

Erratum to: The Controllability of the Gurtin-Pipkin Equation: A Cosine Operator Approach

Original

Erratum to: The Controllability of the Gurtin-Pipkin Equation: A Cosine Operator Approach / Pandolfi, Luciano. - In: APPLIED MATHEMATICS AND OPTIMIZATION. - ISSN 0095-4616. - 64:3(2011), pp. 467-468. [10.1007/s00245-011-9149-6]

Availability:

This version is available at: 11583/2450775 since:

Publisher:

Published

DOI:10.1007/s00245-011-9149-6

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Correction to “The controllability of the Gurtin-Pipkin equation: a cosine operator approach”

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Author version. Paper published in *Applied Mathematics and Optimization* Volume 64, Issue 3 (2011), Page 467-468

Lemma 18 states that $A^{-1}[\mathcal{R}_\infty]^\perp \subseteq [\mathcal{R}_\infty]^\perp$. Its proof is based on Lemma 17 which is not correct since an integral in the (sketched) computations does not cancel out. A proof of Lemma 18 which does not use Lemma 17 is as follows.

Using formula (7), the Laplace transform of $\theta(t)$ with $\theta(0) = 0$ is

$$\hat{\theta}(\lambda) = -A \left(\frac{\lambda}{\hat{b}(\lambda)} I - A \right)^{-1} D\hat{u}(\lambda). \quad (1)$$

Let $u(t) = u_0 e^{-t}$. For every λ (in a right half plane) and $\xi \perp R_\infty$ we have

$$0 = -\langle \xi, \hat{\theta}(\lambda) \rangle = \frac{1}{1 + \bar{\lambda}} \left\langle \xi, A \left(\frac{\lambda}{\hat{b}(\lambda)} I - A \right)^{-1} D u_0 \right\rangle, \quad \forall u_0 \in U.$$

The assumptions on $b(t)$ imply that this equality can be extended by continuity to $\lambda = 0$ and for $\lambda = 0$ we have $\langle \xi, D u_0 \rangle = 0$ for every $u_0 \in U$. Hence, if $\xi \perp R_\infty$ then $\xi \perp \text{im } D$.

Now we use (1). We use $A = A^*$ and we get

$$\begin{aligned}
-\langle A^{-1}\xi, \hat{\theta}(\lambda) \rangle &= -\langle \xi, A^{-1}\hat{\theta}(\lambda) \rangle = \left\langle \xi, A \left\{ A^{-1} \left(\frac{\lambda}{\hat{b}(\lambda)} I - A \right)^{-1} \right\} D\hat{u}(\lambda) \right\rangle \\
&= \left[\frac{\hat{b}(\lambda)}{\lambda} \right] \left\{ \langle \xi, D\hat{u}(\lambda) \rangle + \left\langle \xi, A \left(\frac{\lambda}{\hat{b}(\lambda)} I - A \right)^{-1} D\hat{u}(\lambda) \right\rangle \right\}
\end{aligned}$$

When $\xi \perp R_\infty$, the first addendum is zero since we proved $\xi \perp \text{im } D$. The second addendum is $\langle \xi, \hat{\theta}(\lambda) \rangle = 0$. So, $\langle A^{-1}\xi, \hat{\theta}(\lambda) \rangle = 0$, i.e. $\langle A^{-1}\xi, \theta(t) \rangle = 0$ for every t and every control $u(t)$, as wanted.

References

- [1] L. Pandolfi (2005) The controllability of the Gurtin-Pipkin equation: a cosine operator approach. *Appl Mat Optim* 52: 143-165