Epicyclic gear train dynamics including mesh efficiency

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EPICYCLIC GEAR TRAIN DYNAMICS INCLUDING MESH EFFICIENCY

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ABSTRACT

The paper presents an epicyclic gear train dynamic mathematical model including mesh efficiency, bearings/seals losses and inertial effects. The mathematical model treats separately the mesh between sun and planets gears and the mesh between planets and ring gears. Two different ordinary efficiency values for each gear pair can be specified for forward and reverse power transmission through it. The mesh efficiency is inserted into the dynamic model through a change in the direction of the mean reaction force between tooth surfaces [4]. The extension of the equations valid for ordinary gearing to elementary gear train with epicyclic arrangement is made by using the kinematic inversion [2]. A formula for selecting the correct efficiency value, to be used in the model, depending on the direction of power flows along the epicyclic gear train is presented. Finally, in order to check the validity of the dynamic model proposed, a steady-state working condition is analysed in detail and the mesh efficiency resulting from a numerical simulation of the model is compared with analytical formulas [7].

Keywords: Epicyclic Gear Train, Planetary Gear Set, Mesh Efficiency, Modeling.

1 INTRODUCTION

Epicyclic gear train (EGT) dynamic modeling has recently become relevant to improve the performance of virtual analysis especially in the field of automotive transmissions components [5]. The requirements which currently characterize the transmission design from the point of view of vehicle energy saving ask for gear trains dynamic virtual analysis including a detailed description of the power losses.

As well kwon the main sources of power loss in a gear train are: gear mesh losses, windage and churning losses, bearing and seal losses and lubrication pump losses.

In comparison with ordinary gear trains where mesh efficiency is very often close to unity, the power loss in an epicyclic gear train, depending both on the operating condition and on the gear train geometrical configuration, can become very low [6].

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Several articles were written with the aim of analytically describe the efficiency of such a mechanical device for all the possible power paths through the gearing. Pennestrì and Valentini in [7] summarize some of these analytical approaches, e.g. [4], [8] and [9], for the mesh efficiency computation in a two degrees of freedom (d.o.f.) epicyclic gear train and demonstrate the numerical equivalence of the different formulas presented. The problem of the extension of these analytical expressions for a generic n-link EGT can be dealt with by means of manual (see e.g. [10]) and systematic methods (see e.g. [6]).

A systematic methodology for computing the mechanical efficiency of a generic EGT considering also load dependant power losses and inertia effect is presented in [2]. In that work the multibody formalism is used for solving the inverse dynamic problem in gear train and the approximated method of Anderson and Lowenthal [11], that accounts for sliding, rolling, bearing and windage losses, is adopted for estimating the efficiency of the ordinary gear train.

The present work shows a dynamic model of a two d.o.f. epicyclic gear train that considers the following source of internal power loss: mesh efficiency, seals and bearings. The inertial effects of all the components are also included in the model.

The followed approach consists in the next steps:

- decomposition of the gear train in its basic elements
- insertion of the mesh efficiency for each gear pair through a change in the direction of the mean reaction force between tooth surfaces [4]
- estimation of the ordinary efficiency of the gear pairs as function of gears geometrical parameters and the frictional contact conditions between teeth, see e.g. [1], [3], [4], [11], [12]. This part is not detailed in the paper.
- extension of the formulae valid for ordinary gear train to elementary gear train with epicyclic arrangement [2]
- Coulomb friction model for the resistant torque due to bearings and seals
- dynamic balance equations starting from the free body diagrams and kinematic relations
- dynamic system matrix formulation and solution
- efficiency expression valid for all possible power flow configurations
- graphical and numerical model verification in steady-state conditions

The paper discusses the relations between efficiency and displacement of the meshing force of the gears due to the presence of friction both for external and internal gears and an original formula is proposed to appropriately select the right instantaneous value depending on the specific dynamic situation.

The model presented was implemented in a simulation environment and the correctness of the computed mesh efficiency was verified for all the possible combinations of power flows. The simulated efficiency using the proposed dynamic model and the analytical formulas [7] give exactly the same results for the same steady state operating conditions. A particular case is analysed in detail considering the awaited (graphically) and the computed model variables (analytically) with the aim of verifying the sign of the internal meshing forces and their displacement from the pitch point.
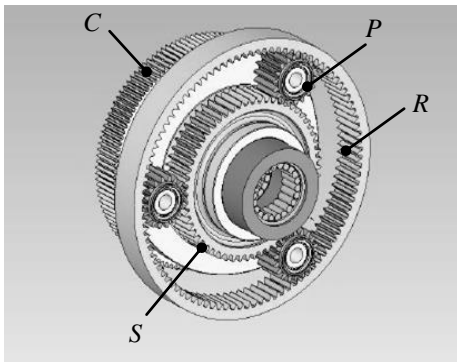

Figure 1- Epicyclic gear train with the adopted nomenclature

2 MESH EFFICIENCY MODELING FOR ORDINARY GEARING

According to the Merritt's model [4], the direction of the meshing force, in presence of friction between teeth, intersects the center distance in a point, which is not the center of instantaneous rotation of the involved pitch circles. Since this distance is not constant, during the passage of the point of contact from end to end of the line of contact, it is useful to compute a mean value of this displacement over a complete cycle of engagement called δ.

Generally speaking, in case of different meshing conditions, i.e. sliding velocities and coefficient of frictions, in approach and recess there can be differences in terms of direct (subscript D) and inverse (subscript I) mesh efficiency. Considering for example the same gears used both as a reduction drive and as a step-up drive, thus replacing the driver with the follower, different values of efficiency can arise. More specifically, as explained in [3], when the recess action of the driver on a reduction drive is greater than the approach action, then the corresponding step-up drive will be less efficient than the reduction drive. Thus, it is important to allow the definition of different efficiency values η_{Dij} and η_{Iij} respectively for forward and reverse power transmission through the gears, that can be generally different. The last two subscripts used both for efficiency and displacement specify the driving gear (i) and the driven gear (j) in the direct configuration.

Figure 1 shows a common planetary gear set assembly with sun (S), planets (P), planet carrier (C) and ring (R). The Figure presents a typical example of gears configuration where there are meshing between both external gears (sun-planet) and internal gears (planet-ring).

The relations between the efficiency and the displacement of the meshing force due to the presence of friction, both for external and internal gears, are now discussed. Considering the mesh that involves the sun and planet gears (external gears), the relation between displacement and efficiency can be formulated as:

$$\delta_{Dsp} = \frac{1 - \eta_{Dsp}}{\frac{\eta_{Dsp}}{R_s} + \frac{1}{R_p}} \qquad \delta_{Isp} = \frac{1 - \eta_{Isp}}{\frac{\eta_{Isp}}{R_p} + \frac{1}{R_s}} \qquad (1)$$

where the variables referred to the sun and to the planets are indicated respectively by subscripts s and p. R_s, R_p and R_r are the pitch circle radius of sun, planets and ring, while R_c is the distance between the center of EGT and the center of planets in the considered epicyclic arrangement. The direct efficiency η_{Dsp} corresponds to the case of planet driven by sun.

The offset displacement of the ordinary gear train composed by the planet (external gear) and the ring (internal gear) can be calculated as:

$$\delta_{Dpr} = \frac{(1 - \eta_{Dpr})}{\frac{\eta_{Dpr}}{R_p} - \frac{1}{R_r}} \qquad \delta_{Ipr} = \frac{(1 - \eta_{Ipr})}{\frac{1}{R_p} - \frac{\eta_{Ipr}}{R_r}} \qquad (2)$$

where subscript r refers to the ring. The direct efficiency η_{Dpr} corresponds to the case of planet as driving gear and ring as the driven one. Once calculated and stored the direct (δ_{Dij}) and inverse (δ_{Iij}) displacement values for each gear pair, it is necessary to appropriately select the right instantaneous value depending on the specific dynamic situation.

3 MESH EFFICIENCY MODELING FOR EPICYCLIC GEAR TRAIN

Equations (1) and (2) are obtained considering the mesh between two gears with their axes fixed in the space, i.e. ordinary gear train, but the same equations can be applied also to a planetary gear set, where there is a carrier that moves the axes of the planet gears. It must be considered in that case, instead of the real absolute velocity, the velocity of the gears relative to the carrier speed; a kinematic inversion in fact cannot affect the power losses [2].

More specifically, if the tangential component of the meshing force exerted by gear j on gear i, indicated in the equations and Figures with F_{tji}, has the same direction as the relative pitch tangential speed (or the same sign of the angular speed of gear i relative to the carrier arm c, i.e. $\omega_{i,rel}$), the i component is the follower of the basic train under a kinematic inversion which makes the gear carrier fixed [7]. Conversely, if they have opposite directions, gear i is the driver.

The expressions proposed in [5] were so extended including some new considerations explained in the following. The non linear expressions of the offset distance, based on the convention of sign of Figure 3, can be computed as follows:

Sun-planet coupling:

$$\delta_{sp} = \text{sgn}\left|F_{tps}(\omega_{p,rel} - \omega_{s,rel})\right| \left(\delta_{Dsp}\, f_-(F_{tps}\, \omega_{s,rel}) + \right. \tag{3}$$
$$\left. -\delta_{Isp}\, f_+(F_{tps}\, \omega_{s,rel})\right)$$

Planet-ring coupling:

$$\delta_{pr} = \text{sgn}\left|F_{trp}(\omega_{r,rel} - \omega_{p,rel})\right| \left(\delta_{Dpr}\, f_-(F_{trp}\, \omega_{p,rel}) + \right. \tag{4}$$
$$\left. -\delta_{Ipr}\, f_+(F_{trp}\, \omega_{p,rel})\right)$$

where

$$f_\pm\left(F_{tji}\, \omega_{i,rel}\right) = \frac{1 \pm \text{sgn}(F_{tji}\, \omega_{i,rel})}{2} \tag{5}$$

and the relative speeds are

$$\begin{cases} \omega_{s,rel} = \omega_s - \omega_c \\ \omega_{r,rel} = \omega_r - \omega_c \\ \omega_{p,rel} = \omega_p - \omega_c \end{cases}$$

The former expressions of the offset distance can be extended to a generic case, considering gears i and j, where i is an external gear and j can be either an external or an internal gear.

The generic formula is:

$$\delta_{ij} = \text{sgn}\left|F_{tji}(\omega_{j,rel} - \omega_{i,rel})\right| \left(\delta_{Dij}\, f_-(F_{tji}\, \omega_{i,rel}) + \right. \tag{6}$$
$$\left. -\delta_{Iij}\, f_+(F_{tji}\, \omega_{i,rel})\right)$$

where

$$\omega_{i,rel} = \omega_i - \omega_k \tag{7}$$
$$\omega_{j,rel} = \omega_j - \omega_k$$

Figure 2 illustrates the two possible cases of meshing between external and internal gears. The carrier is represented as k.

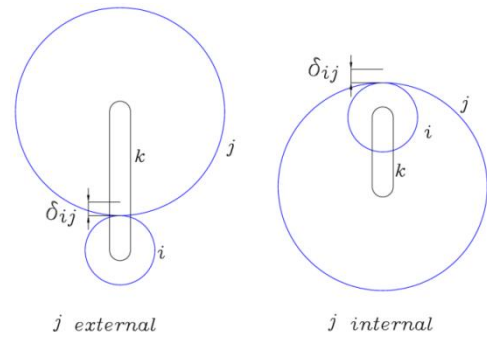Figure 2- Offset distance positive sign convention for external (left) and internal gears (right).

The first part of expression (6), i.e. $\text{sgn}\left|F_{tji}.(\omega_{j,rel} - \omega_{i,rel})\right|$, is used to have null offset distance and consequently a unitary efficiency in two particular cases: when there are no forces exchanged between the gears ($F_{tji} = 0$), e.g. when the gears are completely unloaded, and when all the angular velocities of the gears are the same ($\omega_{j,rel} - \omega_{i,rel} = 0$). For instance, when $\omega_s = \omega_r = \omega_p = \omega_c$, the epicyclic gear train behaves like a unique component and the absence of relative motions involves that the loss in the teeth contacts is null.

The second part of equation (6) accounts for all the other working conditions of the epicyclic gear pair. A specific function, $f_\pm$, is introduced in order to alternatively activate or deactivate one of the two addends (the direct and inverse displacements) depending on the sign of the power of the ordinary gearing obtained through a kinematic inversion which makes the planet carrier fixed.

An example of a possible operating condition covered by equations (3) and (4) appears when ω_s is null, i.e. the sun is blocked ($\omega_{p,rel} \neq \omega_{s,rel}$). Since the sun cannot deliver power at null speed, for sun-planet coupling the only possible driving gear is the planet and so eq. (3) should give the inverse displacement. Indeed, the meshing force of contact that acts on the sun gear causes a moment concordant with the sun relative angular velocity ($f_+ = 1$, $f_- = 0$), then, the offset displacement is negative, i.e. inside of the sun pitch circle: $\delta_{sp} = -\delta_{Isp}$.

4 BEARINGS AND SELAS MODEL

The analytical formulations of mechanical efficiency of ordinary and epicyclic gear trains usually consider only meshing losses. In the proposed dynamics equations further dissipative terms, such as bearings and seals resistant torques, are taken into account for all the four components (sun, planet, carrier and ring). These contributions, called T_B, are defined opposite with respect to the angular velocity difference between the inner and the outer ring of the considered bearing. Thus it is important to correctly define the position of bearings and seals among the epicyclic assembly. A Coulomb friction model is adopted for the torque computation in order to take into account the possible change in the direction of rotation of the specific component. Considering in this case that the bearings of the ring are mounted between the ring gear shaft and the sun gear shaft, then the relative angular velocity between them is introduced as the argument of a sign function in the equations. The sun gear and the carrier are supported through bearings connected to the gearbox housing. Consequently the bearings and seals torques are:

$$\begin{cases} T_{Brs} = |T_{Brs}| \, \text{sgn}(\omega_r - \omega_s) \\ T_{Bs} = |T_{Bs}| \, \text{sgn}(\omega_s) \\ T_{Bc} = |T_{Bc}| \, \text{sgn}(\omega_c) \\ T_{Bpc} = |T_{Bpc}| \, \text{sgn}(\omega_p - \omega_c) \end{cases}$$

where for instance T_{Brs} stands for the bearings and seals torque acting on the ring shaft. The third subscript is introduced only if the bearing is placed between two moving elements of the gearing, otherwise, if is fixed to the ground, it is omitted.

5 EGT KINEMATICS AND DYNAMICS

5.1 SIGN CONVENTIONS

As can be seen in Figure 3, the absolute angular velocities (and accelerations) of sun, ring, planet and carrier are assumed positive if they rotate in clockwise direction and the same applies to the torques.

The tangential component of the meshing force is positive if causes a clockwise moment with respect to the center of the planetary gear set (point O in Figure 3).

The offset distance δ_{sp} for sun-planet coupling, being both external gears, is assumed positive if located on the outside of the sun gear pitch circle and its instantaneous value is computed using equation (3). For planet-ring coupling, internal gears, the offset distance δ_{pr} (see eq. (4)) is assumed positive if located on the outside of the planet gear pitch circle, thus representing the case when it is the driving gear.

5.2 KINEMATIC EQUATIONS

The kinematic relation between the epicyclic gear train speeds can be expressed through the following matricial equation:

$$\begin{Bmatrix} \omega_c \\ \omega_p \end{Bmatrix} = \begin{bmatrix} \dfrac{R_r}{2R_c} & \dfrac{R_s}{2R_c} \\ \dfrac{R_r}{2R_p} & -\dfrac{R_s}{2R_p} \end{bmatrix} \begin{Bmatrix} \omega_r \\ \omega_s \end{Bmatrix} = [K] \begin{Bmatrix} \omega_r \\ \omega_s \end{Bmatrix} \tag{8}$$

5.3 DYNAMIC EQUATIONS

Considering the free body diagrams of ring, sun, carrier and planets represented in Figure 3 the dynamic equations written in matrix form are:

$$[I]\{\dot{\omega}\} = \{T\} - \{T_B\} + n_p[G]\{F_t\} \tag{9}$$

where $[I]$ is the inertia matrix, $\{\dot{\omega}\}$ is the acceleration vector, $\{T\}$ and $\{T_B\}$ are respectively the vector of input torques and the one of bearings and seals torques, n_p is the number of planets, $[G]$ is a geometric matrix containing the radii of all the epicyclic elements and the displacements of the meshing forces, $\{F_t\}$ is the vector of the tangential components of the meshing forces.
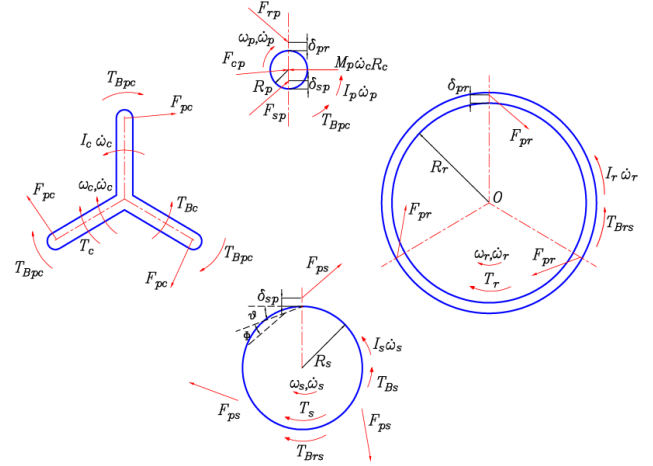

Figure 3- Free body diagrams of the planetary gear train

The additional equation to compute the tangential force exerted by the carrier on each planet is:

$$F_{tcp} = -F_{trp} + F_{tps} + M_p R_c \dot{\omega}_c \tag{10}$$

Equation (9) can be expanded in the following form:

$$\begin{bmatrix} I_r & & & \\ & I_s & & \\ & & (I_c + n_p M_p R_c^2) & \\ & & & n_p I_p \end{bmatrix} \begin{Bmatrix} \dot{\omega}_r \\ \dot{\omega}_s \\ \dot{\omega}_c \\ \dot{\omega}_p \end{Bmatrix} = \begin{Bmatrix} T_r \\ T_s \\ T_c \\ 0 \end{Bmatrix} +$$

$$- \begin{Bmatrix} |T_{Brs}| \, \text{sgn}(\omega_r - \omega_s) \\ |T_{Bs}| \, \text{sgn}(\omega_s) - |T_{Brs}| \, \text{sgn}(\omega_r - \omega_s) \\ |T_{Bc}| \, \text{sgn}(\omega_c) - n_p |T_{Bpc}| \, \text{sgn}(\omega_p - \omega_c) \\ n_p |T_{Bpc}| \, \text{sgn}(\omega_p - \omega_c) \end{Bmatrix} +$$

$$+ n_p \begin{bmatrix} -(R_r + \delta_{pr}) & 0 \\ 0 & R_s + \delta_{sp} \\ R_c & -R_c \\ R_p + \delta_{pr} & R_p - \delta_{sp} \end{bmatrix} \begin{Bmatrix} F_{trp} \\ F_{tps} \end{Bmatrix} \tag{11}$$

5.4 SOLUTION OF THE MOTION EQUATIONS

In order to integrate the system of non-linear differential equations (11), ring and sun angular accelerations are chosen as independent variables.

Therefore, eq. (11) can be separated into two parts:

$$[I_{rs}]\begin{Bmatrix}\dot\omega_r\\\dot\omega_s\end{Bmatrix} = \{T_{rs}\} - \{T_{Brs}\} + n_p[G_{rs}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix} \quad (12)$$

$$[I_{cp}]\begin{Bmatrix}\dot\omega_c\\\dot\omega_p\end{Bmatrix} = \{T_{cp}\} - \{T_{Bcp}\} + n_p[G_{cp}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix} \quad (13)$$

An expression for meshing forces vector can be found starting from equations (8), (12) and (13). By substituting the derivative of eq. (8) into (13) and pre-multiplying the resulting equation by $[K^{-1}I_{cp}^{-1}]$ it yields:

$$\begin{Bmatrix}\dot\omega_r\\\dot\omega_s\end{Bmatrix} = [K^{-1}\,I_{cp}^{-1}]\left(\{T_{cp}\} - \{T_{Bcp}\} + n_p[G_{cp}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix}\right)$$

Then, the previous equation can be pre-multiplied by I_{rs} and substituted into eq. (12):

$$[I_{rs}K^{-1}I_{cp}^{-1}]\left(\{T_{cp}\} - \{T_{Bcp}\} + n_p[G_{cp}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix}\right) = \{T_{rs}\} +$$

$$-\{T_{Brs}\} + n_p[G_{rs}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix}$$

Making some algebraic steps with the aim of isolating the vector of meshing forces it results:

$$n_p[I_{rs}K^{-1}I_{cp}^{-1}G_{cp} - G_{rs}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix} = \Big\{T_{rs} - T_{Brs} +$$

$$-[I_{rs}K^{-1}I_{cp}^{-1}]\{T_{cp} - T_{Bcp}\}\Big\}$$

Then, pre-multiplying by $\left[I_{rs}K^{-1}I_{cp}^{-1}G_{cp} - G_{rs}\right]^{-1}$ and dividing by n_p, the vector of meshing forces can be computed as:

$$\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix} = \frac{1}{n_p}\left[I_{rs}K^{-1}I_{cp}^{-1}G_{cp} - G_{rs}\right]^{-1}\Big\{T_{rs} - T_{Brs} +$$

$$-[I_{rs}K^{-1}I_{cp}^{-1}]\{T_{cp} - T_{Bcp}\}\Big\} \quad (14)$$

Finally, to evaluate the ring and sun gears dynamics, the following equation has to be integrated:

$$\begin{Bmatrix}\dot\omega_r\\\dot\omega_s\end{Bmatrix} = [I_{rs}]^{-1}\left(\{T_{rs}\} - \{T_{Brs}\} + n_p[G_{rs}]\begin{Bmatrix}F_{trp}\\F_{tps}\end{Bmatrix}\right) \quad (15)$$

Dynamic equations (14) and (15) together with kinematic equation (8) are the final equations that can be implemented in a simulation environment for the virtual analysis of the dynamic system. They completely describe the kinematic and dynamic behavior of the epicyclic gear train with mesh efficiency and bearings/seals losses.

It must be noted that the presence of friction between the tooth surfaces generates an algebraic loop in the model. In fact, eq. (6) is an algebraic relation between tangential force and displacement. Since the values of displacements (δ_{sp} and δ_{pr}) must be known in order to compute the geometric matrices (G_{rs} and G_{cp}), and furthermore since these matrices appear also in the final expression of the tangential forces (14), it is evident that the problem exists. Thus, the system of equations can be iteratively solved, starting from the retained force values from the last iteration at the previous time step.

6 STEADY-STATE MODEL VERIFICATIONS

In order to check the validity of the dynamic model proposed in steady-state conditions, in this section an example of epicyclic gear set operation will be analysed in detail. More specifically both the dynamic equations (11) and the offset distance equations (3) and (4) are verified. During the considered tests, all the angular accelerations are set to zero, i.e. the four components are in equilibrium with constant external torques at constant speeds.

Moreover mesh efficiency numeric values, obtained in steady-state conditions, were compared with the results of analytical formulations [7].

6.1 GRAPHICAL MODEL VERIFICATION

The distribution of power between the epicyclic gear train elements for the analysed configuration is resumed in Table I.

Table I – Working condition analysed

Case	Input Gear	Output Gear	Fixed Gear
A	S , R	C	-

The power enters into the system both through the sun and through the ring gear, while it goes out from the carrier. To completely define the working condition of the EGT with the aim of describing its efficiency it is necessary to specify not only the power flow direction through the EGT but also the specific kinematic situation analysed [7]. Here it is assumed that the angular velocities of the sun and the ring have the same positive direction of rotation and that $\omega_r > \omega_s$. The kinematics of this mechanism is illustrated in Figure 4, together with the kinematic inversion that makes fixed the planet carrier; the velocities of the gears relative to the carrier are shown by black triangles.
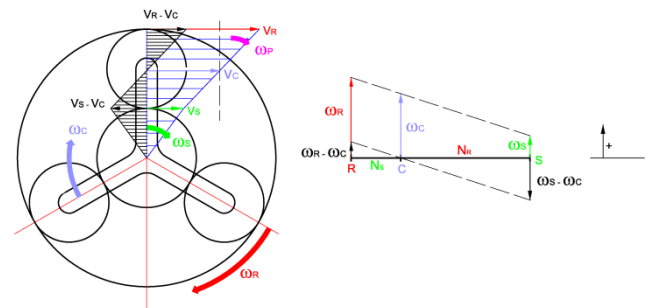

Figure 4 - Kinematics of Case A: Speeds distribution (to the left) and lever analogy (to the right).

The equilibrium of the EGT members is depicted in Figure 5. It is of interest to note that, despite the sun is a driver for the EGT, i.e. $T_s\omega_s > 0$, its relative angular velocity is

negative, therefore the product $T_s(\omega_s - \omega_c) < 0$. This negative power value identifies the driven gear under the kinematic inversion which makes the gear carrier fixed.
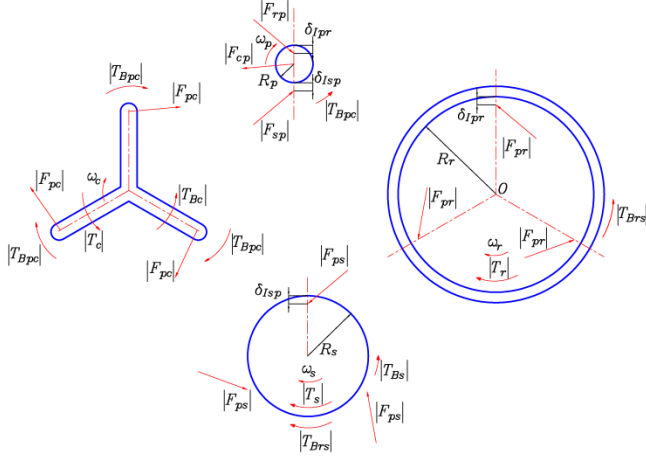

Figure 5 - Equilibrium of the EGT elements considering Case A. The modulus is introduced in order to pick out the real direction of forces, speeds and torques.

It is possible to verify the validity of (3), (4) and (11), by substituting the correct signs of forces, torques and angular velocities and comparing them with the awaited results reported in Figure 5.

It was assumed that ω_s and ω_r have positive sense of rotation (clockwise), than by kinematics relation (8), it results that ω_p and ω_c have positive sense too and $\omega_p > \omega_c$. The tangential force exerted by the planet on the ring can be obtained starting from the dynamic equation of ring, i.e. first row of matrix equation (11), neglecting the inertial contribution and putting in evidence the absolute values of the single terms:

$$F_{tpr} = -F_{trp} = \frac{-|T_r| + |T_{Brs}|}{n_p \, (R_r + \delta_{pr})} \tag{16}$$

It must be observed that the absolute values are introduced, both in equations and in figures, with the aim of facilitating the sign discussion of the results. Since the ring is a driver, the torque T_r has the same direction of its angular velocity and therefore its value is also positive, i.e. $T_r = |T_r|$. Neglecting the dissipative terms, different from the mesh loss, i.e. $T_{Brs} = 0$, consequently F_{tpr} is negative. The graphical equilibrium reported in Figure 5 confirms the analytical solution obtained using the model.

The sun is a driver too, therefore once again torque T_s has to have the same direction of its angular velocity and therefore its value is positive. Starting from the dynamic equation of the sun, i.e. second row of matrix equation (11), the tangential force exerted by each planet on the sun is:

$$F_{tps} = \frac{-|T_s| + |T_{Bs}| - |T_{Brs}|}{n_p \, (R_s + \delta_{sp})} \tag{17}$$

Neglecting the effect of bearing torques, this force is negative too, in agreement with Figure 5.

The meshing forces acting on the planet are the opposite of those calculated before using eq. (16) and (17), that is they both are positive. Thus, they are coherent with the representation in Figure 5.

The third row of matrix equation (11) together with eq.(10) considering also that T_c is negative since it is the follower, i.e. $T_c = -|T_c|$, allows to calculate the tangential component of force that carrier applies to each planet:

$$F_{tcp} = \frac{-|T_c| - |T_{Bc}| + n_p |T_{Bpc}|}{n_p R_c} \tag{18}$$

In accordance with Figure 5 it shows a negative value.

As mentioned before, in order to calculate the offset distances, it's necessary to make the epicyclic gear train into ordinary gear train through a kinematic inversion, i.e. the angular speeds relative to the carrier must be considered. Referring to Figure 4, the relative velocity of the sun is negative, i.e. $(\omega_s - \omega_c) < 0$, and the meshing force exerted by the planet on the sun F_{tps} is negative too. Eq. (5) with $i=s$ and $j=p$ gives:

$$f_\pm\big(F_{tps}\, \omega_{s,rel}\big) = \frac{1 \pm \mathrm{sgn}(F_{tps}\, \omega_{s,rel})}{2} = \begin{cases} \frac{1+1}{2} = 1 \\ \frac{1-1}{2} = 0 \end{cases} \tag{19}$$

Being also

$$F_{tps} \neq 0 \ \& \ \omega_{p,rel} \neq \omega_{s,rel}$$

it follows that

$$\mathrm{sgn}|F_{tps}(\omega_{p,rel} - \omega_{s,rel})| = 1 \tag{20}$$

Thus, substituting these values (19) and (20) into eq. (3) one obtains $\delta_{sp} = -\delta_{Isp}$.

Repeating the same analysis to the contact between planet and ring, it's possible to observe that the relative velocity of planet is positive and the meshing force F_{trp} is positive too. Thus, substituting the values into eq. (4), the offset displacement will be equal to $\delta_{pr} = -\delta_{Ipr}$, as it is represented in Figure 5.

6.2 MESH EFFICIENCY NUMERICAL VALIDATION USING ANALYTICAL METHODS

The EGT dynamic model was implemented in a commercial modeling and simulation environment and the mesh efficiency values, obtained in steady-state conditions, were compared with the results of analytical formulations [7]. Obviously for this purpose the bearing/seals resistant torques were set to zero.

The EGT dynamic model explained in the paper was coupled with one speed controller for each driving gear and one load for each driven gear of the EGT.

Feed-forward terms are calculated and applied to the EGT elements, both to driving and to driven gears. They consist in the torques required to satisfy the equilibrium of the EGT in case of steady-state conditions and unitary efficiency, i.e.

$$\begin{cases} T_{c,ff} = \left(1 + \frac{R_r}{R_s}\right) T_{s,ff} \\ T_{r,ff} = \frac{R_r}{R_s} \, T_{s,ff} \end{cases}$$

In order to compensate for the differences in the equilibrium torques, with respect to the ideal case, due to the internal losses of the gear train, a speed controller for each driving gear and a quadratic load for driven gears are added.

The load torque increases with the square of the difference between the instantaneous speed of rotation and the initial speed:

$$T_{i,load} = T_{i,ff} - k_i (\omega_i - \omega_{i0})^2 \, \text{sgn}(\omega_i - \omega_{i0})$$

where k_i is a proportionality constant.

Speed controller is composed by the sum of the feed-forward term and a PID feedback controller with the aim of keeping the controlled speed (ω_i) equal to its initial value (ω_{i0}):

$$T_{i,control} = T_{i,ff} + k_p (\omega_{i0} - \omega_i) + k_i \int (\omega_{i0} - \omega_i) dt + k_d \frac{d(\omega_{i0} - \omega_i)}{dt}$$

In this way, by specifying the initial conditions according to a specific power configuration to be analysed, thanks to load and speed controller, it is possible to reach the steady-state equilibrium of the system and compare the results of analytical methods with simulation. The efficiency of the EGT, defined as the ratio of the power that comes out of the system by the power that comes in, is computed through the following generic expression:

$$\eta = \frac{|P_{out}|}{|P_{in}|} =$$
$$= \frac{|T_r \, \omega_r| \, f_-(T_r \, \omega_r) + |T_s \, \omega_s| \, f_-(T_s \, \omega_s) + |T_c \, \omega_c| \, f_-(T_c \, \omega_c)}{|T_r \, \omega_r| \, f_+(T_r \, \omega_r) + |T_s \, \omega_s| \, f_+(T_s \, \omega_s) + |T_c \, \omega_c| \, f_+(T_c \, \omega_c)}$$

Once again the usage of function $f_\pm$ allows to define a unique efficiency equation for all the possible EGT working conditions. If the sign of the i-th power is positive, it means that represents power entering into the system and so must be placed at the denominator of the efficiency equation, otherwise if the product between speed and torque is negative it is an output power and so should be at the numerator.

All the possible working conditions in terms of power flows direction along the EGT were tested and the obtained simulation results are exactly the same of those obtainable using one of the method cited in [7].

7 CONCLUSIONS

An epicyclic gear train dynamic model including mesh efficiency, bearings/seals losses and inertial effects were presented in the paper. The validity of the proposed model for a specific steady state working condition was demonstrated by analyzing the internal forces of the mechanism. In addiction the mesh efficiency value of the epicyclic gear train resulting from numerical simulations of the model were compared with analytical formulas; all the possible working conditions in terms of power flows direction along the EGT were tested and the obtained

results using the two methods are exactly the same. The same approach used to insert the dissipative terms into the analysed gearing layout can be extended for the study of more complex epicyclic and ordinary gear train configurations.

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