Erratum to: The Controllability of the Gurtin-Pipkin Equation: A Cosine Operator Approach

Correction to "The controllability of the Gurtin-Pipkin equation: a cosine operator approach"

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Lemma 18 states that $A^{-1}[\mathcal{R}_\infty]^\perp \subseteq [\mathcal{R}_\infty]^\perp$. Its proof is based on Lemma 17 which is not correct since an integral in the (sketched) computations does not cancel out. A proof of Lemma 18 which does not use Lemma 17 is as follows.

Using formula (7), the Laplace transform of $\theta(t)$ with $\theta(0) = 0$ is

$$\hat{\theta}(\lambda) = -A\left(\frac{\lambda}{\hat{b}(\lambda)}I - A\right)^{-1} D\hat{u}(\lambda)\,. \tag{1}$$

Let $u(t) = u_0 e^{-t}$. For every λ (in a right half plane) and $\xi \perp R_\infty$ we have

$$0 = -\langle \xi, \hat{\theta}(\lambda)\rangle = \frac{1}{1+\bar{\lambda}}\left\langle \xi, A\left(\frac{\lambda}{\hat{b}(\lambda)}I - A\right)^{-1} Du_0\right\rangle\,, \qquad \forall u_0 \in U\,.$$

The assumptions on $b(t)$ imply that this equality can be extended by continuity to $\lambda = 0$ and for $\lambda = 0$ we have $\langle \xi, Du_0\rangle = 0$ for every $u_0 \in U$. Hence, if $\xi \perp R_\infty$ then $\xi \perp \operatorname{im} D$.

Now we use (1). We use $A = A^*$ and we get

$$-\langle A^{-1}\xi, \hat{\theta}(\lambda)\rangle = -\langle \xi, A^{-1}\hat{\theta}(\lambda)\rangle = \left\langle \xi, A\left\{A^{-1}\left(\frac{\lambda}{\hat{b}(\lambda)}I - A\right)^{-1}\right\} D\hat{u}(\lambda)\right\rangle$$

$$= \overline{\left[\frac{\hat{b}(\lambda)}{\lambda}\right]}\left\{\langle \xi, D\hat{u}(\lambda)\rangle + \left\langle \xi, A\left(\frac{\lambda}{\hat{b}(\lambda)}I - A\right)^{-1} D\hat{u}(\lambda)\right\rangle\right\}$$

When $\xi \perp R_\infty$, the first addendum is zero since we proved $\xi \perp \operatorname{im} D$. The second addendum is $\langle \xi, \hat{\theta}(\lambda)\rangle = 0$. So, $\langle A^{-1}\xi, \hat{\theta}(\lambda)\rangle = 0$, i.e. $\langle A^{-1}\xi, \theta(t)\rangle = 0$ for every t and every control $u(t)$, as wanted.

References

[1] L. Pandolfi (2005) The controllability of the Gurtin-Pipkin equation: a cosine operator approach. Appl Mat Optim 52: 143-165