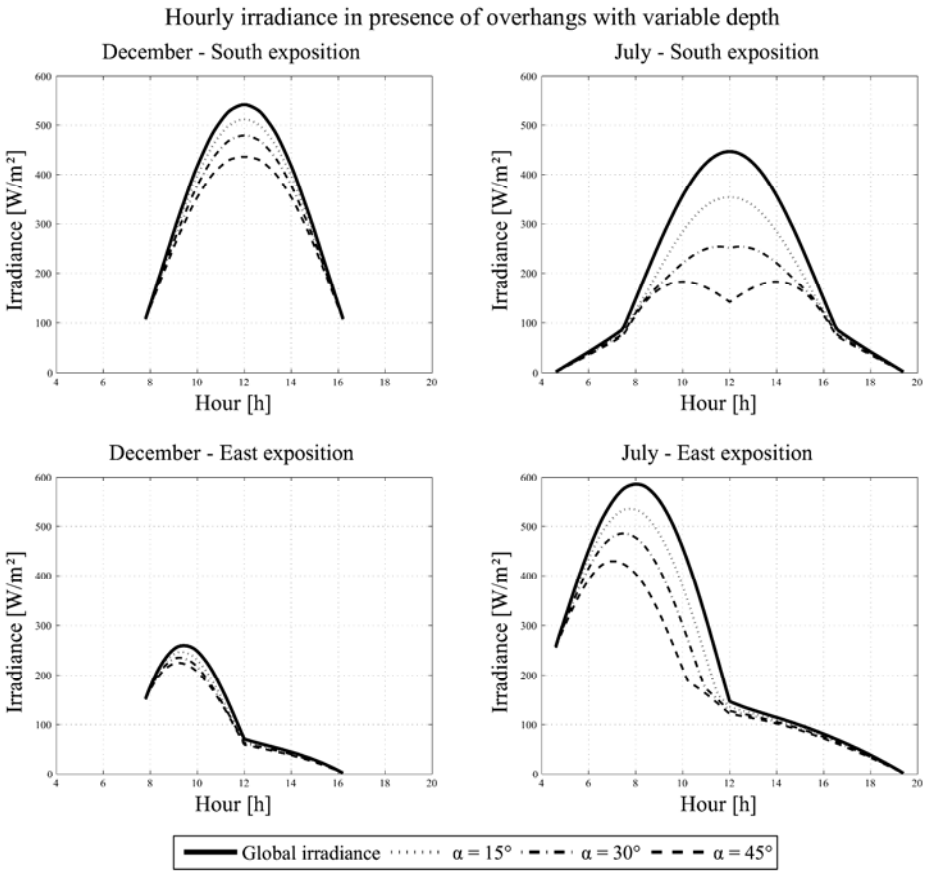


*Figure 3 Hourly irradiance on a squared window in presence of overhangs seen with a 15°, 30° and 45° angle*

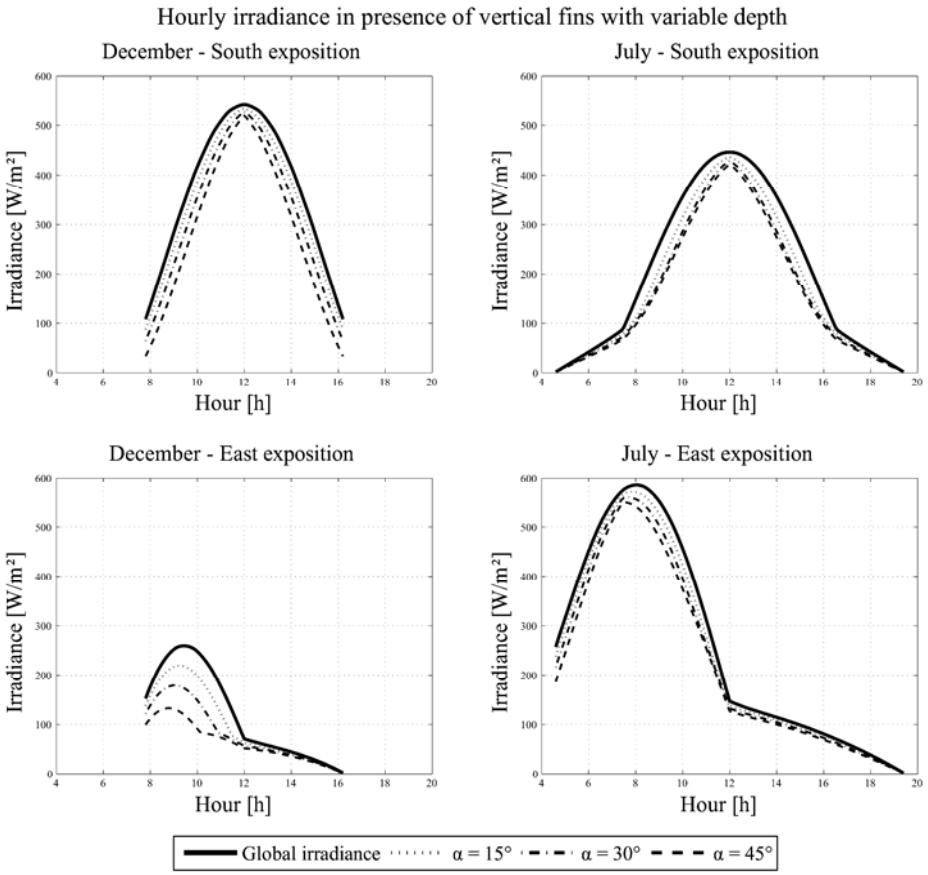


*Figure 4 Hourly irradiance on a squared window in presence of vertical fins seen with a 15°, 30° and 45° angle*