

Two-dimensional shearless turbulent mixing: kinetic energy self diffusion, also in the presence of a stable stratification

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Two-dimensional turbulence is important in many natural and engineering contexts. It presents some special and interesting phenomena that does not occur in three dimensional turbulence. Moreover, it also idealizes geophysical phenomena in the atmosphere, oceans and magnetosphere and provides a starting point for the modeling of these phenomena.

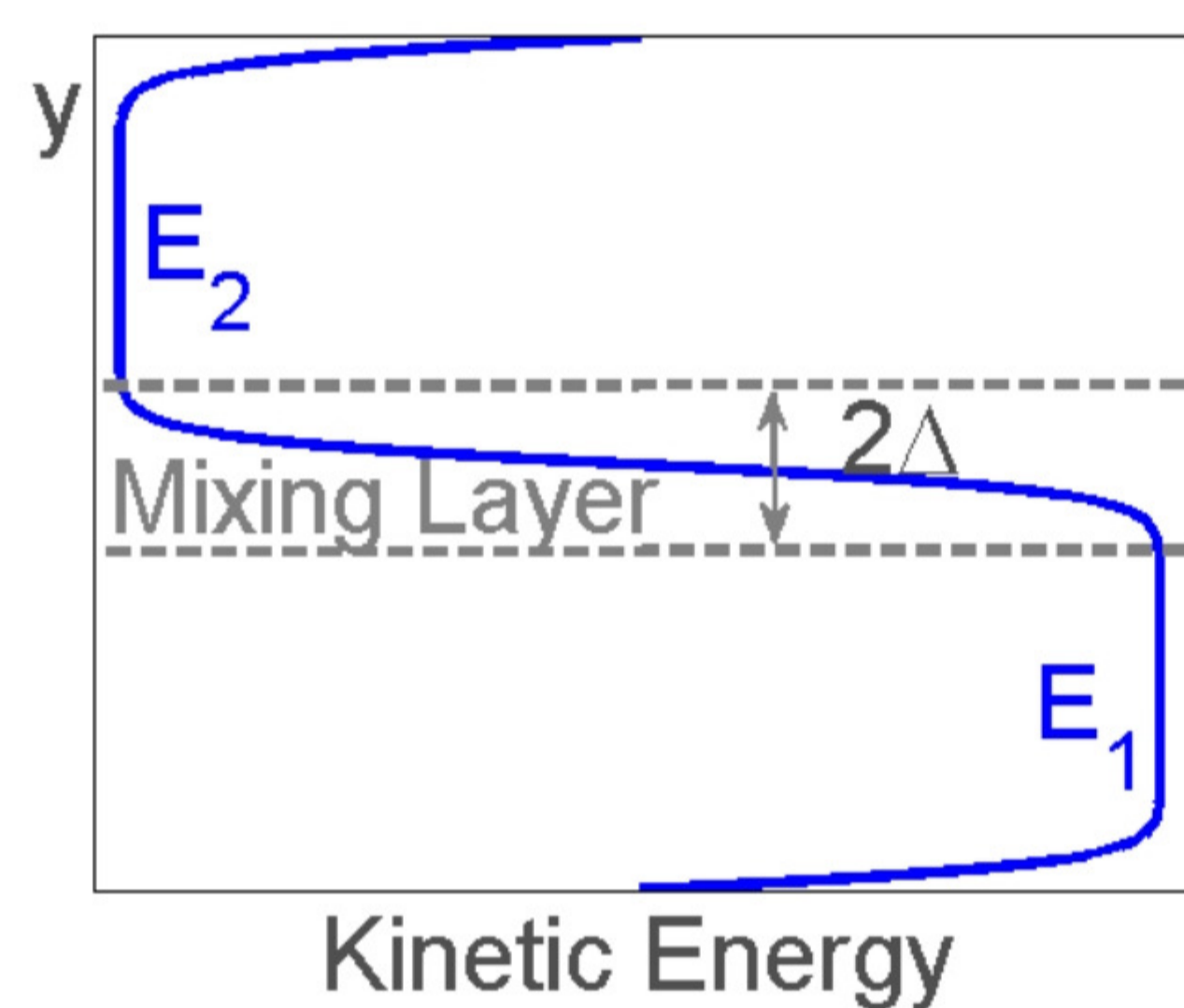
In this contest, we would like to present new results concerning the turbulent energy transport in the simplest kind of two dimensional inhomogeneous flow, a turbulent shearless mixing process generated by the interaction of two isotropic turbulent fields with different kinetic energy but the same spectrum shape. The self diffusion of kinetic energy is numerically (DNS) observed in two cases: with and without a stable density stratification.

Ustratified Mixing

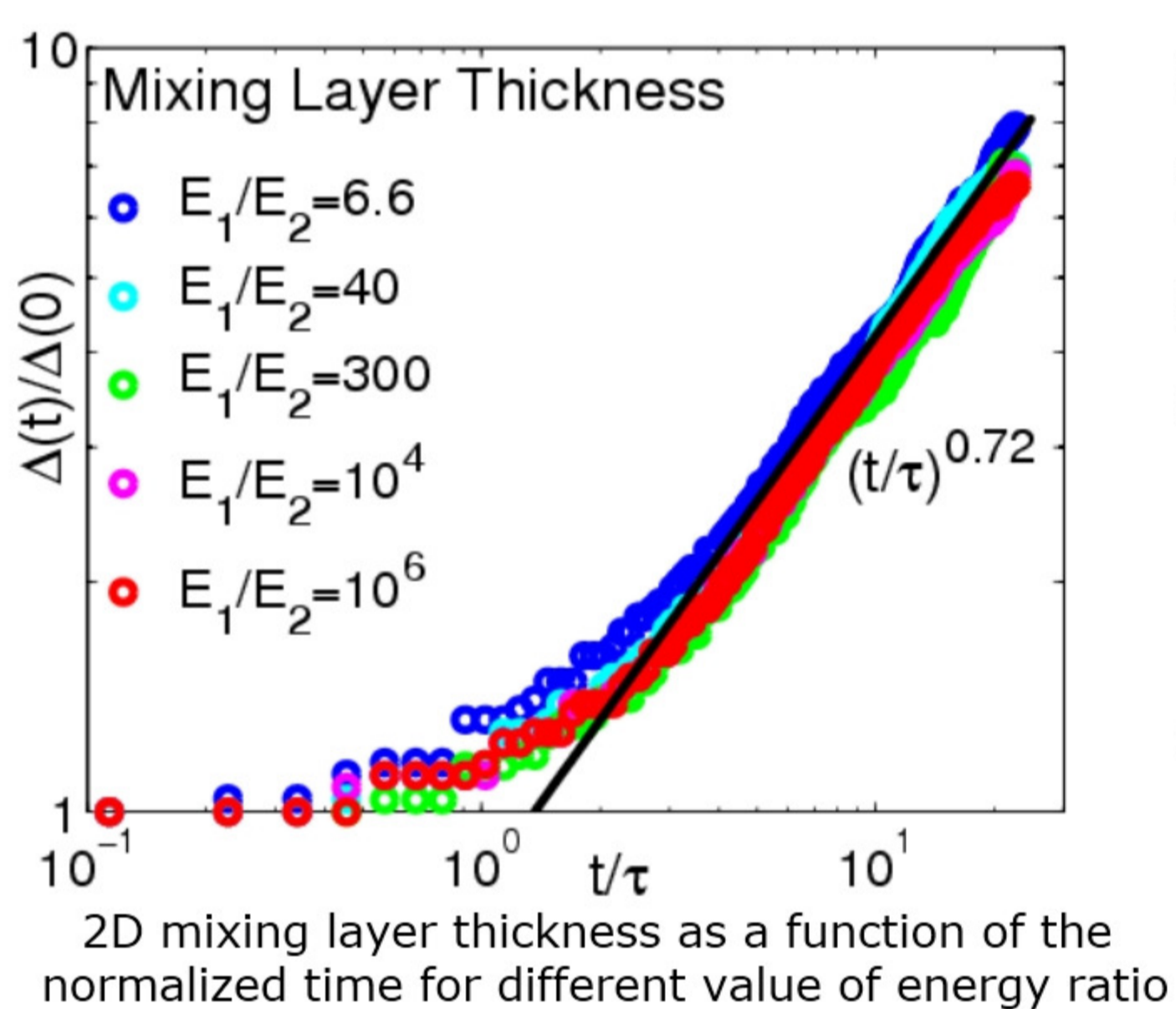
Two decaying turbulent field with Kinetic Energy E_1 and E_2 are matched by means of a hyperbolic tangent function

$$u(x) = u_1(x)p(x) + u_2(x)(1-p(x))$$

$$p(x) = \frac{1}{2} \left[1 + \tanh\left(a\frac{x}{L}\right) \tanh\left(a\frac{x-L/2}{L}\right) \tanh\left(a\frac{x-L}{L}\right) \right]$$



The ratio of the turbulent kinetic energy has been chosen as the sole control parameter



We define a penetration as the position of the maximum of the skewness normalized over the mixing layer thickness

$$\eta = \frac{x_s}{\Delta(t)/\tau}$$

and the diffusion velocity

$$v_D = \frac{dx_s}{dt} = \eta \frac{d\Delta}{dt}$$

$$2D: \frac{\Delta(t)}{\Delta(0)} \propto \frac{t^{0.72}}{\tau}$$

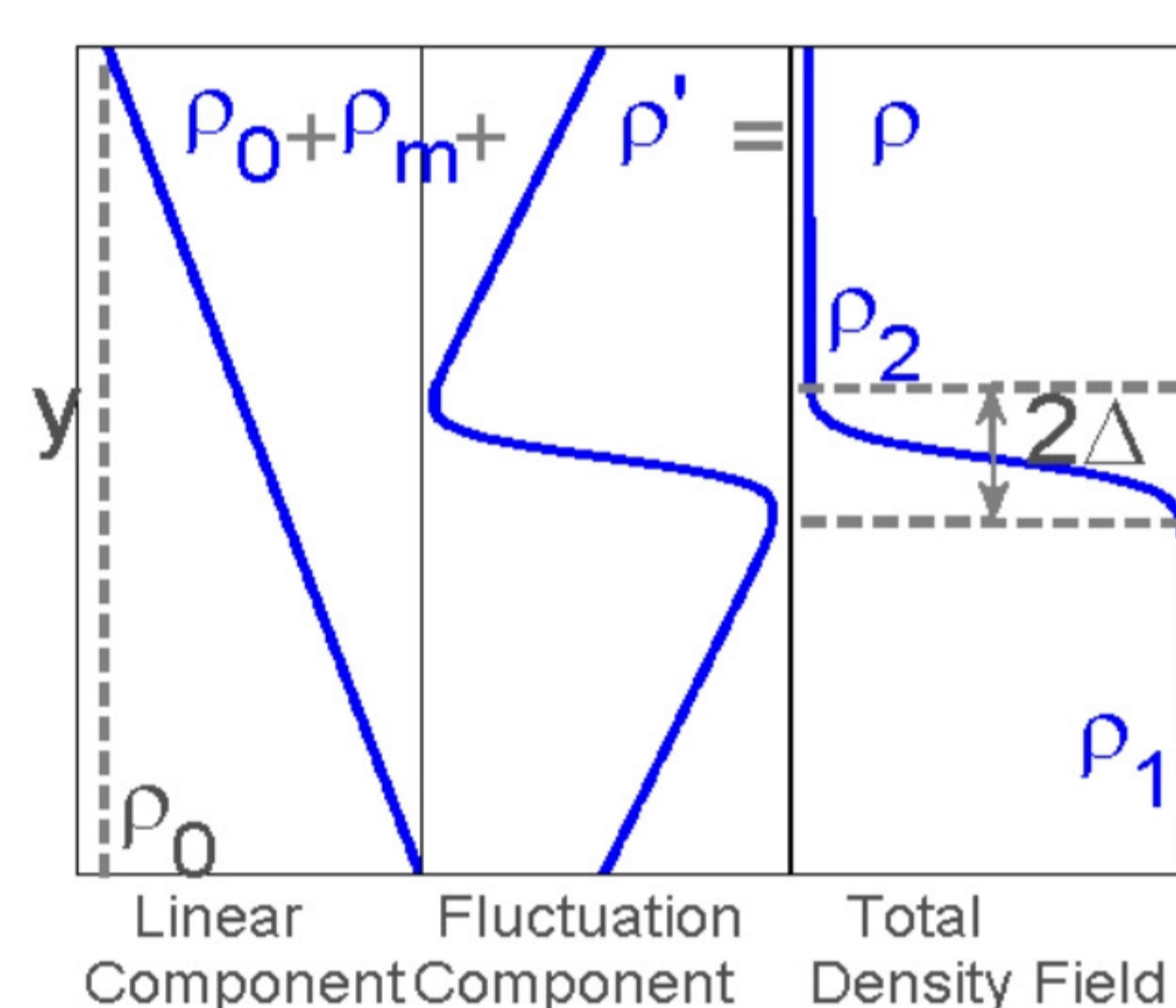
$$3D: \frac{\Delta(t)}{\Delta(0)} \propto \frac{t^{0.43}}{\tau}$$

$$v_D = \frac{\eta d(t^{0.72}/\tau)t}{dt} \propto t^{-0.28}$$

$$v_D = \frac{\eta d(t^{0.43}/\tau)t}{dt} \propto t^{-0.57}$$

The results show that the mixing layer is highly intermittent in the self similar stage of decay: the experiment shows that the presence of a turbulent energy gradient is sufficient for the appearance of intermittency and anisotropy. We can observe an intermittency front advanced in position in respect with the mixing layer thickness

Stratified Mixing



We change the experiment by adding the effect of a stable stratification. We create an initial density field by combining two constant density fields with the same hyperbolic tangent used for the vorticity field.

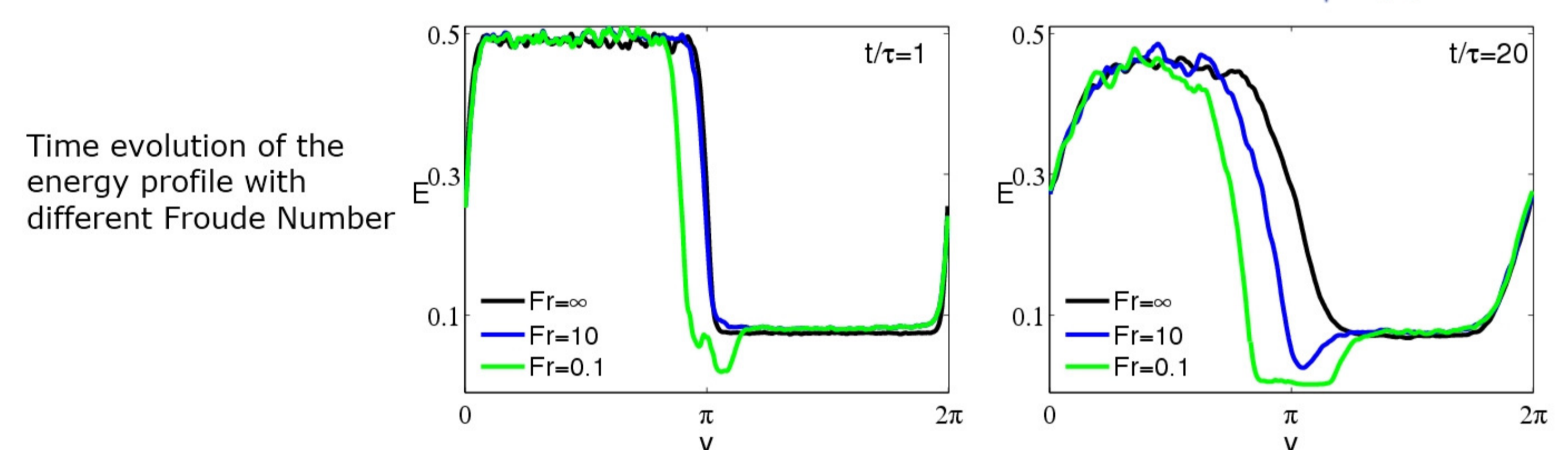
$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho_0} \nabla p - \frac{\rho'}{\rho_0} g + \nu' u$$

$$\frac{\partial \rho'}{\partial t} + (u \cdot \nabla)\rho' + v \frac{d\rho_m}{dy} = k' u$$

The results obtained in this way can be considered as the vertical section of a three-dimensional stratified flow.

The Froude number has been chosen as the sole control parameter.

$$Fr = \frac{U}{\sqrt{-\frac{g}{\rho_0} \frac{\partial \rho_m}{\partial y} L}}$$

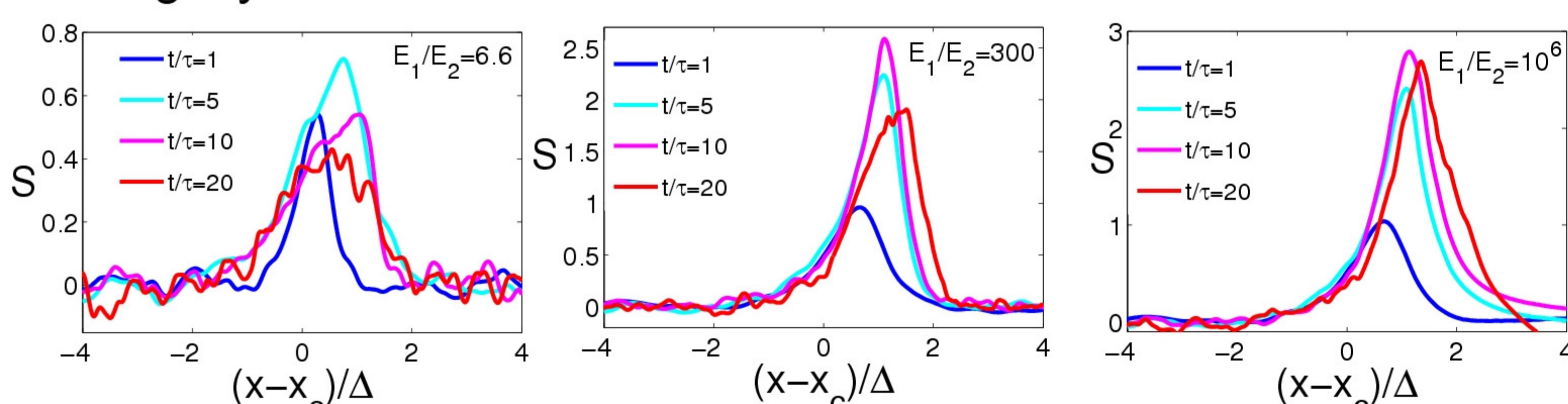


The energy profile in the mixing region is lower than the minimum value imposed by the initial condition, which shows the effect of the buoyancy force work.

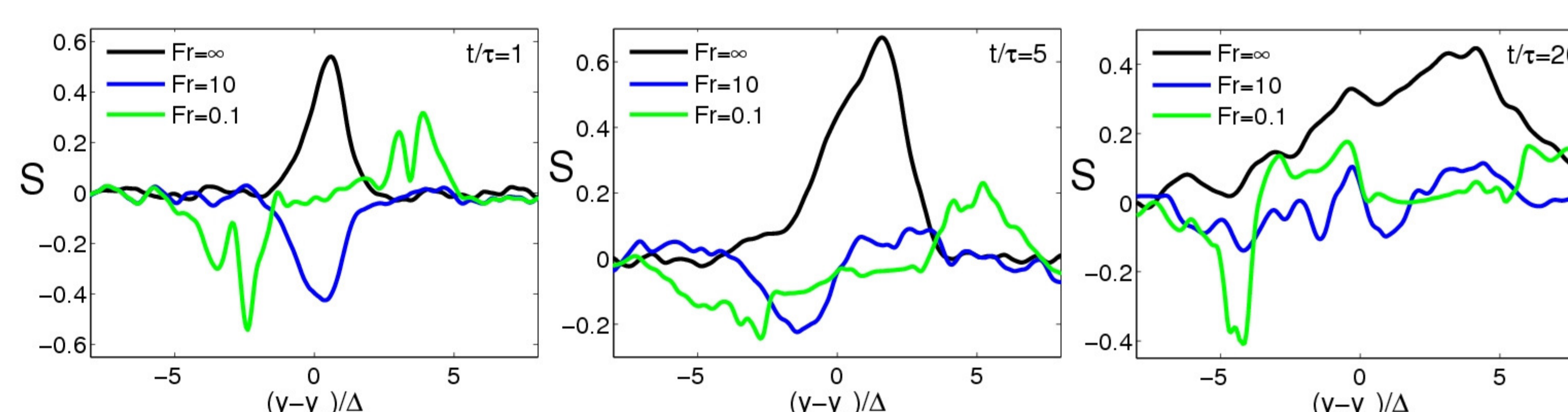
This is reasonable since this is the only area in which a density gradient is present from initial time.

So close to interface the work done by buoyancy forces reaches its maximum value and the interface behaves like a energy hole.

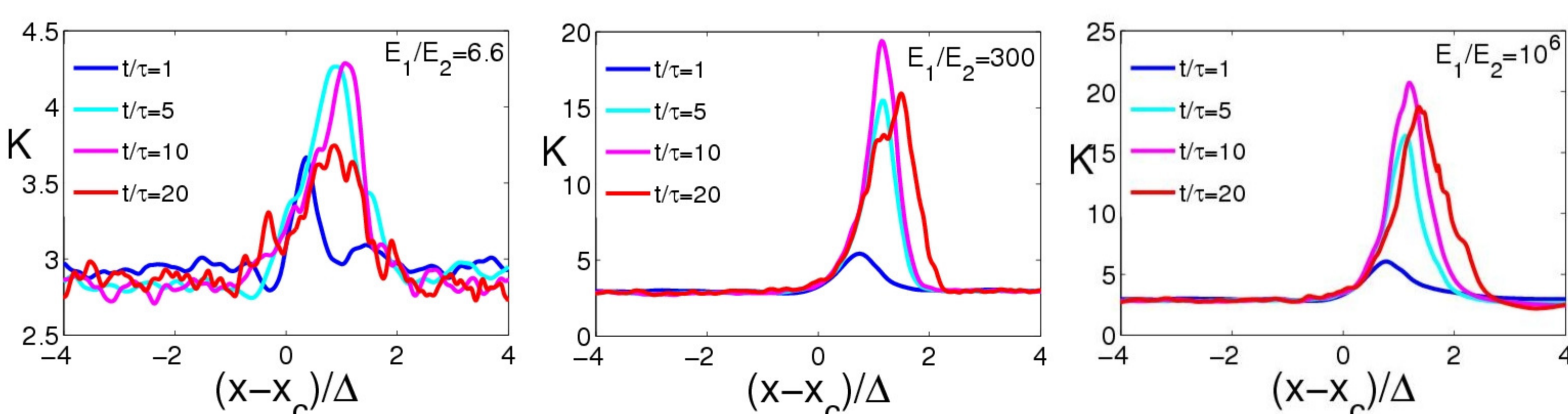
The velocity skewness enlightens the generation of an inverse energy flow and intermittent penetration from the low to the high energy field even in the case of mild stratification.



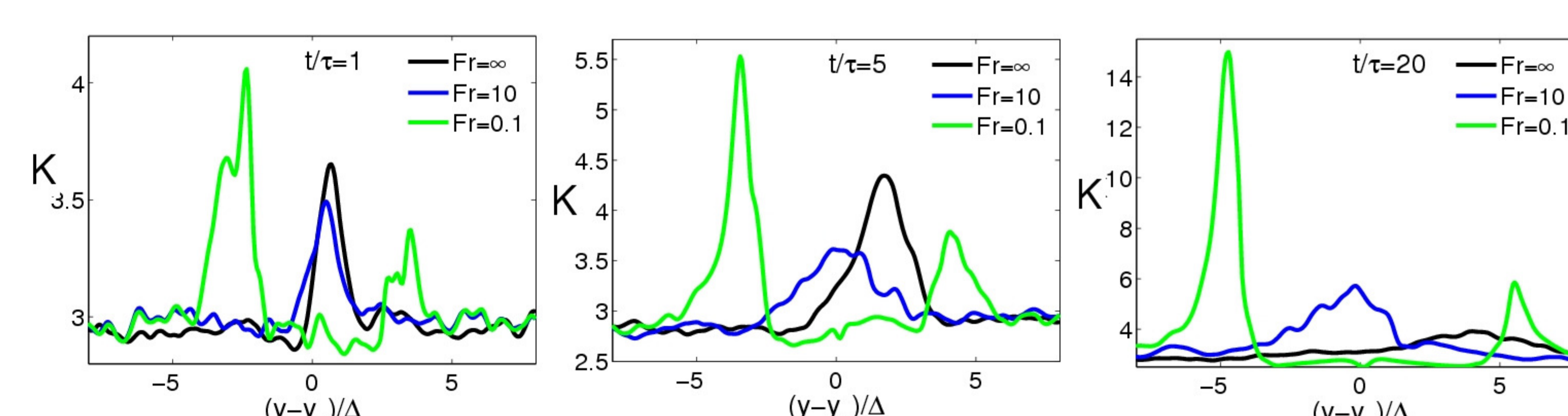
Skewness of the velocity component in the inhomogeneous direction for some energy ratio normalized over the center and the thickness of the mixing layer.



Skewness of the velocity component in the inhomogeneous direction for each Froude Number normalized over the center and the thickness of the mixing layer



Kurtosis of the velocity component in the inhomogeneous direction for some energy ratio normalized over the center and the thickness of the mixing layer



Kurtosis of the velocity component in the inhomogeneous direction for each Froude Number normalized over the center and the thickness of the mixing layer