Physics-based Small-Signal sensitivity analysis for the variability aware assessment of devices and linear analog subsystems

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Program

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Tuesday May 31st

8h15 **Opening**

Session D: Design Tools and Methodologies for Variability

8h30 - 9h15

Chair: Nicolas VERKINDEREN - TI

Keynote : Vassilios GEROUSIS and Igor KELLER – Cadence, USA

Advances and Challenges in Variations-aware Static Timing Analysis

Strong process variations is a characteristic feature of advanced process nodes since 60-nm. A wealth of papers has been published since middle of last decade proposing various techniques of accounting for process and environment variations in Static Timing Analysis (STA). The most prominent approach is Statistical STA which enjoyed strong interest in academia but has so far not been widely adopted in design community. Primary reason for the slow adoption of SSTA is its high runtime and memory costs, nontrivial effort in interpretation of results, inability to adequately model nonlinear or non-Gaussian variations and huge size of statistical timing libraries and extracted parasitics. An alternative approach based on running multiple process corners has prevailing usage in industry, but it also suffers from several setbacks: (i) explosion of corners, (ii) difficulties in generating and managing large set of libraries and RC for various corners, (iii) lack of or inefficient support of MMMC in available industrial STA solutions. Recently several new methods of handling on-chip variations in STA have emerged, including Advanced OCV (AOCV) which became popular due to its low cost yet relatively good accuracy. Being an approximation of SSTA, AOCV is based on applying derating factors which are calculated in advance. At the same time various simplified flavors of SSTA (a Poor Men's SSTA) have been evaluated such as based on extraction of statistical models from corner libraries, different combinations of MMMC and SSTA and others. In this talk we compare the mentioned methods of handling process variations in STA and discuss pro's and con's of each approach.

Vassilios Gerousis, Senior Architect & Technologist, Cadence.
Vassilios Gerousis is veteran of the semiconductor industry with over 25 years working at TI, Motorola and Infineon. Vassilios is now Senior Architect and Technologist of Digital IC Design Implementation at Cadence Design Systems, heavily involved/driving the Cadence SSTA solution. Prior to Cadence, Vassilios worked as a senior technologist for 7 years at Infineon in Germany. After doctorate graduation in EE from Northeastern University in Boston, he joined TI in 1979 for

three years at Houston, Texas. He worked for Motorola for 18 years in both Austin Texas and then Chandler, Arizona. Vassilios has focused in several areas: Physical design for advanced nodes, High Speed Digital Design, 3D IC design and analysis.

Igor Keller, Architect, Cadence Design Systems, Inc.
Igor Keller received his M.A. and PhD degrees in Mechanics and Applied Mathematics from University of Perm, Russia. After that he spent several years in Russian Academy of Sciences and Israel Institute of Technology (Technion) researching complex phenomena in distributed nonlinear systems. For the last 9 years Igor has been with Cadence Designs Systems working on advanced noise analysis, delay calculation and cell modeling technologies. Prior to Cadence, he worked at Intel developing static noise and timing analysis solutions.

9h15

Statistical Timing Characterization of Standard Cells with Semi-Monte-Carlo Method

Zeqin Wu[1], Philippe Maurine[1], Nadine Azemard[1] and Gilles Ducharme[2]

[1] LIRMM
[2] Université Montpellier II

9h35

System Level Analysis and Accurate Prediction of Electromigration

Tushar Cupta[1], Olivier Heron[1], Nicolas Ventroux[1], Thomas Zimmer[1], François Marc[2] and Clement Bertolini[2]

[1] CEA-LIST
[2] Université de Bordeaux

9H55

Circuit-level modelling of SRAM minimum operating voltage Vddmin in the C40 node

Lorenzo Clampolini[1], Siddharth Gupta[1], Olivier Callen[1], Amit Chhabra[1], Dibya Dipti[1], Sebastien Haendler[1], Shishir Kumar[1], Daniel Noblet[1], Pierre Malinge[1], Nicolas Planes[1], David Turgis[1], Christophe Lecocq[1] and Shamsi Azmi[1]

[1] ST

10h15

Nonlinear modelling for sub 62nm IC statistical analysis

Yoann Courant[1], Philippe Hérédia[1] and Firas Mohamed[1]

[1] Infiniscale

10h35

Physics-Based Small-Signal Sensitivity Analysis for the Variability Aware Assessment of Devices and Linear Analog Subsystems

Francesco Bertazzi[1], Fabrizio Bonani[1], Simona Donati Guerrieri[1] and Giovanni Ghione[1]

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Physics-Based Small-Signal Sensitivity Analysis for the Variability Aware Assessment of Devices and Linear Analog Subsystems

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***Abstract*—This paper presents a general framework for the assessment of the physics-based small-change sensitivity analysis of AC device performances as a function of process parameters. The proposed technique is based on the linearization of a partial-derivative physical model (e.g. the drift-diffusion transport system) around a nominal process parameter, and on the evaluation of relevant Green's functions linking the parameter variations in each point of the device to external, circuit-oriented AC performances. This technique allows for a considerable saving of simulation time, even with respect to previously proposed AC sensitivity tools, also based on Green's functions. An example of the evaluation of the Y-parameters variability of a high frequency MESFET device is presented and validated, proving that this technique is the basis for a general variability assessment of analog devices and subsystems in linear conditions.**

I. INTRODUCTION

During the last few years, nanometer CMOS processes have been increasingly exploited in the implementation of RF analog and mixed-signal subsystems [1], [2], such as the receiver and transmitter blocks appearing in most radio applications below the X band. Further, device downsizing currently allows CMOS circuits to reach the microwave and millimeter wave range [3], that are traditionally covered by GaAs- or InP-based transistors such as MESFETs, (P)HEMTs and HBTs. From a technological standpoint, process variability is a serious issue also in Schottky-gate compound semiconductor FETs, where nanometer gate lengths are obtained by lateral etching rather than lithography alone. Process variability affects analog subsystems in a rather different way when compared to digital subsystems. While the complexity of analog circuits is much lower than in the digital case, thus making the variability issue intrinsically less dramatic, process variability may induce fluctuations in the device small-signal parameters (such as those influencing gain and input/output matching) in potentially critical devices, therefore deeply affecting the system performances (e.g., the receiver sensitivity). As a consequence, process variability may influence in a complex way the operation of linear or quasi-linear analog blocks, such as low-noise amplifiers and mixers, leading to a potential deterioration of the overall power or conversion gain and of the noise figure through a combination of transconductance fluctuations and input and output mismatch with respect to the optimum (power or noise) conditions.

When analyzing the variability of analog circuits with respect to process parameters, a basic, first-order tool is provided by the linear sensitivity model, relating small variations of the circuit performances P, ΔP, to small variations of some technological parameter σ, $\Delta\sigma$. The relevant figure of merit is therefore $S_{\sigma}^{P} = \Delta P/\Delta\sigma \approx \partial P/\partial\sigma$. To perform such an analysis, one must evaluate the circuit sensitivity vs. variations of the active device small-signal parameters, and the sensitivity of those vs. the technological parameters. Although well established techniques exist to model and analyze the variability of the analog circuit performances induced by small-signal parameter fluctuations in the active devices on the basis of sensitivity-based linear approaches [4], problems arise whenever such device small-signal parameter fluctuations have to be connected, at a modeling level, to the variations of technological parameters (such as gate length, electrode spacing, doping levels etc.). On the other hand, while the evaluation through physics-based models of the sensitivity of DC parameters such as the drain or collector current with respect to technological parameter variations is available through efficient techniques [5], [6], the computation of the small-signal parameter sensitivity through a physics-based model is in principle a formidable task, since such sensitivities are, in fact, second-order sensitivities of the device short-circuit currents or open-circuit voltages [5].

In the present paper we propose a novel approach to sensitivity analysis that exploits, as a starting point, the large-signal physics-based linearized model already proposed by some of the present authors in [7], based on an efficient Green's function technique. This new approach basically provides, under arbitrary loading conditions, the sensitivities of the output current or voltage harmonics (DC, fundamental, higher harmonics) with respect to the variations of technological parameters. For the sake of definiteness, let us concentrate on a device where frequency conversion is negligible, e.g. the active element of a low-noise or high-gain amplifier. In the presence of a small enough AC input and output excitation superimposed to a DC bias, the first-harmonic output currents under short-circuit loading conditions simply are the (normalized) elements of the device admittance matrix and their sensitivities are the small-signal sensitivities of the Y-parameters. The same idea allows to evaluate the sensitivities of the conversion matrix elements when the device is operated in the so-called small-signal large-signal regime. From small-signal or conversion parameter sensitivities, the sensitivity of the circuit performances can be finally evaluated; notice that since the device-level large-signal based sensitivity analysis requires a mixed-mode simulation including the active device together with the embedding circuit, circuit variability can be in principle evaluated directly through the mixed-mode simulation.

II. PHYSICS-BASED SMALL-SIGNAL SENSITIVITY

The formulation of sensitivity analysis for semiconductor devices described by physics-based partial differential equation (PDE) models, i.e. models solving for the microscopic internal distributions of variables in the device on the basis of conservation equations derived in turn from the moments of the Boltzmann transport equation, can be found in [5], [6]. We summarize here the results, exploiting a general formulation valid for all orders of PDE models, although of course the numerical burden rapidly increases as nonstationary transport is taken into account. For this reason, we make explicit reference to the bipolar drift-diffusion transport description, made of Poisson and the electron and hole continuity equations.

Furthermore, since the aim of sensitivity analysis is the assessment of the variation of terminal electrical variables as a function of spatially localized (deterministic or random) fluctuations inside the device volume, we add to the PDE system (and to the relevant boundary conditions) a set of equations linking the observation variables (here collectively denoted as $\boldsymbol{o}(t)$, where $\boldsymbol{o}$ may contain currents, voltages or any combination of these) to the microscopic PDE system unknowns. After spatial discretization of the PDE part, the resulting system can be expressed as

$$\begin{cases} \boldsymbol{F}\left(\boldsymbol{\psi}, \boldsymbol{n}, \boldsymbol{p}, \dot{\boldsymbol{n}}, \dot{\boldsymbol{p}}; \sigma\right) = \boldsymbol{0} \\ \boldsymbol{\chi}\left(\boldsymbol{\psi}, \boldsymbol{n}, \boldsymbol{p}; \boldsymbol{s}, \sigma\right) = \boldsymbol{0} \\ \boldsymbol{O}\left(\boldsymbol{\psi}, \boldsymbol{n}, \boldsymbol{p}, \dot{\boldsymbol{n}}, \dot{\boldsymbol{p}}; \boldsymbol{o}\right) = \boldsymbol{0} \end{cases} \tag{1}$$

where $\boldsymbol{F} = [F^{(\psi)}, F^{(n)}, F^{(p)}]^{\mathrm{T}}$ includes ($^{\mathrm{T}}$ denotes transpose) the discretized Poisson, electron continuity and hole continuity equations, respectively; $\boldsymbol{\chi} = [\chi^{(\psi)}, \chi^{(n)}, \chi^{(p)}]^{\mathrm{T}}$ corresponds to the discretized boundary conditions; $\boldsymbol{O}$ represents the constitutive equations of the observable terminal electrical variables $\boldsymbol{o}$. Furthermore, $\dot{\alpha}$ denotes the first time derivative of $\alpha(t)$. Finally, $\boldsymbol{s}$ is a set of external generators applied to the device terminals. Notice that adding the set of observation variables to the systems of equations leads to a better formulation of the sensitivity problem, especially within the Green's function approach, with a negligible overhead in terms of matrix size (details will be given elsewhere).

The device discretized equations and boundary conditions depend on the parameter set σ, relative to physical and technological data such as e.g. mobility models, doping, device dimensions (geometry) etc. For the sake of simplicity we consider here a single parameter only: the extension to the vector case (e.g., the doping level spatially discretized on the simulation mesh) is trivial, since linearity guarantees the application of the superposition principle. A constant variation $\Delta\sigma$ with respect to the nominal value σ_0, is the primary cause of the variations we are looking for.

According to the shape of the source term s, different sensitivity analyses are actually possible:

- if $\boldsymbol{s} = \boldsymbol{s}_0$, i.e. the device is driven in time-invariant conditions, the resulting variations (of course proportional to $\Delta\sigma$ if a small-change analysis is carried out) allow to evaluate the device *DC sensitivity* [5];
- if $\boldsymbol{s} = \tilde{\boldsymbol{s}}(t)$ where the impressed generators are time-periodic, the parametric variations are used to estimate the *AC sensitivity* [6]. In case of a small amplitude time-varying component of $\boldsymbol{s}(t)$ (i.e., if $\boldsymbol{s} = \boldsymbol{s}_0 + \tilde{\boldsymbol{s}}(t)$ where $\tilde{\boldsymbol{s}}(t)$ is small enough to drive the device in linearity around $\boldsymbol{s}_0$), the *small-signal (SS) AC sensitivity* is defined [5].

Let us denote with subscript 0 the solution of (1) with the nominal value σ_0 of the parameter. If the parameter undergoes a variation $\Delta\sigma$ and we can assume that the corresponding variation in the observable variables $\Delta\boldsymbol{o}$ is small, linearity implies $\Delta\boldsymbol{o} \propto \Delta\sigma$. The linearity assumption allows to express all variables α as the superposition of the value for nominal parameter α_0 and the induced variation $\Delta\alpha$, and to obtain the equation system providing the link between $\Delta\boldsymbol{o}$ and $\Delta\sigma$ through a linearization of (1) around the nominal solution. For DC sensitivity analysis, the resulting problem is a time-invariant linear system, while in the AC case a *linear, periodically time-varying* (LPTV) system is defined. Notice that linearization [5], [6] implies an input term to the linear system which is proportional to the parameter variation $\Delta\sigma$ multiplied by a function of the unperturbed solution $\boldsymbol{f}^{(\beta)}$ ($\beta = \psi, n, p$ denotes the equation where the input term is placed) corresponding to the Jacobian elements of (1) related to the first derivative with respect to σ.

Because of the linearity of the system, we can provide a full characterization by evaluating the corresponding *Green's function*, which is the resulting fluctuation when the input parameter variation is a unit impulse (as a function of time and space). In spatially discretized form, provided that the generalized boxes discretization scheme is exploited, the unit impulse (a δ function in the continuous formulation) becomes simply a 1 in the node i where the δ function is centered. Therefore, the corresponding variation on observable o_j, i.e. the Green's function, is denoted as $G_{o_j,i}^{(\beta)}$, where $\beta = \psi, n, p$ represents the equation where the impulse source is placed.[1] Discretized spatial superposition allows to finally express the output variation as

$$\begin{aligned} \Delta o_j &= \sum_{\beta=\psi,n,p} \sum_i G_{o_j,i}^{(\beta)} f_i^{(\beta)} \Omega_i \Delta\sigma \\ &= \sum_i s_{\sigma,i}^{o_j} \Omega_i \Delta\sigma = S_\sigma^{o_j} \Delta\sigma \end{aligned} \tag{2}$$

where

$$s_{\sigma,i}^{o_j} = \sum_{\beta=\psi,n,p} G_{o_j,i}^{(\beta)} f_i^{(\beta)} \tag{3}$$

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[1] This simplified formulation holds since $\Delta\sigma$ is time independent.

is the (space dependent) *distributed sensitivity* [5]. Ω_i is the control volume associated to mesh node i. Clearly, the required sensitivity $S_{\sigma}^{o_j}$ is obtained as a spatial integral of the distributed sensitivity $s_{\sigma,i}^{o_j}$, which in turn is easily evaluated once the relevant Green's functions are available.

This formulation is very convenient from a numerical standpoint, since the required Green's functions, linking an external observation variable o_j to the variation of physical properties in each point inside the device, can be efficiently calculated by means of the numerical techniques in [7]–[9], originally devised for physics-based noise analysis and essentially exploiting an adjoint-like approach. The approach proposed in this paper has a major advantage also with respect to the SS sensitivity analysis proposed in [5], also based on Green's functions, but where the linearization with respect to the parameter variation is carried out on the linearized AC response calculated with nominal parameter. As a consequence, the second order derivative of the model equations is required, or rather (to be more precise) their Hessian matrix. Furthermore, the linearized system leading to the SS parameter variations is characterized by a forcing term which in turn depends on the DC variations of the model variables in the entire device volume. The determination of such variations can of course still be carried out exploiting the DC Green's functions, but since the resulting perturbation is required on the entire volume the observation points of the Green's functions should cover all the device, thus preventing the use of the efficient numerical techniques in [8]. Hence, the formulation in [5] does not seem suitable for inclusion into up-to-date TCAD tools, especially beacuse of the implementation effort required for the Hessian matrix calculation. Such technique will not be considered for comparison with other approaches in the following Sec. III. The technique proposed in this work, on the contrary, can be easily included into commercial tools, such as Synopsys Sentaurus, provided that a multi-harmonic mixed-mode simulation environment is available.

An example of SS AC sensitivity is the variation of the Y parameters of the device as a consequence of a variation of the gate length L. This is an example of geometric sensitivity [10] particularly significant in Schottky-gate III-V or III-N (e.g. AlGaN/GaN) (P)HEMTs, the latter being affected by considerable process immaturity. In this case, the device is biased by a DC value of terminal voltages and a small input tone $V_{\text{AC},j}\exp(\mathrm{i}\omega t)$ at frequency ω is superimposed to the DC voltage bias of terminal j. The terminal currents are also composed by the DC and the AC small-amplitude values. Namely, the current at the kth terminal will be given by a DC value $I_{\text{DC},k}$ plus a small amplitude tone $I_{\text{AC},k}\exp(\mathrm{i}\omega t)$. Once converted into the frequency domain, the system of equations will be solved with nominal value of gate length allowing for the simulataneous evaluation of nominal values of $I_{\text{DC},k}$ and $I_{\text{AC},k}$, $\forall k$. Note that the value of the AC phasor allows for the evaluation of the nominal Y parameters as

$$Y_{kj} = \frac{I_{\text{AC},k}}{V_{\text{AC},j}}.$$

Then, the system is linearized to assess the variation with respect to the gate length, and the relevant Green's functions are computed. Since we are interested into the sensitivity of AC parmeters, we have selected as the observation variable the variations of the AC phasors at each terminal: more precisely, the Green's function yields the sensitivity $S_L^{I_{\text{AC},k}}$ of the short-circuit AC current phasor at terminal k with respect to the variation of the gate length L. Since $V_{\text{AC},j}$ is an impressed voltage, the sensitivity of $I_{\text{AC},k}$ and of Y_{kj} are simply related through

$$S_L^{Y_{kj}} = \frac{S_L^{I_{\text{AC},k}}}{V_{\text{AC},j}}. \tag{4}$$

As a final remark, notice that, since the system is solved in frequency domain, keeping both DC and AC phasors, the variation of the DC current will be also be available at the same time as the sensitivity of the AC phasors, thus allowing for a simultaneous DC and AC sensitivity estimation environment.

III. EXAMPLE

As an example of application, we consider the sensitivity of Y-parameters as a function of the gate length of a typical GaAs MESFET for high-frequency applications: the epitaxial layer is 150 nm thick and n-doped with $N_{\text{D}} = 10^{17}$ cm^{-3}. The gate contact nominal length is 0.25 μm, while the gate-source spacing is 0.9 μm and the gate-drain distance is 2.5 μm. The simulation has been performed with a DC bias $V_{\text{GS}} = 0$ V and $V_{\text{DS}} = 5$ V and an AC tone with 0.1 mV amplitude at 1 GHz, so as to drive the system into a linear AC behavior. The device geometry has been varied by changing the device gate length with steps of 5 nm up to 50 nm, i.e. a total variation up to 20% around the nominal 0.25 μm value. First a reference solution has been found by an incremental approach, i.e. by solving the system with varying gate length. Each time the gate length is varied the device grid is adapted in order to fit the given parameter variation [10]. Then the linearized Green's function approach has been carried out and compared to the reference solution in order to validate the proposed sensitivity analysis. Finally a standard small-signal AC solution, with linearization around the DC bias point, and repeated analyses with varying gate length values have also been carried out for further reference. Notice that the latter approach turns out to be the most time consuming in terms of simulation time. Fig. 1 shows the comparison of the incremental and linearized Green's function approach for the DC drain current while Fig. 2–5 show the comparison of the variations of the most relevant Y parameters with respect to gate length variations as resulting from the three proposed approaches. Similar results hold also for Y_{12} and Y_{22}, not shown here. The agreement is remarkable especially for variations of the gate length within 10%, demonstrating the capability of the proposed technique. Further validation has been obtained with different devices and various parameters, such as doping variations. Details will be given elsewhere.

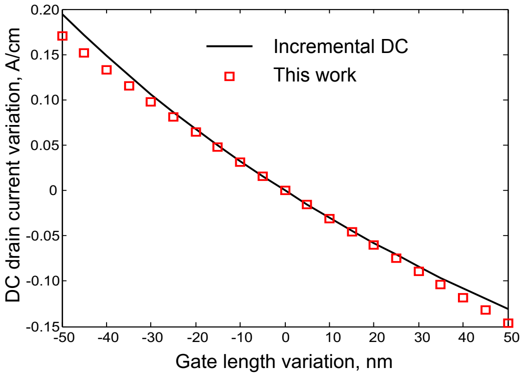

Fig. 1: DC drain current variation as a function of the gate length. Black full line: incremental, full model solution. Squares: this work.

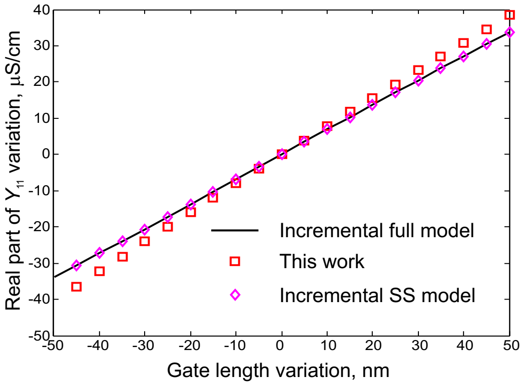

Fig. 2: Variation of the real part of Y_{11} as a function of the gate length. Black full line: incremental, full model solution. Squares: this work. Diamonds: incremental small-signal analysis.

ACKNOWLEDGMENT

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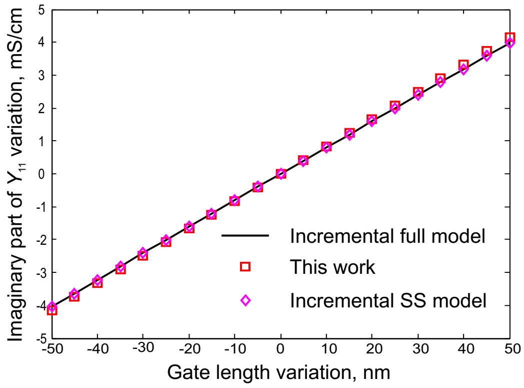

Fig. 3: Variation of the imaginary part of Y_{11} as a function of the gate length. Black full line: incremental, full model solution. Squares: this work. Diamonds: incremental small-signal analysis.

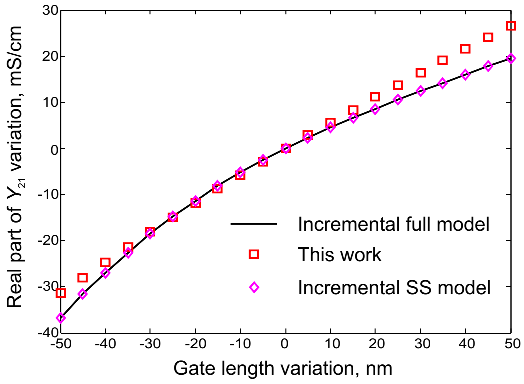

Fig. 4: Variation of the real part of Y_{21} as a function of the gate length. Black full line: incremental, full model solution. Squares: this work. Diamonds: incremental small-signal analysis.

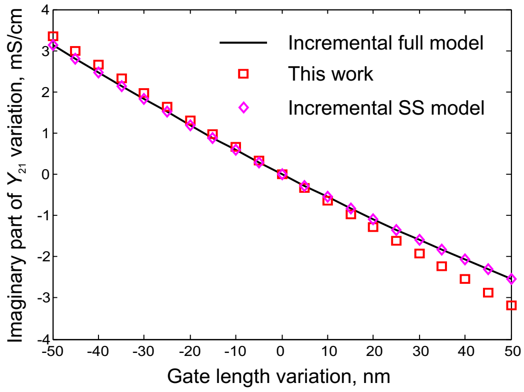

Fig. 5: Variation of the imaginary part of Y_{21} as a function of the gate length. Black full line: incremental, full model solution. Squares: this work. Diamonds: incremental small-signal analysis.

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