

Ant colony system-based applications to electrical distribution system optimization

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Ant colony system-based applications to electrical distribution system optimization / Chicco, Gianfranco - In: Ant Colony Optimization - Methods and Applications / Ostfeld A.. - ELETTRONICO. - Rijeka : Intech, 2011. - ISBN 9789533071572. - pp. 237-262

*Availability:*

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Intech

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DOI:

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# Ant Colony System-based Applications to Electrical Distribution System Optimization

Gianfranco Chicco  
*Politecnico di Torino*  
*Italy*

## 1. Introduction

Electrical distribution networks are structurally weakly meshed, but are typically operated in radial configurations to simplify the network protection schemes. This implies the need to carry out suitable selection of the redundant network branches to open in normal conditions, as well as to define the control variables to set up in order to guarantee effective system operation and voltage quality. Furthermore, the structure of the distribution networks has to be upgraded to meet future needs, according to economic objective functions defined for distribution system planning. Finally, distribution systems need to face possible interruptions through the formulation of appropriate service restoration strategies, based on switching on and off a number of branches, reducing the effects of faults occurring in the networks by defining a time-dependent path to restore electricity supply to all consumers. All these aspects can be addressed by formulating and solving suitable optimization problems for distribution network operation and planning. Typically these problems are defined within discrete domains for the decision variables. However, for large real distribution systems the highly-dimensional and/or combinatorial nature of the optimization problems make it impracticable to resort to exhaustive search, since the number of combinations would be excessively high to be processed in reasonable computation times. Global optimizations can then be solved in terms of adopting suitable meta-heuristics.

In this context, ant colony optimization (ACO) provides viable solutions to the various distribution system optimization problems. Key features of ACO, such as parallel search, shortest path finding, adaptability to changes in the search space, long-term memory and information sharing, have been fully exploited in their classical formulations, in advanced versions such as the hyper-cube ACO framework, as well as in hybrid formulations combining the ACO properties with those of other meta-heuristics.

This chapter summarizes the formulation and use of ACO algorithms and variants to solve different types of electrical distribution system optimization problems, namely:

- *Distribution system optimal reconfiguration and voltage support.* Various objective functions to minimize can be set up in the presence of a set of structural and operational constraints. The objectives include total distribution system losses, voltage deviations with respect to their reference values, maximum load on system components, load balancing, and capacitor placement. Performing network configuration changes and setting up the level of insertion of compensating devices or distributed generation and

resources connected to the network nodes are the main actions needed, according to which the decision variables are defined.

- *Optimal planning of distribution networks and resources.* The classical objective function is composed of the total (fixed and variable) costs for network reinforcement, taking into account structural network components, as well as possible introduction of distributed energy resources. For the purpose of application of an ACO algorithm to a minimization problem, the inverse of the total costs can be considered as objective function.
- *Optimization of service restoration and reliability.* The typical objective in this case is maximization of distribution network reliability under specific constraints. For this purpose, composite reliability indices are formulated by taking into account frequency and duration of the interruptions. Multi-objective formulations with reliability and economic terms have also been adopted. The decision variables for distribution system upgrade are defined with the purpose of increasing the level of automation or exploiting the potential of distributed generation and resources to assist service restoration.

The specific aspects concerning the implementation of ACO algorithms are illustrated and discussed in the next sections, taking into account the representation of the search space with definition of the variables, the parameters and their initialization, the formulation of global and local heuristic rules for pheromone update, and finally the stop criterion. The dedicated solutions adopted to take into account the nature of the distribution system optimization problems addressed are highlighted, as well as the role of ACO parameters within hybrid formulations of the solution algorithms.

Dedicated reference is made to literature papers, mainly to those appeared in scientific journals. Uniform notation and symbols are used throughout this chapter. As a consequence, the symbols may differ with respect to the ones included in the original referenced papers.

## 2. Ant colony optimization characteristics and general framework

### 2.1 Basic features of the ant colony optimization algorithms

ACO algorithms are stochastic search procedures based on a parametrized probabilistic model called *pheromone model* (Dorigo & Blum, 2005). ACO algorithms adopt an iterative solution process in which each iteration corresponds to the parallel search operated by  $M$  ants. The pheromone quantity is typically initialized to a constant positive value and is updated during the solution process, enhancing its relative concentration in the most promising regions of the search space, in such a way to increase the probability of accessing these regions during the successive phases of the stochastic search.

Dorigo et al., 2006, indicate that the most widely adopted ACO algorithms in general applications are the ant system (Dorigo et al., 1996), the max–min ant system (Stützle & Hoos, 2000), and the ant colony system (Dorigo et al., 1996). The latter is the most widely used for solving electrical distribution system optimization problems. The general scheme of ACO algorithms adopted for these problems is summarized in the flow-chart shown in Fig. 1. Besides the user-defined number of ants  $M$ , the general parameters include a number of specific parameters that are introduced in the following subsections. At the beginning of the solution process, the ants are placed in locations randomly selected or defined with specific criteria. At each iteration, the ants move within the search space on the basis of a state

transition rule. The ant movement and the objective functions corresponding to the solution points reached during the search determine the pheromone update according to a global heuristic rule. An optional local heuristic rule can be introduced to further impact on the search process.

As for any stochastic search process, the solutions reached depend on the extraction of random numbers, typically from a uniform probability distribution in the range  $[0, 1]$ . When repeatability of the results under the same conditions is needed, the seed for random number extraction is set to a fixed value at the beginning of the execution of the algorithm.

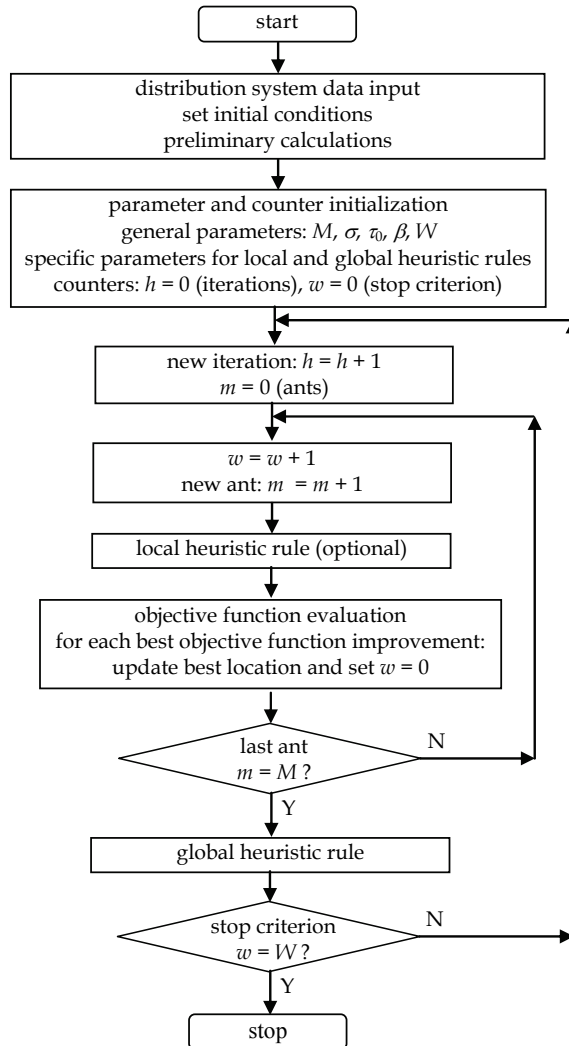


Fig. 1. General flow-chart for ant-colony optimization application to electrical distribution systems.

## 2.2. Weighted probabilistic selection mechanism

At each iteration, each ant  $m = 1, \dots, M$  moves along a path selecting the new locations according to a *state transition rule*: for ant  $m$ , the probability  $p_{(i,j)}^{(m)}$  of moving from location  $i$  to a location  $j$  (belonging to the set  $\mathbf{S}_i^{(m)}$  of the locations that can be visited by ant  $m$  starting from location  $i$ ) is expressed as

$$p_{(i,j)}^{(m)} = \frac{\tau_{(i,j)} \left( d_{(i,j)} \right)^{-\beta}}{\sum_{v \in \mathbf{S}_i^{(m)}} \tau_{(i,v)} \left( d_{(i,v)} \right)^{-\beta}} \quad (1)$$

where  $\tau_{(i,j)}$  is the amount of pheromone existing in the connection between the locations  $i$  and  $j$ , the term  $d_{(i,j)}$  represents the *distance* between the locations  $i$  and  $j$ , and the exponent  $\beta > 0$  quantifies the relative importance of the pheromone and distance entries<sup>1</sup>. In practice, a transition from  $i$  to  $j$  becomes most probable when the distance between the locations is relatively low and there is a relatively high amount of pheromone deposited on the connection between the locations  $i$  and  $j$ .

On the practical point of view, after calculating the probabilities  $p_{(i,j)}^{(m)}$ , the transition of ant  $m$

from location  $i$  to a location belonging to the set  $\mathbf{S}_i^{(m)}$  is identified by using a *weighted probabilistic selection* mechanism based on the construction of the cumulative distribution function (CDF) (Carpaneto & Chicco, 2008), analogous with a biased roulette wheel mechanism. In fact, since the sum of the probabilities satisfies the basic probability law

$\sum_{v \in \mathbf{S}_i^{(m)}} p_{(i,v)}^{(m)} = 1$ , it is possible to construct the CDF of the probabilities associated to the state

transition rule. An example in which ant  $m$  starts from location  $i$  and can reach four possible destinations (numbered as 1, 2, 3 and 4) with corresponding probabilities equal to 0.2, 0.3, 0.4 and 0.1, respectively, is shown in Fig. 2. Then, taking a random number  $r$  extracted from a uniform probability distribution in  $[0, 1]$  and entering it onto the vertical axis, it is possible to identify the destination to reach by determining the corresponding step on the CDF. In the example of Fig. 2, if the random number  $r = 0.35$  is extracted the destination is  $j = 2$ .

## 2.3 Local heuristic rule

This rule is optional and can be formulated in different ways. A typical formulation is the one of the ant colony system (Dorigo et al., 1996), that considers a small positive constant  $\tau_c$  not lower than the initial pheromone  $\tau_0$  and not higher than a quantity depending on the objective function values. The amount of pheromone existing in the connection between the

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<sup>1</sup> In some applications the pheromone terms indicated in (1) are associated to a further exponent. However, for the sake of reducing the overall parameters, a single exponent can be sufficient for tuning the relative importance of the pheromone quantity and distance terms. In the applications shown in this chapter, the double exponent representation used in some papers is reported to the single-exponent scheme.

locations  $i$  and  $j$  is changed from  $\tau_{(i,j)}^{previous}$  to  $\tau_{(i,j)} = \rho \tau_c + (1 - \rho) \tau_{(i,j)}^{previous}$ , where  $\rho$  is a user-defined parameter. This rule changes dynamically the characteristics of the path from  $i$  to  $j$ , thus modifying the extent to which this path is likely to be successively visited by the ants.

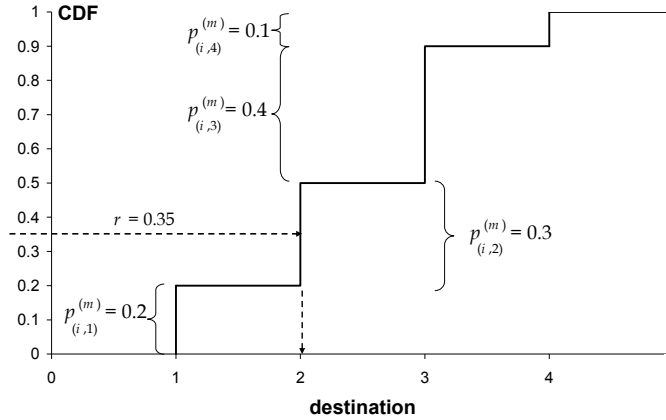


Fig. 2. Selection of the destination based on the construction of the CDF.

**2.4 Global heuristic rule**

This rule is applied at the end of each iteration  $h$ . During the iteration, the path followed by one of the ants exhibited the best performance. The global heuristic rule is implemented by considering two terms: (i) pheromone *evaporation* applied to all connections, driven by the parameter  $\sigma \in [0, 1]$ ; (ii) pheromone *reinforcement* limited to the connections among the locations reached by the best path, depending on a specifically defined function  $\psi^{(h)}$ :

$$\tau_{(i,j)}^{(h)} = (1 - \sigma) \tau_{(i,j)}^{(h-1)} + \sigma \psi^{(h)} \tag{2}$$

Generally, the function  $\psi^{(h)}$  is inversely proportional to the distance covered by the best path. According to Dorigo & Blum, 2005, in the *iteration-best update rule* the best path can be the one found at iteration  $h$ , that is,  $\psi^{(h)} = \kappa_\tau / d_{best}^{(h)}$ , where  $d_{best}^{(h)}$  represents the distance and  $\kappa_\tau$  is a user-defined factor. Alternatively, in the *best-so-far update rule* the best distance  $\widehat{d}_{best}^{(h)}$  found so far is used, thus obtaining  $\psi^{(h)} = \kappa_\tau / \widehat{d}_{best}^{(h)}$ . The update rule could also change during the iteration process, for instance using the iteration-best update rule in the first iterations and best-so-far update rule in the successive iterations.

**2.5 Pheromone bounds and hypercube framework**

Successful rules for driving the pheromone update have been set up by limiting the minimum and maximum amounts of pheromone. The ant colony system algorithm (Dorigo et al., 1996) applies to all pheromone values a non-fixed lower bound, possibly depending

on the objective function values. The max-min ant system (Stützle & Hoos, 2000) applies to all pheromone values an explicit lower bound and a non-fixed upper bound depending on the objective function values. The hypercube framework (Blum & Dorigo, 2004) provides an automatic scaling of the objective function values used in the search process, and results in a more robust and easier way to implement computational procedure. The initial pheromone quantity  $\tau_0 \in [0, 1]$ , and the pheromone values are always limited within the interval  $[0, 1]$  by properly formulating the pheromone update rules.

## 2.6 Elitism

In the basic version of the heuristic, each iteration starts with a set of locations, and these locations are then changed during the iteration according to the rules applied. As such, there is no certainty that the best solution found at a given point is maintained for the whole iterative process. In the *elitistic* version of the heuristic, the best solution found in the previous iteration is maintained without being subject to any change during the current iteration. In this case, a monotonically increasing path for the objective function to be maximized (e.g. fitness), or, correspondingly, a monotonically decreasing path for the objective function to be minimized, is generated in the successive iterations. The elitistic version of a heuristic is generally convenient to apply in terms of effectiveness of the solution process (Reeves, 1993).

In ACO, a specific technique is used to implement elitistic considerations. Considering the set  $S_i^{(m)}$  of locations that can be visited by ant  $m$  starting from location  $i$ , the selection process of the destination is driven by a user-defined parameter  $q_0 \in [0,1]$ . After extracting a random number  $r \in [0,1]$ , if  $r > q_0$  the selection is made by using the weighted probabilistic selection mechanism (Section 2.2) with probabilities defined as in (1), obtaining the destination  $\hat{j}$ . Conversely, for  $r \leq q_0$  the destination with relatively high pheromone and relatively low distance in the connection is sought. In summary, the destination  $j \in S_i^{(m)}$  to be reached by ant  $m$  is selected by applying the following rule:

$$j = \begin{cases} \arg \max_{u \in S_i^{(m)}} \{ \tau_{(i,u)} d_{(i,u)}^{-\beta} \} & \text{if } r \leq q_0 \\ \hat{j} & \text{otherwise} \end{cases} \quad (3)$$

The more the parameter  $q_0$  is close to unity, the more the selection becomes driven by elitistic considerations.

## 2.7 Stop criterion

The most convenient stop criterion for a heuristic refers to conclude the iterative process when no objective function improvement occurs after a predefined number  $W$  of successive iterations. This criterion is by far more effective than the simple use of a maximum number of iterations  $H$ . In fact, setting up a maximum number of iterations could result either in premature stop of the solution process when substantial improvements are still occurring, or in unnecessary computational overburden when no change in the best solution is found for a large number of the last successive iterations. The maximum number of iterations  $H$  can be

used as a last-resource option, mainly for algorithm testing. However, many references still adopt the simple stop criterion based on the maximum number of iterations.

## 2.8 Parameter values

The choice of the ACO parameters is made by the user, who can conduct dedicated parameter analyses for assessing the most suitable parameter values to be used to solve the specific problem on the specific system under analysis. In order to summarize the parameters used in various applications to distribution system optimization, Table 1 shows the numerical values of the parameters adopted in the reference papers recalled in this chapter. Multiple indications are included to represent ranges of values used for parametric analysis.

Reference	Parameters					
	$M$	$\sigma$	$\tau_0$	$\beta$	$W$	$H$
Ahuja et al., 2007	-	0.05	100	-	-	100
Carpaneto et al., 2004	20	0.02	10	1	10	-
Carpaneto & Chicco, 2004	100	0.1	10	1	5	-
Carpaneto & Chicco, 2005	10÷600	0.01÷0.2	10	1	10	-
Carpaneto & Chicco, 2008	10	0.04	1	1	5	-
Chang, 2008	5	0.05÷0.2	0.1	0.1, 0.15	-	450
Chiou et al., 2004	5	]0, 1[	(*)	1	-	500
Falaghi et al., 2009	50	0.1	0.1 (lower bound)	(-)	-	100
Favuzza et al., 2006	50	0.00001	0.2	2	20	-
Favuzza et al., 2007	50	0.00001	0.2	2	20	-
Gomez et al., 2004	4÷50	0.2÷1	random	1÷4	-	30
Niknam, 2008	10÷200	(*)	(*)	0.01÷0.1	-	100
Su et al., 2005	5	0.1÷0.5	(*)	0.1÷0.25	-	30
Teng & Liu, 2003	50÷250	0.1÷0.6	> 0 (small)	1	-	100
Tippachon & Rerkpreedapong, 2009	(*)	0.2	not fixed	2	-	(*)
Wang & Singh, 2008	200	0.27	> 0 (small)	1	-	200
Zamboni & Monteiro, 2009	5	-	0.01	0.15	-	200

Table 1. Numerical values of the parameters used in reference papers. A dash indicates that the parameter is not used. An asterisk (\*) indicates that a numerical value is not available.

## 3. Optimal reconfiguration and voltage support

### 3.1 Distribution system topology and network calculations

In the weakly-meshed distribution system structure, the network topology typically lacks of regularity. As such, it is not possible to exploit concepts like network symmetry or uniformly distributed connection among the network nodes to simplify the computational procedures for network topology analysis.

For a generic weakly-meshed distribution system structure, let us assume the following notation: the set of nodes  $N$  contains  $i = 1, \dots, N$  nodes, the subset of supply nodes  $S \subset N$  contains  $s = 1, \dots, S$  supply nodes, and the set of branches  $B$  contains  $b = 1, \dots, B$  branches.

Furthermore, the set  $T \subset B$  contains the transformers located in the distribution network, and the set  $X$  contains all the possible radial configurations of the distribution system.

A general property is that each radial network obtained from the weakly-meshed network structure has the same number  $A = B - N + S$  of open branches. However, opening  $A$  branches is a necessary but not a sufficient condition for obtaining a radial network, since by arbitrarily opening  $A$  branches it could be possible to isolate one or more network nodes and to maintain loops in the network. As such, the identification of the radial structures is not straightforward. Even the enumeration and generation of all radial configurations that can be obtained for a given weakly meshed structure is a difficult task, that can be addressed by using specific algorithms (Andrei & Chicco, 2008). However, the size of the real distribution systems is typically large, and the corresponding number of possible radial configurations is high enough to make exhaustive search practically intractable in terms of computational burden.

For a radial configuration  $X \in X$ , the distribution network calculations are carried out by solving the power flow with a specific iterative method like the backward-forward sweep (Shirmohammadi et al., 1988; Baran & Wu, 1989; Andrei et al., 2008). The power flow results provide the complex voltages at every distribution system node. The branch currents and losses in the distribution system are then computed on the basis of the power flow results. In specific cases approximate solutions for loss calculation (Baran & Wu, 1989b) are adopted (for instance in Chiou et al., 2004; Su et al., 2005; Chang, 2008).

## 3.2 Objective functions and constraints

### 3.2.1 Optimization problem formulations

The optimal reconfiguration of the distribution system is a *discrete* problem, consisting of the search of the radial configuration that minimizes (or maximizes) a specified objective function  $f(X)$  under various operational constraints. The general formulation for a minimization problem is  $\min_{X \in X} \{f(X)\}$ , subject to a set of equality and inequality constraints.

The *equality constraints* are the power flow or approximate loss calculation equations.

Concerning the *inequality constraints*, the constrained variables are the voltage magnitude at each node (a bilateral constraint with maximum and minimum limits implemented independently of each other), the phase-to-ground current at each node (with maximum limit), the three-phase short circuit current at each node (with maximum limit), the thermal current at each branch (with maximum limit). Other operational constraints (Carpaneto et al., 2003) may reflect the need to avoid the presence of multiple cascade circuit breakers in any path of the radial network (being the circuit breakers non-selective), the need to avoid simultaneous interruption of two (or more) supply points for the customers requiring enhanced supply availability in their contractual provision (according to which independent supply points have to be maintained, that is, connected to radial networks starting from different substations), and, in case, a constraint on the maximum number of switching operations occurring to move from the standard distribution system configuration to the new configuration found in the solution process.

There are two ways to introduce the inequality constraints in the optimization problem:

1. checking the constraints for each solution and *discarding* any solution that does not satisfy *all* constraints;
2. embedding the inequality constraints into a *penalized* objective function, by adding penalty terms to the specific objective function.

The use of the penalized objective function is commonplace for heuristic methods. In this way, it is possible to obtain a numerical value of the objective function in any condition, provided that the network structure is acceptable (in our case, the network is radial), without the need of detecting and eliminating infeasible solutions that do not meet the operational constraints. This allows for reducing the limits on the size of the search space. Using the penalized objective function together with a dedicated mechanism to avoid the generation of non-radial networks (Section 3.3), the check for the existence of valid solutions indicated in the framework of a basic ACO algorithm in Dorigo & Blum, 2005, is not necessary, and thus it is not represented in the flow-chart of Fig. 1.

Let us consider an objective function composed of a specific term  $f'(X)$  based on the objective to be assessed. Let us further assume that the objective function has to be *minimized*. In this case, positive penalty terms are added to  $f'(X)$  in case of violations, obtaining the *penalized* objective function expressed for instance in the form:

$$\min_{X \in X} \{f(X) = f'(X) + \mathbf{w}^T \Delta \mathbf{u}(X)\} \quad (4)$$

where:

- $\mathbf{u}(X)$  is a column vector containing the variables subject to constraints, and  $\Delta \mathbf{u}(X)$  is a column vector whose components are positive in case of violations and zero elsewhere; these components can be the absolute deviations with respect the corresponding constraints, in case elevated to an user-defined exponent (e.g., squared);
- $\mathbf{w}$  is a column vector containing positive user-defined weights for the variables belonging to  $\mathbf{u}(X)$  for which a violation occurs, and null values for the components without violation;
- the superscript T denotes transposition.

### 3.2.2 Specific objective functions

The specific objective function  $f'(X)$  can be written as follows:

- For *total distribution system losses*, considering the branch series resistance  $R_b$  and the branch current  $I_b$ :

$$f'(X) = P_L^{tot}(X) = \sum_{b \in \mathbf{B}} R_b I_b^2 \quad (5)$$

- For *voltage deviations* with respect to the rated value, considering the voltage magnitude  $V_k$  at node  $k$  and the rated voltage  $V^{(r)}$ :

$$f'(X) = \Delta V^{\max}(X) = \max \left\{ \left| V^{(r)} - \max_{k \in \mathbf{K}} \{V_k\} \right|, \left| V^{(r)} - \min_{k \in \mathbf{K}} \{V_k\} \right| \right\} \quad (6)$$

- For *maximum load* on branches (feeders or transformers), the objective function is the maximum ratio between the branch current  $I_b$  and the corresponding current rating  $I_b^{(r)}$ :

$$f'(X) = L^{\max}(X) = \max_{b \in \mathbf{B}} \left\{ \frac{I_b}{I_b^{(r)}} \right\} \quad (7)$$

- For *transformer load balancing*, the rationale of this objective is to make the load sharing of each transformer (expressed in relative values with respect to the transformer capacity) proportional to the capacity of that transformer. The objective function is the total unbalance, expressed in terms of the total network apparent power load  $A_{LL}^{tot}$  (including losses), of the apparent power  $A_b(X)$  of each transformer  $b \in T$  in the configuration  $X$  under study, and the ratio  $\ell_b = A_b^{(r)} / \sum_{v \in T} A_v^{(r)}$  between the transformer capacity and the sum of capacities of all network transformers:

$$f'(X) = \Delta U_T^{tot}(X) = \sum_{b \in T} \frac{A_b(X) - \ell_b A_{LL}^{tot}}{\ell_b A_{LL}^{tot}} \quad (8)$$

Alternatively, it is possible to define a transformer load balancing index (Wu et al., 2010).

- For *feeder load balancing*, the objective is to make the branch loading as uniform as possible throughout the network, comparing the apparent power of each branch  $b \in B$  to the corresponding MVA limit  $A_b^{\max}$ :

$$f'(X) = \Delta B^{tot}(X) = \sum_{b \in B} \left( \frac{A_b(X)}{A_b^{\max}} \right)^2 \quad (9)$$

- For *capacitor placement*, the objective function can be set up as the sum of the total system losses and of the annual cost of the capacitors installed, in order to highlight the convenient solutions in which the loss reduction makes it possible at least to compensate the capacitor costs. Considering the specific cost  $\rho_c$  in monetary units per kvar, and the rated power  $Q_{C_k}^{(r)}$  of the capacitor to be installed at node  $k \in K$ :

$$f'(X) = P_L^{tot}(X) + \sum_{k \in K} \rho_c Q_{C_k}^{(r)} \quad (10)$$

In the capacitor placement problem, an additional constraint is set up in order to limit the size of the capacitor to be installed at each node to the reactive power load  $Q_{L_k}$  at that node, i.e.,  $Q_{C_k}^{(r)} \leq Q_{L_k}$  for  $k = 1, \dots, K$ .

A specific aspect adopted in the optimal reconfiguration problem formulation in Carpaneto & Chicco, 2008, refers to the consideration that each branch has two switches, one at each terminal, and for any open branch it is possible to choose which switch has to be maintained open. If the switches have different levels of automation, the switch-on/switch-off operations are convenient to be done only at the terminal with higher level of automation, because of easier manoeuvrability. The on/off position of the switches is then specified in detail in the problem formulation for each open branch.

The above objective functions are expressed in terms of *power* quantities, that is, they are valid for a single snapshot in time. However, with the progressive introduction of distributed resources in the energy networks, the focus is now shifted to distribution system operation in a given time interval. For this purpose, the distribution network objectives have to be reformulated by explicitly considering the dependence on time of the relevant variables.

Let us represent the time domain with a set of regularly spaced discrete points  $j = 1, \dots, J$ , as in Carpaneto & Chicco, 2010. Without taking into account the possibility of making configuration changes in the time interval of analysis, the general form of the objective function is expressed as:

$$\min_{X \in X} \left\{ f(X) = f'(X) + \sum_{j=1}^J \mathbf{w}^T \Delta \mathbf{u}_j(X) \right\} \quad (11)$$

and the specific objective functions are written taking into account the results of the power flow calculations at the various points in time. As an example, the voltage deviations with respect to the reference value can be assessed by considering, for  $j = 1, \dots, J$ , the voltage magnitude  $V_{kj}$  at node  $k \in K$ , the rated voltage  $V^{(r)}$ , the consumer damage constant  $C_k$  (Strezoski et al., 2001), and the consumer energy  $E_{kj}$  introduced as a weighting factor to enhance the importance of nodes and time intervals with relatively high energy consumption (Carpaneto et al., 2004; Carpaneto & Chicco, 2010):

$$f'(X) = \sum_{j=1}^J \sum_{k \in K} \left( C_k E_{kj} \left( V_{kj} - V^{(r)} \right)^2 \right) \quad (12)$$

Similar formulations can be obtained by considering, instead of a regular discretization in time, a reduced number of load variation steps, as in the minimum electrical energy generation cost problem defined in Niknam, 2008, applied to a fixed radial network configuration in which the decision variables refer to capacitors, transformer tap changers, distributed generators and voltage regulators.

Multi-objective functions can be formulated as well, by simultaneously considering two or more objectives (Ahuja et al., 2007).

### 3.3 Approaches for ant colony optimization applications

#### 3.3.1 Classification of the alternative approaches

For the purpose of constructing *radial* system configurations, alternative approaches can be exploited. A first classification can be made on the basis of the starting conditions used:

1. *constructive* approach: the initial network is only composed of the supply points, and the network branches are successively added until a radial network is formed;
2. *loop-based* approach: all branches are initially considered in their closed state, and the  $A$  branches to open are determined in such a way to open  $A$  loops identified in the network;
3. *variation-based* approach: the initial network is radial and a new configuration is obtained by closing and opening some branches according to a specific branch-exchange rule formulated to preserve radial configurations after each variation.

The details of these approaches and the corresponding ACO algorithms used are illustrated in the sequel.

#### 3.3.2 Constructive approach

The constructive approach uses a pheromone-driven application of the Prim-Jarník algorithm (Prim, 1957) to form the radial network by successively adding edges that do not

form any loop. In the presence of multiple supply (root) points, with  $S > 1$ , there are  $S$  radial networks simultaneously growing by adding one branch at a time to one of these networks. A key aspect is the selection of the next branch to be added.

In the constructive scheme illustrated in Carpaneto & Chicco, 2004, the initial network is composed of the only substation nodes. The specific objective function (5) is used. The pheromone is applied to all the branches of the weakly-meshed distribution system structure. The initial pheromone is set to  $\tau_b^{(0)} = \tau_0$  for any branch  $b \in B$ . An additional parameter  $\xi > 1$  is used to represent the pheromone reinforcement factor and is set to  $\xi_b^{(h)} = 1$  at the beginning of each iteration for any branch  $b \in B$ . At each iteration, each ant  $m = 1, \dots, M$  builds a radial system by adding  $K-S$  branches to the initial network. At each step of the branch addition, a list of candidate nodes is formed, containing all the nodes to which it is possible to connect a further open branch without closing a loop (at the beginning, this list contains only the substation nodes). Then, one of the open branches converging to a node included in the list has to be added to the radial network under formation. For this purpose, a branch selection mechanism is applied. The selection is driven by the definition of a suitable distance in the state transition rule (1). For instance, branch resistance (or impedance) can be used as distance. In the version illustrated in Carpaneto & Chicco, 2004, a specific selection mechanism is implemented, in order to enable a relatively balanced growth of the radial networks supplied by different substations. The series resistance of the branches already belonging to the path from each node contained in the list and the corresponding supply substation is used as the distance related to that node<sup>2</sup>. Then, the state transition rule based on the weighted probabilistic selection mechanism (Section 2.2) is applied to extract the node belonging to the list formed to which an open branch has to be connected. Once the node has been selected, if there are multiple open branches that can be connected to its terminals without forming any loop, another extraction is carried out by using again the weighted probabilistic selection mechanism, in which the distances now consist of the resistances of the branches that can be connected. At the end of the radial network construction for the current iteration  $h$ , the pheromone reinforcement factor is set to  $\xi_b^{(h)} = \xi$  at each branch belonging to the radial network formed, and to zero at every other branch.

The global heuristic rule is applied to each branch by using an expression similar to (2), in which the function  $\psi^{(h)}$  multiplied by the evaporation factor  $\sigma$  contains the pheromone reinforcement factor, as well as a further component given by the ratio between the best solution  $f(X_{best})$  found so far and the best solution  $f(X_{best}^{(h)})$  found at the current iteration (excluding the best solution found in previous iterations and reproduced in the current iteration according to the application of the elitism principle):

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<sup>2</sup> For the substation nodes not yet connected to any other node, the distance is set to a very low default value (e.g.,  $10^{-8}$ ), thus increasing at these nodes the probability of being extracted by the weighted probabilistic selection mechanism with respect to other nodes already connected to the corresponding supply substations through a series of branches.

$$\tau_b^{(h)} = \sigma \frac{\tau_b^{(h-1)} f(X_{best}^{(h)})}{f(X_{best}^{(h)})} + (1 - \sigma) \tau_b^{(h-1)} \quad (13)$$

The ratio between the two best solutions is equal to unity if the best solution found so far has been obtained in the current iteration, otherwise is lower than unity, being the objective function to be minimized. In the latter case, the pheromone reinforcement at the current iteration  $h$  is in some way mitigated with respect to the case in which the best solution was updated during the iteration.

### 3.3.3 Variation-based approach

The initial distribution system configuration  $X^{(0)} \in \mathbf{X}$  is chosen as a radial network, typically corresponding to the standard configuration adopted by the distribution company. The solution algorithm proceeds by progressively introducing *variations* in the configuration according to specific local and global heuristic rules. Each variation is constrained by the need of forming a radial structure, thus adopting the branch-exchange mechanism (Civanlar et al., 1988; Baran & Wu, 1989b) to maintain network radiality.

Two types of branch-exchange mechanisms can be defined, based on inverse strategies:

- close-open branch exchange*: consists of closing an open branch, detecting the *loop* formed in the system, and restoring the radial configuration by opening a branch inside the loop;
- open-close branch exchange*: consists of opening a closed branch, detecting the *island* formed in the system and all the possible connections among any node of the island and the remaining part of the network, and restoring the radial configuration by closing one of these connections.

In both mechanisms, the way in which the branches to open and close are selected depend on the characteristics of the solution strategy. For this purpose, it is possible to make a distinction between *deterministic* methods (in which specific rules are set up to apply the branch-exchange mechanism, with the possibility of trapping into local optima, Carpaneto et al., 2003) and *probability-based* methods (in which the selection is performed on the basis of random number extractions dependent on probability values, such as in the weighted probabilistic selection mechanism described in Section 2.2). ACO algorithms belong to the latter type of methods.

In the approach presented in Carpaneto & Chicco, 2008, the pheromone is associated to each branch (more specifically, to each switch, but general reference will be made here to branches, for the sake of comparison with the other methods). The rationale is to enhance the pheromone at the branches that have to remain open in the configuration corresponding to the best objective function. This approach operates in the hyper-cube ACO framework (Blum & Dorigo, 2004), so that the pheromone has to be initialized and vary in the  $[0,1]$  range. The objective function (5) is used. Two additional parameters are introduced to be used in the local heuristic rule: the reduced neighbourhood size  $A$  and the number  $\zeta$  of successive configuration changes produced by each ant. The value  $\zeta$  cannot be too low, in order to allow for exploring the solution space by introducing relatively large variations, but at the same time it cannot be too high, to avoid the introduction of many changes to the reference configuration used.

At each iteration  $h$ , the reference configuration is set to the one corresponding to the best solution obtained so far. The  $M$  ants are initially located on randomly selected branches.

Each ant  $m = 1, \dots, M$  introduces a number of successive configuration changes  $\zeta_m^{(h)}$  randomly selected from 1 to  $\zeta$  by applying the local heuristic rule. The changes are operated by applying  $\zeta_m^{(h)}$  times the close-open branch-exchange mechanism. A further variant embedded in the local heuristic is the identification of a reduced neighbourhood of size  $A$ , such that the choice of the branch to open after closing a selected branch is limited to the  $A$  branches adjacent to the closed branch in the loop formed. Within the neighbourhood formed, the branch to open is chosen by applying the weighted probabilistic selection mechanism (Section 2.2) by using as weight the pheromone associated to each branch. After  $\zeta_m^{(h)}$  successive configuration changes have been introduced, the objective function is computed. The best configuration  $X_{best}^{(h)}$  of the current iteration  $h$  and the best configuration  $X_{best}$  found so far are updated in case new best solutions are found during the calculations. In the global heuristic rule, the pheromone is updated according to the following relation:

$$\tau_b^{(h)} = \sigma \nu_b^{(h)} \frac{f(X_{best})}{f(X_{best}^{(h)})} + (1 - \sigma) \tau_b^{(h-1)} \quad (14)$$

where  $\nu_b^{(h)} = 1$  if branch  $b$  is open in the best configuration found at iteration  $h$ , otherwise  $\nu_b^{(h)} = 0$ . The considerations on the ratio between the two best solutions, corresponding to the function  $\psi^{(h)}$  in (2) are the same as the ones discussed in (13). Since the first addend in (14) cannot be higher than unity, and the initial pheromone cannot exceed unity by definition of the hyper-cube framework, the pheromone updated by the global heuristic rule always remains in the range  $[0, 1]$ , thus maintaining full consistency with the hypotheses of the hyper-cube ACO framework.

The multi-objective optimization presented in Ahuja et al., 2007, considers a set  $\Omega$  of objectives, such as total loss minimization (5), voltage deviations with respect to the rated value (6) and transformer load balancing (8), and searches for the best-known Pareto front, composed of the non-dominated solutions for which it is not possible to find other solutions with better values of all individual objective functions (Shukla & Deb, 2007). The initial solutions are generated through the Prim-Jarník constructive algorithm (Prim, 1957), then configuration changes are determined by exploiting cloning principles and pheromone-based hypermutation. Pheromone is used to store information on the solution effectiveness, in order to use this information during the solution process. A separate pheromone table is initialized (with equal values  $\tau_0$ ) and used for each individual objective. Each pheromone element is denoted as  $\tau_{b,\omega}^{(h)}$ , with reference to iteration  $h$ , branch  $b$  and objective function  $\omega \in \Omega$ . Each non-dominated solution is subject to cloning to obtain a number of copies of it. Each clone is then subject to hypermutation on the basis of the pheromone values. To do that, the open-close branch-exchange mechanism is used, driven by weighted probabilistic selection (Section 2.2) to select the branch to open (using the pheromone weights) and to further select the branch to close among the remaining open branches (using equal weights). The pheromone update is performed by considering pheromone evaporation at all branches according to the second addend of (2), then including an addend providing pheromone reinforcement only on the branches corresponding to non-dominated solutions. This addend contains a constant  $\chi$  and the ratio between two terms: the upper-side term is the minimum value of the objective function for objective  $\omega$  in the set  $X^{(h)}$  of configurations providing

non-dominated solutions at the current iteration, and the lower-side value is the objective function for objective  $\omega$  in the current non-dominated solution:

$$\tau_{b,\omega}^{(h)} = \chi \frac{\min_{X \in X^{(h)}} \{f_{\omega}(X)\}}{f_{\omega}(X)} + (1 - \sigma) \tau_{b,\omega}^{(h-1)} \quad (15)$$

The update formula (15) is applied at each iteration to each branch and to each objective function.

### 3.3.4 Loop-based approach

The loop-based approach exploits a *loop-opening* principle. The weakly-meshed distribution system structure is initially considered with all branches closed, thus containing a number of loops. The ants are initially located at random positions in the network. The search space contains a number of loops  $A = B - N + S$  equal to the number of branches to be open (Section 3.1). Each ant selects one switch to be opened for each loop. The loops are processed one at a time, by listing for each loop the branches it contains and selecting among them the branch to open with a probabilistic transition rule based on the pheromone located on these branches.

Minimum loss reconfiguration is addressed from the loop-based viewpoint in Su et al., 2005, Chang, 2008, and Wu et al., 2010. The approximate solution for loss calculation introduced in Baran & Wu, 1989b, is used in Su et al., 2005, and in Chang, 2008, on any radial network constructed during the solution process, while Wu et al., 2010, includes as objectives both total losses and load balancing, performing full power flow calculation. The iterative process follows standard ACO concepts. Pheromone is located on the system branches and is initialized to the value  $\tau_0$ . Pheromone update is made according to the local heuristic rule (Section 2.3). The initial ants are located on the distribution system nodes. Each ant selects the next node in which it has to move on the basis of the state transition rule (1). Elitism is introduced by saving the best configuration found during the process, or by applying the concepts indicated in Section 2.6. In addition to the maximum number of iterations, in some cases the stop criterion includes as a rule the fact that all ants selected the same tour.

In Chang, 2008, the optimal reconfiguration is addressed by taking into account capacitor placement, either as a single solution and combined with loop-based open branch selection. In this case, the search space is extended to include, for each node, different capacitor sizes. Each ant operates in this extended search space, by including in a single tour both the loops and the set of nodes containing the capacitors. The addition of capacitor placement to open branch selection gives the possibility to set up additional problem variables and to enhance the performance of the optimal reconfiguration process.

Chiou et al., 2004, address the optimal capacitor placement problem by using an ant direction hybrid differential evolution method. The objective function to be minimized is the overall cost of power losses and capacitor placement (10). In this method, different mutation operators are available, and ACO is used to select the most convenient mutation operator to be used during the solution process. Each ant selects a mutation operator on the basis of a transition rule of the type (1) driven by a distance calculated on the diversity among the individuals (genes) generated with respect to the best individual found at the previous generation. A mutation operator is considered to be proper if the objective function of the current generation is better than the best objective function of the previous generation:

$$\ell^{(h,m)} = \left| \frac{f'(X^{(h,m)})}{f'(X_{best}^{(h-1)}) - f'(X^{(h,m)})} \right| \quad (16)$$

For ant  $m$ , the pheromone update is performed by summing up a term containing the remaining pheromone after evaporation and a *fluctuant* pheromone quantity added if the proper mutation operator has been chosen by the ant:

$$\tau^{(h,m)} = \Delta\tau^{(h,m)} + (1 - \sigma)\tau_b^{(h-1,m)} \quad (17)$$

where, considering the user-defined constant value  $\kappa$ :

$$\Delta\tau^{(h,m)} = \begin{cases} \kappa / \ell^{(h,m)} & \text{if the mutation operator is proper} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Zamboni & Monteiro, 2009, start with the weakly meshed structure with all branches closed, and adopt the classical weighted probabilistic selection mechanism as in (1), in which the branch resistance is considered as distance. The pheromone is associated to the branches, and all branches are assigned the same initial pheromone quantity  $\tau_0$ .

The local pheromone update at iteration  $h$  is driven by the square of the current required to transfer the active power  $P_i$  and reactive power  $Q_i$  from node  $i$  to node  $j$ , being  $i$  and  $j$  the terminals of branch  $b$ :

$$\Delta\tau_b^{(h)} = \sqrt{\left(\frac{P_i}{R_b}\right)^2 + \left(\frac{Q_i}{X_b}\right)^2} \quad (19)$$

The additional parameter  $q_0$  is introduced to further bias the selection mechanism by applying the elitistic criterion indicated in Section 2.6, allowing an ant located in a given node to move with probability  $q_0$  to the node with highest pheromone among the ones reachable, and partitioning the remaining  $1-q_0$  probability of selection by using the selection mechanism (1). The authors indicate to set  $q_0 \leq 0.5$  in order to avoid giving excessive weight to the local information.

Ants are initially located in some nodes, and each ant tends to move to a neighbouring node. The paths already visited are stored in order to avoid visiting them again. The ant transition from one node to another is repeated for a specified number  $p$  of steps. If the ant selects visiting an already visited path, it remains at the initial location. Any ant reaching a substation node remains in that node for the following steps. In this way, each ant collects the contributions of the loads served along the path from a node to a substation.

The solution method exploits the concept of *élite ants*, as the ones that cover the lowest distance paths. A maximum number  $M^{(e)}$  of élite ants (set up as a parameter) is considered in the global pheromone update at iteration  $h$ , where pheromone reinforcement is done in each branch belonging to the corresponding paths  $\mathbf{B}^{(e)} \subset \mathbf{B}$  containing the subset of nodes  $\mathbf{K}^{(e)} \subset \mathbf{K}$ , by introducing an extra pheromone quantity expressed as:

$$\Delta \tau_b^{(h)} = \sqrt{\left( \frac{R_b \sum_{k \in K^{(e)}} P_k}{\sum_{v \in B^{(e)}} R_v} \right)^2 + \left( \frac{X_b \sum_{k \in K^{(e)}} Q_k}{\sum_{v \in B^{(e)}} X_v} \right)^2} \quad (20)$$

where the active power  $P_k$  and reactive power  $Q_k$  refer to the nodes encountered in the path. In this way, the extra pheromone is somehow proportional to the total load encountered in the path traced by each elite ant. When at a given iteration no elite ant is formed, the procedure moves to the next iteration.

The stop criterion is based on reaching a predefined maximum number of iterations. At the end of the iterative process, the pheromone matrix contains different values, characterizing each branch. Considering the initial weakly-meshed structure, in order to decide which branches to open to form the best radial network, an algorithm is used which starts from a substation and proceeds until forming a mesh. Then, the branch belonging to that mesh having the least amount of pheromone is open. This procedure is run for  $A = B - N + S$  times, then the final radial configuration is found.

Voltage control optimization is addressed in Carpaneto et al., 2004, for distribution systems with distributed generation, by using the objective function (12) formulated for time-varying loads and generations. The control variables are partitioned into time-independent and time-dependent, and are all expressed through discrete values. The solution is carried out by using nested heuristics, with a primary loop solved with a genetic algorithm to find pseudo-optimal combinations of the time-independent variables, and a secondary loop solved with ACO to find the best path for the time-dependent variables corresponding to the assigned values of the time-independent ones. In the secondary loop, the limits of the path are represented by a rectangle, with time and voltage magnitude as coordinates. Within this rectangle, a number of discrete points are identified, and these points are assigned an initially equal pheromone quantity. An iterative process is then started, in which  $M$  ants are sent at each iteration to generate a path. The transition from a given point to the successive one (in time) is driven by a local heuristic rule, based on the weighted probabilistic selection mechanism (Fig. 2) applied to a neighbourhood of the current voltage magnitude, using the pheromone of the candidate points as weights. Once an ant has constructed an entire path, the objective function (12) is evaluated. After  $M$  ants have been sent, the global heuristic rule is applied by evaporating part of the current pheromone at every point, and by reinforcing the pheromone in the points belonging to the path producing the best objective function at that iteration through the use of a pheromone amplification factor. If the path is also the best one found so far, an higher value of the pheromone amplification factor is used. The secondary loop is concluded when the objective function exhibits no improvement after a predefined number of successive iterations. For the distribution system optimal reconfiguration problem, ACO algorithms showing better performance than other heuristics are indicated in some references, for instance in comparison with the simulated annealing algorithm (Su et al., 2005; Carpaneto & Chicco, 2008; Chang, 2008) and with genetic algorithms (Su et al., 2005; Chang, 2008).

## 4. Distribution system optimal planning

### 4.1 Formulation of the optimal planning problems

In a planning problem, the objective function is generally set up in *economic* terms. A basic distinction occurs between *operational* planning (carried out at constant load) and *expansion* planning (carried out at variable load).

A predefined set  $J$  of alternative planning actions (e.g., addition/removal of branches, distribution automation enhancement, addition of local generation units), each of which has to maintain a radial distribution network configuration after its activation, is chosen by the decision maker, in order to assess the most effective solution among the possible combinations of these alternatives. Since in real systems the number of possible combinations can be so high to make exhaustive search intractable, the use of heuristics like ACO can conveniently assist the decision-making process.

Starting from the current state of operation of the distribution system, it is possible to formulate the problem by considering the variations that can be introduced in the systems, thus calculating the corresponding effects. Considering a combination  $\mathbf{R} \subseteq J$  of planning actions activated at the stage of the solution process under analysis, in Carpaneto & Chicco, 2005, the objective function for the operational planning problem is formulated as:

$$f(\mathbf{R}) = C_A(\mathbf{R}) + \Delta C_R(\mathbf{R}) + \Delta C_M(\mathbf{R}) + \Delta C_V(\mathbf{R}) \quad (21)$$

where  $C_A(\mathbf{R})$  is the cost of activation of the combination  $\mathbf{R}$  of planning actions, and the terms  $\Delta C_R(\mathbf{R})$ ,  $\Delta C_M(\mathbf{R})$  and  $\Delta C_V(\mathbf{R})$  are the *variations* of the cost components referred to reliability (power and energy not served), maintenance and annual investment, respectively, with respect to the present system operation. The cost component  $\Delta C_V(\mathbf{R})$  can also be constrained by a budget limit.

Favuzza et al., 2006, address the distribution system reinforcement with distributed generation, using the annual cost of distribution systems reinforcement strategy as objective function in a multi-year planning approach. The objective function is expressed as:

$$f(\mathbf{R}) = C_L(\mathbf{R}) + C_M(\mathbf{R}) + C_V(\mathbf{R}) - R_D(\mathbf{R}) - R_I(\mathbf{R}) \quad (22)$$

where, in addition to the previously defined terms,  $C_L(\mathbf{R})$  is the cost of losses,  $R_D(\mathbf{R})$  is the saving referring to the energy not bought from the transmission system because of distributed generation exploitation, and  $R_I(\mathbf{R})$  is the revenue resulting from incentives (e.g., for production from renewable sources).

In Favuzza et al., 2007, the objective function for distribution system reinforcement with gas microturbines is formulated by taking into account also the costs  $C_E(\mathbf{R})$  of the electrical energy bought and the revenues  $R_E(\mathbf{R})$  from selling electricity and heat:

$$f(\mathbf{R}) = C_L(\mathbf{R}) + C_M(\mathbf{R}) + C_V(\mathbf{R}) + C_E(\mathbf{R}) - R_E(\mathbf{R}) \quad (23)$$

Gomez et al., 2004, formulate expansion planning as a mixed nonlinear integer optimization problem by considering the total (fixed and variable) costs and partitioning these costs among existing and proposed distribution system circuits and substations, with a given maximum load condition. The same partitioning is used to express the capacity constraints. Bi-directional voltage magnitude constraints are introduced as well. Other constraints refer to the energy balance and to consider only radial networks. A maximization problem is solved, in which the objective function is the inverse of the previously mentioned total costs.

## 4.2 Ant colony optimization applications

In the ACO application presented in Carpaneto & Chicco, 2005, the pheromone is associated to the individual planning actions. Initially, all planning actions  $j \in J$  are associated to the same quantity of pheromone  $\tau_0$ . During the iterative process, each ant  $m = 1, \dots, M$  activates a

number of planning actions chosen at random by applying the weighted probabilistic selection mechanism (Fig. 2) according to the probabilities (1), depending on the distance formulated for the specific problem. For instance, the application presented in Carpaneto & Chicco, 2005, deals with the placement of remote controllers in the distribution system nodes, considering as planning action the placement of an individual remote controller in a node belonging to a set of candidate nodes. In this case, the distance referring to a given node is the inverse of the product of the power not served times the equivalent failure rates in the upstream and downstream portions of the network with respect to that node. At the end of each iteration, the global heuristic rule is applied by evaporating part of the pheromone and increasing the pheromone quantity for the planning actions providing the best objective function during the iteration (if the solution is the best one found so far the amplification factor is higher than the one used for the best solution at the current iteration). In Favuzza et al., 2006, and Favuzza et al., 2007, the problem is represented as a graph, in which the (reinforced) distribution system configurations at the different years are the nodes, and the distances between the nodes are the actualized transition costs (including investment and operation) from the reference configuration to the new one. In this application, the set of nodes to be visited is not unique, but it is in principle infinite. The solution algorithm is then set up as a dynamic ant colony search, in which the algorithm creates at each year a finite number of candidate solutions, forming a dynamic tree structure. Basically, a parameter  $N_s$  is introduced to set up the number of candidate solutions generated (corresponding for instance to different sizes of the local generation units), and a mechanism to replace the worst search directions with most convenient ones (in terms of pheromone value and distance) is included to avoid excessive expansion of the dynamic tree during the solution process. Each resulting multi-year path is associated to a cost determined as the sum of the actualized transition costs.

For a given ant  $m$ , starting from the configuration  $X_i^{(m,y)}$  at a given year  $y$ , the configuration  $X_j^{(m,y+1)}$  reached at the successive year is selected among the set  $S_i^{(m,y+1)}$  of the configurations generated dynamically. The selection is carried out by using elitistic considerations as in Section 2.6. The user-defined parameter  $q_0$  is adapted during the solution process, decreasing it when the number of successive solutions without improvement increases, thus reducing the importance of elitistic choice in favour of the weighted probabilistic selection. The pheromone is updated according to the standard local heuristic rule (Section 2.3), and to the global heuristic rule (2) applied at the end of each iteration, in which the function  $\psi^{(h)}$  is the inverse of the cost of the best path found during the iteration.

In Gomez et al. 2004, initially a random pheromone quantity is associated to each branch of the network. Then, each ant explores the network to reach all its nodes with a radial network, with a search guided by the weighted probabilistic selection mechanism illustrated in Section 2.2. The selection is carried out by using elitistic considerations as in Section 2.6. Classical local and global heuristic rules are used for pheromone update. In the global heuristic rule (2), the function  $\psi^{(h)} = \kappa_r / C_{\min}^{(h)}$  is given by the ratio between the user-defined factor  $\kappa_r$  and the minimum cost of the best path found during the iteration. The distance used for pheromone update takes into account the branch length, the magnitude of the load at the end of the path, and the incremental cost of the network.

## 5. Optimization of service restoration and reliability

### 5.1 Formulation of the service restoration and reliability optimization problems

Faults in a distribution system are generally classified as *temporary* (when the protection trip is followed by successful reclosure and final normal operation) and *permanent* (requiring successive restoration phases after the protection trip, with remote-controlled operations, additional manual operations, on-site repair and eventual service restoration (Carpaneto & Chicco, 2004b)).

Distribution system reliability is represented by using *local* indices (referring to a single load point) and *global* indices (characterizing the overall distribution system). The relevant quantities are frequency and duration of the interruptions, and the corresponding global indices (Billinton & Allan, 1984) are the system average interruption frequency index (SAIFI) and the system average interruption duration index (SAIDI).

The relevant functions of the devices installed in distribution systems for reliability and service restoration purposes are switch and protection. The switch function refers to isolate a fault occurring downstream with respect to the switching device location, and can be operated by reclosers (with protection capability) or switches (with no protection capability). The protection function can be performed by circuit breakers, fuses and reclosers. The reclose function reduces the interruption duration after a temporary fault.

Different objective functions have been formulated for the optimization problems involving service restoration and reliability. In Teng & Liu, 2003, the objective function consists of the customer interruption costs (CIC) per year, taking into account voltage drop and feeder load transfer capability constraints. In Ahuja et al., 2007, service restoration is addressed starting from the solutions obtained for the multi-objective optimal reconfiguration problem (Section 3.3.3), considering only the voltage deviation objective as relevant for restoration purposes. Wang & Singh, 2008, address the location of reclosers and distributed generation units for improving distribution system reliability. The objective function to minimize is written as:

$$f = w_1 \frac{SAIFI}{SAIFI^{(T)}} + w_2 \frac{SAIDI}{SAIDI^{(T)}} \quad (24)$$

where  $w_1$  and  $w_2$  represent the weights associated with the interruption frequency and duration terms, respectively.  $SAIFI^{(T)}$  and  $SAIDI^{(T)}$  are the target values for SAIFI and SAIDI. Falaghi et al., 2009, formulate a fuzzy optimization procedure to solve the multiobjective optimization with the two conflicting objectives of improving service reliability (assessed through the calculation of the annual expected energy not supplied) and minimizing the cost of sectionalizing switches. The objective function is composed of the weighted sum of the two objectives,  $\tau'_0 = w_1 \bar{\mu}_R + w_2 \bar{\mu}_C$ , where  $\bar{\mu}_R$  and  $\bar{\mu}_C$  are the fuzzy membership function used for reliability improvement and cost of sectionalizing switches, respectively, and the coefficients  $w_1$  and  $w_2$  satisfy the condition  $w_1 + w_2 = 1$ . This function is used as a quality function in the heuristic rules for pheromone addition to the best paths.

Tippachon & Rerkpreedapong, 2009, address the multi-objective optimization problem concerning the optimal placement of reclosers and fuses. The objectives considered are SAIFI, SAIDI, and the total cost  $C^{tot}$ , and are addressed with the aim of searching for the non-dominated solutions belonging to a Pareto front (Shukla & Deb, 2007). The latter includes the average cost of temporary and permanent interruptions, and the fixed cost for investment and installation of switches and protective devices. The constraints refer to the

possibility of installing only reclosers and switches on the main distribution system feeders, the avoidance of fuse location upstream of a recloser, and the maximum number of fuses to be placed in a series path. The max-min operator is applied to select the best multiobjective solution.

**5.2 Ant colony optimization schemes and solutions**

In Ahuja et al., 2007, the pheromone table corresponding to the voltage deviation objective is used in order to obtain a feasible configuration for the service restoration problem after a contingency. The distribution network is progressively constructed starting from scratch by using the Kruskal’s algorithm (Kruskal, 1956) to connect branches not forming any loop until all nodes are reached. As relatively high pheromone values indicate the quality of a branch to remain open, the inverse of the pheromone values (added to a user-defined constant to avoid biasing the search towards the solution found in normal conditions) are used in the weighted probabilistic selection mechanism (Section 2.2) in order to extract the branch to be connected to the network. Falaghi et al., 2009, apply a classical version of ACO. An efficient scheme for addressing reliability optimization through ant ACO algorithms is illustrated and used in some references (Teng & Liu, 2003; Wang & Singh, 2008; Tippachon & Rerkpreedapong, 2009). This scheme adopts a multi-stage representation of the search space, in which the number of stages  $D$  is equal to the number of devices to be located in the distribution system (Fig. 3). Furthermore, at each stage the information dimension includes the candidate locations  $\xi = 1, \dots, \Xi$ . In a given stage  $\delta = 1, \dots, D$ , a state  $\gamma_\delta$  in the search space is defined by the pair (location  $\xi$ , device type).

Each ant starts from the nest and proceeds on the various stages until reaching the destination. At each stage, an ant selects one state to go in the next stage, on the basis of a function of the pheromone and of the distance  $d_{(i,j)}$  of the connection from the present state  $\gamma_\xi^{(\delta)}$  to a state  $\gamma_\xi^{(\delta+1)}$  in the following stage.

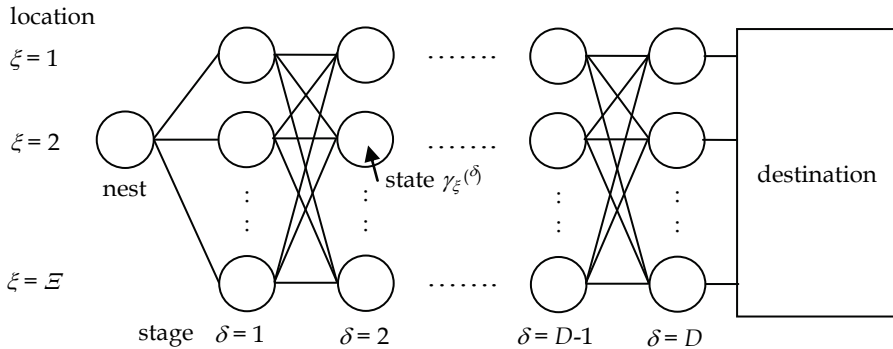


Fig. 3. Multi-stage representation of the search space.

In Teng & Liu, 2003, the max-min ant colony system (Stützle & Hoos, 2000) is used to address the switch relocation problem. For an ant  $m$  moving from the state  $i = \gamma_\xi^{(\delta)}$  at stage  $\delta$  to the point  $j = \gamma_\xi^{(\delta+1)}$  at stage  $\delta+1$ , the transition probability is expressed by considering as distance  $d_{(i,j)}$  the inverse of the CIC reduction between stage  $\delta$  and  $\delta+1$ , using the set  $S_i^{(m,\delta)}$  of the states that can be reached at the successive stage to obtain:

$$p_{(i,j)}^{(m,\delta)} = \frac{\tau_{(i,j)}^{(m,\delta)} d_{(i,j)}^{-\beta}}{\sum_{u \in S_i^{(m,\delta)}} \tau_{(i,u)}^{(m,\delta)} d_{(i,u)}^{-\beta}} \tag{25}$$

Alternatively, equation (25) can be rewritten by avoiding the use of the distance information to make calculations faster.

In Wang & Singh, 2008, ACO is applied by initially positioning the ants in randomly selected starting nodes. The pheromone is associated to the connections among the nodes, with equal initial values  $\tau_0$ . The multi-stage scheme of Fig. 3 is used. The distance is proportional to the inverse of the improvement of the objective function in two successive stages. Classical local and global heuristic rules are used. Elitism is applied as in Section 2.6.

In Tippachon & Rerkpreedapong, 2009, the multi-stage representation of Fig. 3 is extended to solve a multi-objective optimization problem through the multiple ant colony system (MACS) (Gambardella et al., 1999). MACS uses a single pheromone matrix and several heuristic information functions. The pheromone matrix values are initialized by using as initial estimation the value  $\tau_0 = (SAIFI_0 \cdot SAIDI_0 \cdot C_0^{tot})^{-1}$ .

Starting from the state  $i$  on a given stage, let us consider the set  $S_i$  of the states that can be reached at the successive stage, the distances  $d_{(i,j)} = \{SAIFI_{(i,j)}, SAIDI_{(i,j)}, C_{(i,j)}^{tot}\}$  when an ant  $m$  reaches the state  $j \in S_i^{(m)}$ , the user-defined parameter  $q_0 \in [0, 1]$  to introduce elitism (Section 2.6), and a random number  $r \in [0, 1]$  extracted each time a selection has to be made. The state  $j$  to be reached is then selected by applying the following rule:

$$j = \begin{cases} \arg \max_{u \in S_i^{(m)}} \left\{ \tau_{(i,u)} \prod_{v=1,2,3} \left( d_{(i,u)}^{(v)} \right)^{-\lambda_v \beta} \right\} & \text{if } r \leq q_0 \\ \hat{j} & \text{otherwise} \end{cases} \tag{26}$$

where the term  $\lambda_v = \text{mod}(m / M + \lambda_v - 1, 1)$  is introduced to force the ants  $m = 1, \dots, M$  to search in different directions of the Pareto front, and the state  $\hat{j}$  is chosen by applying the weighted probabilistic selection mechanism by computing the probabilities in a way similar to (1) for  $j \in S_i^{(m)}$ , namely:

$$p_{(i,j)}^{(m)} = \frac{\tau_{(i,j)} \prod_{v=1,2,3} \left( d_{(i,j)}^{(v)} \right)^{-\lambda_v \beta}}{\sum_{u \in S_i^{(m)}} \tau_{(i,u)} \prod_{v=1,2,3} \left( d_{(i,u)}^{(v)} \right)^{-\lambda_v \beta}} \tag{27}$$

After each ant movement from  $i$  to  $j$ , the local heuristic rule (Section 2.3) is applied to update the pheromone  $\tau_{(i,j)}$ .

The pheromone  $\tau_0$  may be changed during the solution process. After the generation of new solutions, it is verified which solutions are non-dominated (i.e., belong to the set  $P'$ , and are potential solutions to belong to the Pareto front  $P$ ). For each solution  $p \in P'$ , the value  $\tau'_0$

is calculated as  $\tau'_0 = (SAIFI_p \cdot SAIDI_p \cdot C_p^{tot})^{-1}$ . If  $\tau'_0 > \tau_0$  the pheromone is reinitialized to  $\tau'_0$ , otherwise the global update rule is applied to all connections  $(i, j)$  of each solution  $p \in P'$ :

$$\tau_{(i,j)} = \frac{\sigma}{SAIFI_p \cdot SAIDI_p \cdot C_p^{tot}} + (1 - \sigma)\tau_{(i,j)}^{previous} \quad (28)$$

## 6. Conclusions

This chapter has summarized the framework of application referring to some typical distribution system optimization problems. While some applications use classical versions of the ACO algorithms, specific adaptations of the algorithms to specific problems led to customized versions with ACO variants or hybridizations with other heuristics.

Future work can be developed to improve the effectiveness of the solutions of the ACO algorithms, also benefiting from recent theoretical findings, such as the ones concerning the choice of an appropriate pheromone model to avoid the so-called second-order deception with decreased algorithm performance over time (Blum & Dorigo, 2005; Dorigo & Blum, 2005), the development of ACO algorithms functionally equivalent to the original ones, with pheromone independent of the scale of the problem instance considered (Birattari et al., 2007), the possible ACO application to continuous optimization problems (Blum, 2005), and the analysis of the convergence time of the ACO algorithms (Huang et al., 2009).

## 7. References

- Ahuja, A.; Das, S. & Pahwa, A. (2007). An AIS-ACO hybrid approach for multi-objective distribution system reconfiguration. *IEEE Trans. on Power Systems*, Vol. 22, No. 3, (August 2007) 1101–1111, ISSN 0885-8950.
- Andrei, H.; Caciula, I. & Chicco, G. (2008). A comprehensive assessment of the solutions of the distribution system minimum loss reconfiguration problem. *Proc. 7th World Energy System Conference*, ISSN 1198-0729, Iași, Romania, 30 June - 2 July 2008.
- Andrei, H. & Chicco, G. (2008). Identification of the radial configurations extracted from the weakly meshed structures of electrical distribution systems. *IEEE Trans. on Circuits and Systems. Part I: Regular Papers*, Vol. 55, No. 4, (May 2008) 1149-1158, ISSN 1549-8328.
- Baran, M.E. & Wu, F.F. (1989). Optimal capacitor placement on radial distribution systems. *IEEE Trans. on Power Delivery*, Vol. 4, No. 1, (January 1989) 725-732, ISSN 0885-8977.
- Baran, M.E. & Wu, F.F. (1989b). Network reconfiguration in distribution systems for loss reduction and load balancing. *IEEE Trans. on Power Delivery*, Vol. 4, No. 2, (April 1989) 1401-1407, ISSN 0885-8977.
- Billinton, R. & Allan, R.N. (1984). *Reliability Evaluation of Power Systems*. Plenum, New York, 1984, ISBN 978-0306452598.
- Birattari, M.; Pellegrini, P. & Dorigo, M. (2007). On the Invariance of Ant Colony Optimization. *IEEE Trans. on Evolutionary Computation*, Vol. 11, No. 6, (December 2007) 732-741, ISSN 1089-778X.
- Blum, C. (2005). Ant colony optimization: Introduction and recent trends. *Physics of Life Reviews*, Vol. 2, No. 4, (December 2005) 353-373, ISSN 1571-0645.

- Blum, C. & Dorigo, M. (2004). The hyper-cube framework for ant colony optimization. *IEEE Trans. on Systems, Man and Cybernetics: Part B*, Vol. 34, No. 3, (April 2004) 1161-1172, ISSN 1083-4419.
- Blum, C. & Dorigo, M. (2005). Search bias in ant colony optimization: on the role of competition-balanced systems. *IEEE Trans. on Evolutionary Computation*, Vol. 9, No. 2, (April 2005) 159-174, ISSN 1089-778X.
- Carpaneto, E.; Chicco, G. & Roggero, E. (2003). Comparing deterministic and simulated annealing-based algorithms for minimum losses reconfiguration of large distribution systems. *Proc. IEEE Bologna PowerTech*, Vol. 2, ISBN 0-7803-7967-5, Bologna, Italy, 23-26 June 2003.
- Carpaneto, E.; Chicco, G.; De Donno, M. & Napoli, R. (2004). Voltage controllability of distribution systems with local generation sources. *Proc. Bulk Power System Dynamics and Control - VI*, pp. 261-273, ISBN 88-87380-47-3, Cortina d'Ampezzo, Italy, August 22-27, 2004.
- Carpaneto, E. & Chicco, G. (2004). Ant-colony search-based minimum losses reconfiguration of distribution systems. *Proc. IEEE Mediterranean Electrotechnical Conference (Melecon 2004)*, Vol. 3, ISBN 0-7803-8271-4, Dubrovnik, Croatia, 971-974, 12-15 May 2004.
- Carpaneto, E. & Chicco, G. (2004b). Evaluation of the probability density functions of distribution system reliability indices with a characteristic functions-based approach, *IEEE Trans. on Power Systems*, Vol. 19, No. 2, (May 2004) 724-734, ISSN 0885-8950.
- Carpaneto, E. & Chicco, G. (2005). Optimal operational planning of large distribution systems with Ant Colony Search, *Proc. 15<sup>th</sup> Power System Computational Conference (PSCC 2005)*, Liège, Belgium, 22-26 August 2005.
- Carpaneto, E. & Chicco, G. (2008). Distribution system minimum loss reconfiguration in the hyper-cube ant colony optimization framework. *Electric Power Systems Research*, Vol. 78, No. 12, (December 2008) 2037-2045, ISSN 0378-7796.
- Carpaneto, E. & Chicco, G. (2010). Steady-state assessment of the DG impact on voltage control and loss allocation. Chapter 3 in Gaonkar, N.D. (ed.), *Distributed generation*, In-Teh, Vukovar, Croatia, February 2010, 51-76, ISBN 978-953-307-046-9.
- Chang, C.-F. (2008). Reconfiguration and capacitor placement for loss reduction of distribution systems by ant colony search algorithm. *IEEE Trans. on Power Systems*, Vol. 23, No. 4, (November 2008) 1747-1755, ISSN 0885-8950.
- Chiou, J.-P.; Chang, C.-F. & Su, C.-T. (2004). Ant direction hybrid differential evolution for solving large capacitor placement problems. *IEEE Trans. on Power Systems*, Vol. 19, No. 4, (November 2004) 1794-1800, ISSN 0885-8950.
- Civanlar, S.; Grainger, J.; Yin, H. & Lee, S. (1988). Distribution feeder reconfiguration for loss reduction. *IEEE Trans. on Power Delivery*, Vol. 3, No. 3, (July 1988) 1217-1223, ISSN 0885-8977.
- Dorigo, M.; Maniezzo, V. & Coloni, A. (1996). Ant system: optimization by a colony of cooperating agents. *IEEE Trans. on Systems, Man and Cybernetics: Part B*, Vol. 26 (February 1996) 29-41, ISSN 1083-4419.
- Dorigo, M. & Gambardella, L.M. (1997). Ant colony system, a cooperative learning approach to the traveling salesman problem. *IEEE Trans. on Evolutionary Computation*, vol. 1, no. 1, (April 1997) 53-66, ISSN 1089-778X.

- Dorigo, M. & Blum, C. (2005). Ant colony optimization theory: A survey. *Theoretical Computer Science*, Vol. 344, No. 2-3, (November 2005) 243-278, ISSN 0304-3975.
- Dorigo, M.; Birattari, M. & Stützle, T. (2006). Ant colony optimization. *IEEE Computational Intelligence Magazine*, Vol. 1, No. 4, (November 2006) 28-39, ISSN 1556-603X.
- Falaghi, H.; Haghifam, M.-R. & Singh, C. (2009). Ant Colony Optimization-Based Method for Placement of Sectionalizing Switches in Distribution Networks Using a Fuzzy Multiobjective Approach. *IEEE Trans. on Power Delivery*, Vol. 24, No. 1, (January 2009) 268-276, ISSN 0885-8977.
- Favuzza, S.; Graditi, G. & Riva Sanseverino, E. (2006). Adaptive and Dynamic Ant Colony Search Algorithm for Optimal Distribution Systems Reinforcement Strategy. *Applied Intelligence*, Vol. 24, No. 1, (February 2006) 31-42, ISSN 1573-7497.
- Favuzza, S.; Graditi, G.; Ippolito, M.G. & Sanseverino, E.R. (2007). Optimal Electrical Distribution Systems Reinforcement Planning Using Gas Micro Turbines by Dynamic Ant Colony Search Algorithm. *IEEE Trans. on Power Systems*, Vol. 22, No. 2, (May 2007) 580-587, ISSN 0885-8950.
- Gambardella, L.M.; Taillard, E. & Agazzi, G. (1999). MACS-VRPTW: A Multiple Ant Colony System for Vehicle Routing Problems with Time Windows. In: Corne, D.; Dorigo, M. & Glover, F. (ed.), *New Ideas in Optimization*. McGraw-Hill, UK, 1999, 63-76, ISBN 978-0077095062.
- Gomez, J.F.; Khodr, H.M.; De Oliveira, P.M.; Ocque, L.; Yusta, J.M.; Villasana, R. & Urdaneta, A.J. (2004). Ant colony system algorithm for the planning of primary distribution circuits. *IEEE Trans. on Power Systems*, Vol. 19, No. 2, (May 2004) 996-1004, ISSN 0885-8950.
- Huang H.; Wu, C.-G. & Hao, Z.-F. (2009). A Pheromone-Rate-Based Analysis on the Convergence Time of ACO Algorithm. *IEEE Trans. on Systems, Man, and Cybernetics, Part B: Cybernetics*, Vol. 39, No. 4, (August 2009) 910-923, ISSN 1083-4419.
- Kruskal, J.B. (1956). On the shortest spanning subtree of a graph and the traveling salesman problem. *Proceedings of the American Mathematical Society*, Vol. 7, No. 1, (February 1956) 48-50, ISSN 0002-9939.
- Niknam, T. (2008). A new approach based on ant colony optimization for daily volt/var control in distribution networks considering distributed generators. *Energy Conversion and Management*, Vol. 49, No. 12, (December 2008) 3417-3424, ISSN 0196-8904.
- Prim, R.C. (1957). Shortest connection networks and some generalizations. *Bell System Technical Journal*, Vol. 36, No. 6, (1957) 1389-1401, ISSN 0005-8580.
- Reeves, C.R. (Ed.) (1993). *Modern heuristic techniques for combinatorial problems*. Blackwell Scientific Publications, Oxford, UK, 1993, ISBN 0-470-22079-1.
- Shirmohammadi, D.; Hong, H.W.; Semlyen, A. & Luo, G.X. (1988). A compensation-based power flow method for weakly meshed distribution and transmission networks. *IEEE Trans. on Power Systems*, Vol. 3, No. 2, (May 1988) 753-762, ISSN 0885-8950.
- Shukla, P.K. & Deb, K. (2007). On finding multiple Pareto-optimal solutions using classical and evolutionary generating methods. *European Journal of Operational Research*, Vol. 181, No. 3, (September 2007) 1630-1652, ISSN 0377-2217.
- Strezoski, V.C.; Katic, N.A. & Janjic, D.S. (2001). Voltage control integrated in distribution management system. *Electric Power Systems Research*, Vol. 60, No. 2, (December 2001) 85-97, ISSN 0378-7796.

- Stützle, T. & Hoos, H.H. (2000). Max-min ant system. *Future Generation Computer Systems*, Vol. 16, No. 8, (June 2000) 889-914, ISSN 0167-739X.
- Su, C.-T.; Chang, C.-F. & Chiou, J.-P. (2005). Distribution network reconfiguration for loss reduction by ant colony search algorithm. *Electric Power Systems Research*, Vol. 75, (August 2005) 190-199, ISSN 0378-7796.
- Teng, J.-H. & Liu, Y.-H. (2003). A novel ACS-based optimum switch relocation method. *IEEE Trans. on Power Systems*, Vol. 18, No. 1, (February 2003) 113-120, ISSN 0885-8950.
- Tippachon, W. & Rerkpreedapong, D. (2009). Multiobjective optimal placement of switches and protective devices in electric power distribution systems using ant colony optimization. *Electric Power Systems Research*, Vol. 79, No. 7, (July 2009) 1171-1178, ISSN 0378-7796.
- Wang, L. & Singh, C. (2008). Reliability-Constrained Optimum Placement of Reclosers and Distributed Generators in Distribution Networks Using an Ant Colony System Algorithm. *IEEE Trans. on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, Vol. 38, No. 6, (November 2008) 757-764, ISSN 1094-6977.
- Wu, Y.-K.; Lee, C.-Y.; Liu, L.-C. & Tsai, S.-H. (2010). Study of reconfiguration for the distribution system with distributed generators. *IEEE Trans. on Power Delivery*, Vol. 25, No. 3, (July 2010) 1678-1685, ISSN 0885-8977.
- Zamboni, L. & Monteiro, L.H.A. (2009). Optimization of the topology of electric energy distribution networks by using algorithm inspired on ant behavior. *IEEE Latin America Transactions*, Vol. 7, No. 1, (March 2009) 85-91, ISSN 1548-0992.