Energy spectrum power-law decay of linearized perturbed shear flows

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Abstract:
The -5/3 power-law scaling of the energy spectrum in the inertial range is a well-known notion in the phenomenology of turbulence (in the sense of Kolmogorov 1941), and it is confirmed both in laboratory and numerical simulations (see for instance [1]). We propose to study the state that precedes the onset of instability and transition to turbulence to: (a) understand whether the nonlinear interaction among different scales in fully developed turbulence can affect the energy spectrum, and to (b) quantify the level of generality on the value of the energy decay exponent of the inertial range. In this condition, the system shows all the features (e. g. linearized convective transport, linearized vortical stretching, and molecular diffusion) as those characterizing the turbulent state, except the nonlinear interaction. The perturbative transient dynamics, which is governed by the initial-value problem related to the linearized perturbative Navier-Stokes equations, is very complicated and shows a great variety of different behaviours [2, 3]. We ask whether the linearized perturbative system is able to show a power-law scaling for the energy spectrum in an analogous way to the Kolmogorov argument [4].

We determine the decay exponent of the energy spectrum for arbitrary three-dimensional perturbations acting on three different typical shear flows - i.e. the bluff-body wake, the cross-flow boundary layer and the plane Poiseuille channel flow - for stable and unstable configurations. Then, we evaluate the energy spectrum of the linearized perturbative system - as the wavenumber distribution of the perturbation kinetic energy density in asymptotic condition - and we compare it with the well-known -5/3 Kolmogorov power-law scaling.

In general, we observe about a decade ($k \in [2, 20]$ and $k \in [15, 150]$ for the bluff-body wake and the plane Poiseuille flow, respectively) where longitudinal and oblique perturbations present a power-law decay which is close to -5/3 (violet curves), while purely three-dimensional waves have a decay of about -2 (red curves). For larger wavenumbers ($k > 20$ and $k > 150$ for the bluff-body wake and the plane Poiseuille flow, respectively), all perturbations show a power-law decay very close to -2 (red curves). For longer waves ($k < 1 - 2$ and $k < 10$ for the bluff-body wake and the plane Poiseuille flow, respectively), results do not seem to reveal any characteristic nor universal behavior. The energy spectrum strongly depends, here, on initial and boundary conditions.

This scenario is confirmed by extensive data concerning the bluff-body wake (see (a) and (b) parts of Fig. 1) as well as preliminary results for the plane Poiseuille flow (part (c) of Fig. 1).

So far, we think it possible to formulate the hypotheses that the nonlinear interaction is maybe not the main factor responsible of the specific value of the -5/3 decay exponent in the energy spectrum and the spectral power-law scaling of intermediate waves is a general dynamical property of the Navier-Stokes solutions which encompasses the nonlinear interaction.

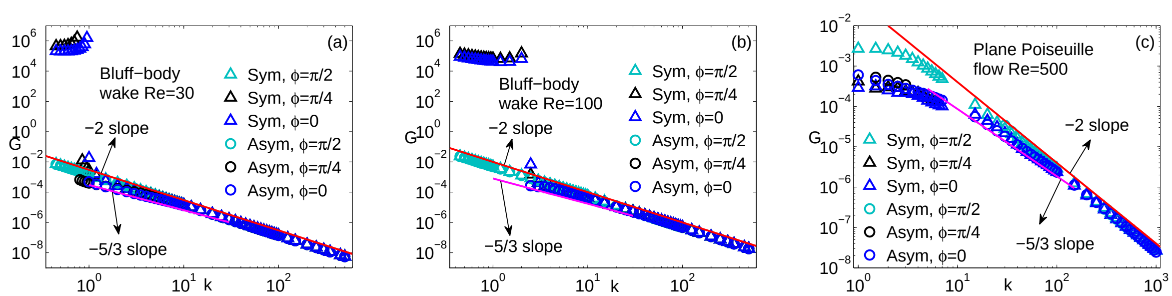

Figure 1: Energy spectrum G of symmetric (triangles) and asymmetric (circles) perturbations (blue: $\phi = 0$, black: $\phi = \pi/4$, light blue: $\phi = \pi/2$). (a)-(b) Bluff-body wake at $Re = 30$ (stable) and $Re = 100$ (unstable), respectively. (c) Plane Poiseuille flow at $R = 500$ (stable). Red and violet curves: -2 and $-5/3$ slopes, respectively.

References

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