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New Families of Valid Inequalities for the Two-Echelon Vehicle Routing Problem

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Abstract

Multi-echelon distribution systems are quite common in supply-chain and logistic management. They are used by public administrations in their transportation and traffic planning strategies as well as by companies to model their distribution systems. In the literature, most studies address issues related to the movement of flows throughout the system from the origins to their final destinations.

In this paper we consider the Two-Echelon Vehicle Routing Problem (2E-CVRP), the two-echelon variant of the well known Capacitated Vehicle Routing Problem,

where the delivery from one depot to the customers is managed by routing and consolidating freight through intermediate depots, called satellites. Valid inequalities based on the TSP and CVRP, the network flow formulation, and the connectivity of the transportation system graph are presented.

Extensive computational results on instances with up to 50 customers show an improvement of the best known results between 4% and 15%.

Keywords: Multi-Echelon VRP, Valid Inequalities, Lower Bounds.

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1 Introduction

In Multi-Echelon Vehicle Routing Problems the delivery from one or more depots to the customers is managed by routing and consolidating the freight through intermediate depots, called satellites. This family of problems differs from Multi-Echelon Distribution Systems present in the literature, where the attention is focused on the flow assignment among the levels only. In our case we also consider the fleet management and the overall distribution system routing. The routing in Multi-Echelon Distribution Systems is challenging, both in theory and in practice (e.g., City Logistics problems), where keeping big trucks far from the city, while using efficiently small and environmental friendly vehicles in the historical city centers is one of the main goals [9].

In this work we consider the Two-Echelon Vehicle Routing Problem (2E-VRP), the variant of Multi-Echelon Vehicle Routing with one depot and a fixed number of satellites. First level routing manages depot-to-satellites delivery, while the second level deals with the satellites-to-customers delivery. The fleet size is fixed and the trucks are homogeneous inside each level. Vehicles and satellites are capacitated, while neither synchronization between the vehicles in each satellite, nor time-windows of the customers are considered. The literature on 2E-VRP, due to the recent introduction of the problem itself, is limited. A model for the 2E-VRP, able to solve to optimality instances with up to 32 customers, has been presented in [6] and [9]. In [9], the authors also derived two math-heuristics able to solve instances up to 50 customers.

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Concerning larger size instances, a fast cluster-based heuristic method has been proposed by Crainic et al. [2] able to heuristically solve instances up to 150 customers. For an application of the 2E-VRP on freight distribution system we refer the reader to [3], where advanced freight distribution systems are provided, and [4].

The main goal of this work consists in detecting and defining new classes of valid inequalities for strengthening the continuous linear formulation of the 2E-VRP. Starting from the flow-based model presented in [9], we first extend the model giving a stronger formulation for it. Moreover, several valid inequalities are presented, derived from the TSP and CVRP literature, the network flow formulation, and the connectivity of the network.

2 Problem definition and existing cuts

Defined the central depot set $V_0 = \{v_0\}$, a set V_s of intermediate depots called satellites and a customer set V_c, wherein each customer $i \in V_c$ has a positive demand d_i associated, the problem consists in minimizing the total transportation costs, calculated by considering arc costs c_{ij} for shipping goods from one point to the other of the transportation network, while satisfying the demand of all the customers with a limited fleet of vehicles. Differing from classical VRP problems, the freight stored in V_0 must transit through intermediate depots, called satellites, and then be delivered to the customers. The demand of each customer has to be satisfied by only one satellite, and there are no thresholds on minimum and maximum number of customers served by a single satellite. This assumption induces, for each 2E-VRP feasible solution, a partition of V_c set in, at most, $|V_s|$ subsets, each one referring to a different satellite. Customer-satellite assignments are not known in advance, not allowing to solve the problem by decomposition into $|V_s| + 1$ VRPs. Two distinct fleet of vehicles m_1 and m_2, with different capacity size K^1 and K^2, are available to serve first and second network level, respectively.

In order to guarantee the feasibility of the 2E-VRP solutions, a *two commodity model* formulation has been introduced. Any 2E-VRP problem is expressed by the following variables:

- first level arc activation variables x_{ij}, $i, j \in V_0 \cup V_s$ and second level ones y^k_{lm}, $l, m \in V_c \cup V_s$, $k \in V_s$, for routing information on directed graph;
- first level flow variables Q^1_{ij}, $i, j \in V_0 \cup V_s$ and second level ones Q^2_{lmk}, $l, m \in V_c \cup V_s$, $k \in V_s$, each one related to a direct arc, describing freight quantities running on the two networks;

- customer-satellite assignment variables z_{kj}.

The mathematical formulation is mainly composed by a set of equations, related to the unique assignment of each customer to one satellite, and conservative flow equations, avoiding the presence of subtours in the second network level. Additional arc capacity constraints are introduced, due to the presence of size constraints on the vehicles (see [9] for details).

In [9] two families of valid inequalities are introduced in order to strengthen the 2E-VRP formulation: the *edge cuts*, subtour elimination constraints derived from Traveling Salesman Problem (TSP)

$$\sum_{i,j \in S_c} y_{ij}^k \leq |S_c| - 1, \quad \forall S_c \subset V_c, \quad 2 \leq |S_c| \leq |V_c| - 1, \quad k \in V_s \tag{1}$$

and *flow cuts*

$$Q_{ijk}^2 \leq \left(K^2 - d_i\right) y_{ij}^k \quad \forall i, j \in V_c, \forall k \in V_s, \tag{2}$$

strengthening the logical constraints linking arc flows to arc usage variables in the 2E-VRP formulation.

3 New families of cuts for the 2E-VRP

Several classes of valid inequalities can be introduced in the 2E-VRP formulation by extending the CVRP literature (for a survey on last advances and trends, see [1]). Under the assumption 2E-VRP feasible solutions, restricted to the second level network, can be seen as solutions of $|V_s|$ VRPs, any valid inequality class for the VRP can be reformulated for the 2E-VRP. Considering the subtour elimination feature concerning edge cuts for the 2E-VRP, the following inequalities restricted to a candidate subset of customer $S \in V_c$

$$\sum_{k \in V_s} \sum_{\substack{i,j \in S \\ i \neq j}} y_{ij}^k \quad \leq |S| - r\left(S\right) \quad \forall S \subset V_c \quad 2 \leq |S| \leq |V_c| - 1 \tag{3}$$

are valid for the 2E-VRP, where $r\left(S\right)$ is the minimum number of 2nd level vehicles required to serve customers in S. The separation is performed by a heuristic algorithm based on the corresponding heuristic for the CVRP problem presented in [8].

In the same way, strengthened comb inequalities and multistar inequalities for the VRP ([7], [8]) can be introduced as cutting planes classes also in 2E-VRP formulation. The heuristic separation procedure is a specialization of the algorithm from [8] (see [10] for further details).

The existence of the network flows in the mathematical formulation lets us define new classes of valid inequalities, based on the interaction between routing and arc activation variables related to particular sets of arcs. Upper bound variable constraints on arc capacity

$$d_j y_{ij}^k \leq Q_{ijk}^2 \quad \forall i \in V_c \cup V_s, \forall j \in V_c, \forall k \in V_s \tag{4}$$

and special cases of node feasibility inequalities

$$Q_{ijk}^2 - \sum_{\substack{l \in V_c \cup V_s \\ l \neq i}} Q_{jlk}^2 \leq d_j y_{ij}^k \quad \forall i \in V_c \cup V_s, j \in V_c, \forall k \in V_s \tag{5}$$

are cuts for the 2E-VRP and link routing information of LP fractional solution to its arc activation variables.

Other classes of valid inequalities are derived from considering connectivity and feasibility properties of any feasible solution of routing problems, through z_{kj} customer-satellite assignment variables. The following inequality

$$z_{ki} \geq y_{ij}^k + y_{ji}^k \quad \forall i, j \in V_c, i \neq j, \quad \forall k \in V_s, \tag{6}$$

stated as constraint on the activation of two arcs incident to the same couple of customer nodes, can be seen as a special case of the simple connectivity condition on subroutes not containing the satellites.

Both (5) and (6) can be trivially separated by direct inspection in polynomial time.

Concerning route feasibility assumptions, let us define the $i-th$ partition of the customer set V_c defined as follows

$$P_i = \left\{ S_j \subset V_c, j = 1, ..., m_2 : \bigcup_{j=1}^{m_2} S_j = V_c \wedge S_j \cap S_k = \emptyset, k \neq j \wedge S_j \neq \emptyset \right\}.$$

Then, considered the set $\mathcal{P}$ containing all the possible partitions P_i. An element in $\mathcal{P}$ may not correspond to a feasible solution. A simple rule to exclude a partition P_i from the set of possible solutions is considering if the demand associated to one of its subsets is greater than the capacity size of second level vehicle, i.e., $d(S_j) > K^2$. Another rule is based on considering partitions for which a given subset with a restricted number of customers does exist. If it exists a customer set $S \in V_c$ such that complementary demand is equal to zero, i.e.,

$$m_2 - \left\lceil \frac{d(S)}{K^2} \right\rceil = 0 \tag{7}$$

then the following inequalities

$$\sum_{j \in S} \left(y^k_{jk} + y^k_{kj}\right) + y^k\left(E(S)\right) \leq \sum_{j \in S} z_{kj} \quad \forall k \in V_s \tag{8}$$

are valid for the 2E-VRP. The separation procedure is done by a heuristic algorithm building S by iteratively considering the customers in non-increasing order of their demand. For further details, please refer to [10].

4 Computational results

This section is devoted to present the computational results of the valid inequalities presented in the paper. In order to give a better insight of the 2E-CVRP and the valid inequalities themselves, they have been inserted in a Branch & Cut framework. The Branch & Cut has been implemented in SYMPHONY [11] interfaced with XPress 2008 [5]. As in [9], tests have been performed using a personal computer Intel 3.0 GHz with 1 GB RAM where a time limit of 10000 seconds has been imposed on each instance.

In order to compare the results with the literature we use the same instances of [9] and [2]. The instances cover up to 50 customers and 5 satellites and are grouped in two sets, Set 2 from [9] and Set 4 from [2]. All the cuts presented in Section 2 and 3 are applied at the root node in same order we presented them in this paper. For the other nodes, due to the computational effort in the largest instances, we only apply the Lifted cover cuts and Lift-and-Project separation implemented in the XPress 2008 package. The branching is prioritized on z_{jk} variables and a pseudocost-based rule for choosing the variable on which to branch is enabled on this variables. For a detailed discussion about the effect of the different families of cuts, please refer to [10].

The results of Set 2 and 4 are summarized in Tables 1 and 2, respectively. Columns 1 and 2 report the instance details, i.e. instance name and number of satellites in the instance. The SOA columns reports the best results taken from the literature, while columns BC show the behavior of our Branch & Cut. For each method, we give the best solution, the lower bound at the end of the optimization and the gap between them. Optimal values are reported in bold. The computational time is not reported, being fixed a priori to 10000 seconds for both methods.

From the results, we can notice how the Branch & Cut overcomes the cuts and the model introduced in [9] both in accuracy and number of optimal solutions. Our Branch & Cut is able to solve to optimality all the instances in

		SOA			BC		
Instance	Satellites	Final Solution	Best Bound	Gap	Final Solution	Best Bound	Gap
E-n22-k4-s6-17	2	**417.07**	**417.07**	0.00%	**417.07**	**417.07**	0.00%
E-n22-k4-s8-14	2	**384.96**	**384.96**	0.00%	**384.96**	**384.96**	0.00%
E-n22-k4-s9-19	2	**470.60**	**470.60**	0.00%	**470.60**	**470.60**	0.00%
E-n22-k4-s10-14	2	**371.50**	**371.50**	0.00%	**371.50**	**371.50**	0.00%
E-n22-k4-s11-12	2	**427.22**	**427.22**	0.00%	**427.22**	**427.22**	0.00%
E-n22-k4-s12-16	2	**392.78**	**392.78**	0.00%	**392.78**	**392.78**	0.00%
E-n33-k4-s1-9	2	730.16	725.50	0.64%	**730.16**	**730.16**	0.00%
E-n33-k4-s2-13	2	714.63	701.04	1.94%	**714.63**	**714.63**	0.00%
E-n33-k4-s3-17	2	707.62	683.42	3.54%	**707.41**	**707.41**	0.00%
E-n33-k4-s4-5	2	787.29	764.80	2.94%	**778.73**	**778.73**	0.00%
E-n33-k4-s7-25	2	766.49	739.24	3.69%	**756.84**	**756.84**	0.00%
E-n33-k4-s14-22	2	779.19	764.38	1.94%	**779.05**	**779.05**	0.00%
E-n51-k5-s2-17	2	599.20	576.97	3.85%	597.51	556.55	7.36%
E-n51-k5-s4-46	2	561.80	513.09	9.49%	530.76	529.34	0.27%
E-n51-k5-s6-12	2	593.71	526.91	12.68%	554.80	541.17	2.52%
E-n51-k5-s11-19	2	646.66	550.99	17.36%	584.09	558.27	4.63%
E-n51-k5-s27-47	2	538.22	524.00	2.71%	538.22	535.04	0.59%
E-n51-k5-s32-37	2	553.64	540.14	2.50%	**552.27**	**552.27**	0.00%
E-n51-k5-s2-4-17-46	4	694.83	502.82	38.19%	542.37	515.75	5.16%
E-n51-k5-s6-12-32-37	4	571.80	509.35	12.26%	539.02	516.02	4.46%
E-n51-k5-s11-19-27-47	4	724.09	506.99	42.82%	589.14	511.09	15.27%
Mean				8.70%			1.92%

Table 1
Comparison with State-of-the-art on Set 2

		SOA			BC		
Instance	Satellites	Final Solution	Best Bound	Gap	Final Solution	Best Bound	Gap
Instance50-s5-37.dat	5	1548.07	1355.8	14.18%	1548.07	1434.54	7.91%
Instance50-s5-39.dat	5	1525.24	1365.78	11.68%	1525.24	1423.48	7.15%
Instance50-s5-41.dat	5	1775.06	1453.03	22.16%	1775.06	1580.8	12.29%
Instance50-s5-43.dat	5	1455.4	1259.55	15.55%	1455.4	1341.04	8.53%
Instance50-s5-45.dat	5	1497.91	1246.26	20.19%	1497.91	1331.59	12.49%
Instance50-s5-47.dat	5	1621.48	1405.34	15.38%	1621.48	1487.09	9.04%
Instance50-s5-49.dat	5	1499.52	1297.08	15.61%	1499.52	1370.83	9.39%
Instance50-s5-51.dat	5	1436.3	1265.66	13.48%	1436.3	1289.45	11.39%
Instance50-s5-53.dat	5	1571.34	1385.72	13.40%	1571.34	1483.19	5.94%
Mean				16.75%			9.35%

Table 2
Comparison with State-of-the-art on Set 4

Set 2 with 32 customers and one of the instances with 50 customers, for a total of 7 new instances solved to optimality. The trend is confirmed by the mean gap, which is less than 2%. In particular, notice how the gap is reduced in the instances with 4 satellites, which where the most problematic in literature.

The same behavior is confirmed also in Set 4, where the mean gap is reduced by more than 7 percentage points. Finally, notice how the mean of Set 4, which considers instances with 5 satellites, is similar to the mean of instances with 4 satellites in Set 2, which is about 8%, prooving that, even increasing the number of satellites, the mean gap obtained by the Branch & Cut is stable. Moreover, the reduction of the gap from 16.75% to 9.35% in Set 4 is due only to the valid inequalities. In fact, the best integer solutions

obtained in literature and by the Branch & Cut are the same.

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