Surface-wave analysis for building near-surface velocity models —
Established approaches and new perspectives

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ABSTRACT

Today, surface-wave analysis is widely adopted for building near-surface S-wave velocity models. The surface-wave method is under continuous and rapid evolution, also thanks to the lively scientific debate among different disciplines, and interest in the technique has increased significantly during the last decade. A comprehensive review of the literature in the main scientific journals provides historical perspective, methodological issues, applications, and most-promising recent approaches. Higher modes in the inversion and retrieval of lateral variations are dealt with in great detail, and the current scientific debate on these topics is reported. A best-practices guideline is also outlined.

INTRODUCTION

Since Lord Rayleigh first predicted their existence in 1885 (Rayleigh, 1885), surface waves have attracted the interest of a constantly increasing number of researchers from different disciplines, including solid-state physics, microwave engineering, geotechnical engineering, nondestructive testing, seismology, geophysics, material science, and ultrasonic acoustics. Despite marked differences in scales and methods, these disciplines share the goal of exploiting the surface waves that propagate along the boundary of a domain to obtain information on one or more scalar fields inside that domain.

Surface waves are interesting because they can be used to develop noninvasive techniques for characterizing a medium at a small scale (e.g., engineers use ultrasonic surface waves to identify material defects), at a large scale (e.g., seismologists use surface waves to investigate the structure of the earth’s crust and upper mantle), and at an intermediate scale (e.g., geophysicists and geotechnical engineers use surface waves to characterize near-surface geomaterials). All of these applications share the same principles: they use the geometric dispersion of surface waves to infer the properties of the medium by identifying the model parameters.

In near-surface applications, most surface-wave tests estimate the shear-wave velocity profile. This is usually accomplished by adopting a strategy based on estimating the experimental dispersion curve from field data and subsequently solving an inverse problem. This latter step implies the choice of a reference model for the interpretation, which in most cases is a stack of homogeneous linear elastic layers. Surface-wave analysis is usually performed using Rayleigh waves because they are easy to generate and detect on the ground surface; however, Love waves, Scholte waves, and other kinds of guided waves that may be generated in specific stratigraphic conditions can also be analyzed.

Regardless of the type of surface wave used, the standard procedure for surface-wave analysis can be divided into three main steps:

1) acquire the experimental data
2) process the signal to obtain the experimental dispersion curve
3) solve the inverse problem to estimate model parameters

Each step can be performed using different approaches, according to the scale of the problem, the target, the complexity of the subsoil property distribution, and the available equipment and budgets. For applications on an engineering scale, the acquisition is conducted with a multichannel layout of vertical low-frequency geophones and an impact source in an off-end configuration. The processing is performed with automatic picking of the frequency/wavenumber f-k or frequency/slowness w-p spectral maxima, which are then transformed to the dispersion curve. The inverse problem is usually solved with linearized algorithms that use a 1D forward model and yield a 1D S-wave velocity profile.

This basic scheme is used extensively and is suitable for many near-surface applications. Alternatively, the analysis may be performed adopting the full-waveform inversion approach, in which extracting the dispersion curves is unnecessary. Nevertheless, the full-waveform approach requires a realistic simulation of the dynamics of the propagation that accounts for source, attenuation phenomena, and soil-receiver coupling and requires complex computational approaches. For these reasons, the full-waveform approach is seldom applied.
In recent years, the increasing popularity of the method has led to significant methodological improvements, with the aim of supplying the shear-wave velocity $V_S$ distribution in complex structures. In geophysical exploration, interest in the use of surface-wave analysis to build near-surface velocity models is recent but rapidly growing, as can be witnessed from workshops held in major scientific conferences during the last 6–7 years. Several examples of the analysis of ground roll or mud roll present in seismic reflection data to estimate near-surface velocity models and use them for static corrections, geohazard studies, and ground-roll prediction and removal bear witness to the growing attention surrounding this method, even for oil and gas applications.

This paper provides an overview of surface-wave methods and highlights the open problems and current research topics. Special attention is paid to the interaction of different research fields in which surface-wave analysis is used for near-surface characterization.

We begin with a brief and synthetic description of surface-wave properties, paying particular attention to the features relevant for retrieving near-surface velocity models. Then we focus on the historical perspective of surface-wave applications in different fields, referring to seminal works that introduced the use of surface waves into global seismology, seismic exploration, and near-surface characterization. Even though we have neglected the field of global seismology in the analysis of the literature, it is very important to remark that surface waves are used routinely to infer crustal velocity models at a regional or global scale.

In the subsequent section, we move to the present with a look toward the future, and we provide a detailed analysis of the existing literature, focusing on analyzed phenomena (types of surface waves), the target of the analysis (the retrieved model in terms of depth of investigation and geometry), and analysis techniques concerning acquisition, processing, and inversion. To provide a comprehensive overview of the evolution and state of the art of the method, we analyze contributions to all major scientific journals in the fields of geophysical exploration, engineering and environmental geophysics, engineering seismology, and geotechnical engineering. We also give a quick look at the field of nondestructive testing and global seismology. Countless papers in the field of geophysical exploration deal with surface-wave removal from seismic records. We have omitted this theme and have focused on the use of surface waves to build near-surface velocity models.

We then consider two particularly relevant topics: the inclusion of higher modes and lateral variations — the focus of lively scientific debate. We deal with them in depth by concentrating on open problems and possible solutions. The last section is a discussion that aims to define a general guideline for best practices in surface-wave analysis. It starts with the characteristics of the ideal results and back propagates them to acquisition, processing, and inversion.

The paper covers various disciplines in which surface waves are used for different purposes, so we have also added a glossary (Appendix A), partly based on Sheriff (2002), with the most relevant definitions and some domain-specific terms.

**SURFACE-WAVE PROPERTIES**

Many different approaches have been reported in the literature and can be used to model the propagation of surface waves in heterogeneous media. A description of modeling algorithms is not the scope of this paper, but it is useful to outline the main surface-wave propagation features that are relevant for building near-surface velocity models.

Surface waves are seismic waves that propagate parallel to the earth’s surface without spreading energy through the earth’s interior. Their amplitude decreases exponentially with depth, and most of the energy is contained within one wavelength from the surface. The wave propagation is therefore influenced only by the mechanical and geometric properties of that portion of the subsoil. The different harmonics of the propagating surface wave have different wavelengths that propagate at different depths. Consequently, the propagation of surface waves in a vertically heterogeneous medium is characterized by geometric dispersion, i.e., different frequencies propagate with different phase velocities. The geometric dispersion can be retrieved experimentally and inverted to yield the seismic properties of the stratified medium. Surface-wave propagation is particularly sensitive to S-wave seismic properties, so the inversion yields the vertical profile of the S-wave velocities. For seismic sources located on or close to the surface, surface waves are more energetic than body waves. Moreover, they exhibit less loss of energy from geometric spreading than body waves. Thus, surface waves are dominant events in seismic records and are easy to acquire.

Surface-wave propagation in vertically heterogeneous media is actually a multimodal phenomenon: for a given layered system, each frequency component can travel with different velocities. Hence, different phase velocities are possible at each frequency, each of which corresponds to a mode of propagation, and different modes can exist.

![Figure 1](image1.png)

Figure 1. For a synthetic S-wave velocity profile in (a), we show (b) example modal curves and modal displacements for two frequency values: (c) 10 Hz and (d) 20 Hz.
ist simultaneously (Figure 1). The modal curves are only related to the kinematics of wave propagation. They are characteristic of the layered solid and theoretically can be computed considering only the mechanical and geometric properties of the model.

If we consider Rayleigh waves, the most commonly used surface waves in practical applications, the equation of motion for a laterally homogeneous medium can be written assuming a plane strain field, imposing the boundary conditions of the waves in a half-space with a free surface (no stress at the free surface and no stress and strain at infinity) and imposing the continuity of strain and stress at layer interfaces as a linear differential eigenvalue problem (Aki and Richards, 1980).

The vector $\mathbf{f}$, made up of two displacement eigenfunctions and two stress eigenfunctions, and the $4 \times 4$ matrix $\mathbf{A}$, which depends on the vertical distribution of the soil properties, are related by the equation

$$\frac{d\mathbf{f}(z)}{dz} = \mathbf{A}(z)\mathbf{f}(z),$$  

where $z$ is the vertical axis. Equation 1 represents a linear differential eigenvalue problem that has a nontrivial solution for only special values of the wavenumber. The resulting equation is known as the Rayleigh secular equation. It can be written in implicit form as

$$F_R[\lambda(z), G(z), \rho(z), k_j, f] = 0,$$

where $\lambda$ and $G$ are the Lamé parameters, $\rho$ is the mass density, $k_j$ is the wavenumber of the $j$th mode of propagation, and $f$ is the frequency. In vertically heterogeneous media, the wavenumber is a multi-valued function of frequency that represents the modal curves. In general, it is impossible to solve equation 2 analytically; a numerical solver is needed (for an example, see Rix and Lai, 2007).

The eigenvalue problem can be established for a stratified medium with homogeneous linear elastic layers, using a matrix formulation for a single layer and then building the global matrix that governs the problem. Many versions of this general procedure, which are also known as propagator-matrix methods (Gilbert and Backus, 1966), have been formulated. They differ according to the principles on which the single-layer matrix formulation is based and, consequently, according to the assembly process. The oldest and probably most famous method is the transfer-matrix method, originally proposed by Thomson (1950) and modified by Haskell (1953). The stiffness-matrix method proposed by Kausel and Roesset (1981) is essentially a reformulation of the transfer-matrix method, but it offers the advantage of a simplified procedure for assembling the global matrix, according to the classical scheme of structural analysis. The third possibility involves constructing reflection and transmission matrices, which account for the partition of energy as the wave propagates. The wavefield is then given by the constructive interference of waves traveling from one layer to another (Kennett, 1974; Kennett and Kerry, 1979; Kerry, 1981). A comparison of the different approaches is reported by Buchen and Ben-Hador (1996).

The effective energy associated with the different modes during propagation depends on the stratigraphy and on the depth and properties of the source. The fundamental mode is often dominant over a wide frequency band, but in many situations, higher modes play an important role and are actually dominant; they should therefore not be neglected. The different modes have different phase velocities; hence, in the time domain, they separate at a great distance from the source. Otherwise, they superimpose onto one another. The velocity of different modal curves can be quite similar in certain frequency bands; therefore, the separation and identification of modal curves can be very difficult, even in the frequency domain.

### HISTORICAL PERSPECTIVE

Surface-wave propagation can be analyzed at different scales to characterize a wide variety of materials. The goal of this section is to provide an overview of the spread of this survey method, with reference to some selected seminal works that triggered interest in surface-wave analysis in different research fields.

Surface-wave data interpretation requires the availability of digital records and many computationally demanding tasks. It is therefore not surprising that most of the advancements in surface-wave analysis and their widespread application are closely linked to progress that has been made in electronics regarding acquisition equipment and computers. The advent of inexpensive data loggers and personal computers, in particular, has led to the use of surface-wave analysis in near-surface geophysics and engineering characterization. Most of the tools used to analyze seismic records and solve the forward and inverse Rayleigh problems originate from the seismologic research field but have been transferred to engineering characterization. Nevertheless, the spread of surface-wave methods in near-surface applications has produced several developments specifically related to the particular nature of the small scale at which the problem is analyzed.

### Global seismology

Energy released by earthquakes travels to teleseismic distances mainly in the form of long-period surface waves, which, at a great distance from the epicenter, represent by far the largest components of seismic records. Surface waves have been studied in seismology for characterizing the earth’s interior since the 1920s, but their widespread use only started during the 1950s and 1960s thanks to the increased possibilities of numerical analysis (e.g., Press, 1968) and to improvements in instrumentation for recording seismic events connected to earthquakes (Aki and Richards, 1980; Ben-Menahem, 1995). The spread of long-period and broadband networks, which started in the 1970s, has led to large-scale and global studies on the upper mantle structure (Trampert and Woodhouse, 1995; Romanowicz, 2002).

An important problem that had to be solved was related to identifying the different modes of propagation in the recorded signals, which is necessary for a correct interpretation. This led to the development of several sophisticated filtering techniques, based on group velocity features, that attempted to separate the modal components in teleseismic signals (Dziewonski et al., 1969; Levshin et al., 1994).

Approaches based on a time-domain waveform inversion began to be developed at the end of the 1970s with the formulation of a first-order perturbation theory to compute synthetic seismograms for a reference model and the evaluation of derivatives (Woodhouse, 1974). The waveform approach allows model parameters to be evaluated with a single-step procedure directly from seismograms (Nolet et al., 1986), but the necessity of correcting for the crustal structure poses some challenges (Romanowicz, 2002).

Nowadays, the most common approach consists of surface-wave tomography, which is applied at regional and global scales to describe the crust and mantle structure of the earth (Ritzwoller and Levshin, 1998; Kennett and Yoshizawa, 2002; Yoshizawa and Kennett, 2002; Yao et al., 2006; Yao et al., 2008). In this context, the dis-
persion curves can be extracted from ambient noise or from earthquake events through processing techniques for near-surface characterization using microtremor analysis (see below).

The regional and global surface-wave tomography procedure has been reformulated as a three-stage process that works with surface-wave dispersion (Kenneit and Yoshizawa, 2002). The first step in constructing the 3D $V_p$ model is to acquire dispersion curves for several paths crossing the region of interest. Once the phase-dispersion information is assembled for a variety of paths, the next step is to assemble 2D maps of the geographic distribution of the phase velocities for individual periods. The final step of the inversion scheme exploits the localization of phase-velocity information with local inversions for 1D $V_p$ profiles. The full set of multimode phase-dispersion maps is assembled as a function of the frequency, and then some form of cellular inversion is used to extract a 3D model.

Exploration geophysics

The exploration applied to defining the regional structure and evaluating mining and hydrocarbon resources is based mainly on seismic reflection. Although exploration is based on body-wave data, seismograms gathered for deep exploration surveys can contain a large amount of surface waves (called ground roll) that mask the reflections. For this reason, surface waves can constitute a tedious background noise in seismic reflection surveys. Moreover, ground roll is a coherent signature in seismic signals, so it is quite difficult to eliminate its contribution from the shot gathers. Several techniques have been developed specifically to attenuate surface waves in seismic reflection records. Some of the tools used to recognize and remove ground roll can also be used profitably to extract information related to surface-wave dispersion. This is the case of the analysis in the $f-k$ (Nolet and Panza, 1976; Tselentis and Delis, 1998) or the $\omega-p$ (McMechan and Yedlin, 1981) domains.

The geophysical exploration community's interest in exploiting surface waves in seismic gatherings collected for seismic reflection surveys is increasing as the value of the information they can provide is being recognized. The seminal work of Mari (1984) proposes using surface-wave inversion for static computation.

Near-surface applications

Engineering applications of surface-wave analysis were first proposed in the 1950s with the steady-state Rayleigh method (Jones, 1958), but they only became popular and widely used after the introduction of the spectral analysis of surface waves (SASW) method (Nazarian and Stokoe, 1984), which had the merit of making the test faster and theoretically sound by taking advantage of the increasing potential of electronic equipment and personal computers. The term SASW is rather general and could be used for any surface-wave method, but it is almost always identified with the two-station procedure because of the large popularity it achieved.

SASW is based on a two-receiver configuration and basic signal-analysis tools. The dispersion curve is evaluated by estimating, for each frequency component, the time delay between the arrivals at the two receivers of the wave generated by a point source acting on the ground surface. Because the two-receiver approach suffers from some limitations on the frequency band, the experimental dispersion curve is estimated at a site by repeating the acquisition with several receiver configurations and assembling the branches of the experimental dispersion curve to obtain a single curve that is then inverted.

The use of multiple receivers enhances the production rate in the field, makes data processing much faster and less sensitive to operator choices, and supplies more robust and accurate dispersion curves. Applications of multistation tests for surface-wave data for near-surface characterization were introduced in the 1980s (McMechan and Yedlin, 1981; Gabriels et al., 1987), but they were not used extensively until the late 1990s. Today, most near-surface applications incorporate multistation approaches. They are often identified by the phrase multistation analysis of surface waves (MASW), introduced by researchers at the Kansas Geological Survey (Park et al., 1999a; Xia et al., 1999). Multistation approaches commonly adopt a spread of geophones in line with an impulsive or sweep-sine source. Several techniques can be used to process the data, the most widespread being transform-based approaches (McMechan and Yedlin, 1981; Gabriels et al., 1987; Park et al., 1999a). The data collected in the time-space domain are transformed to another domain (e.g., the $f-k$ domain), where the phase-velocity values associated with different frequencies are evaluated by picking the spectral maxima.

The above applications for near-surface characterization are based on active tests in which the waves are generated on purpose by a seismic source acting on the ground surface. The analysis of microtremors generated by natural events or human activities and recorded with geophone networks gathers low frequencies, which are necessary to characterize the medium at greater depths, without the need for heavy active sources. Several processing techniques have been developed to extract dispersion curves from passive data. The most commonly used are spatial autocorrelation (SPAC; Aki, 1957) and frequency-domain beam forming (FDDB; Lacoss et al., 1969) or similar higher-resolution techniques such as minimum-variance distortionless look (MVDL; Capon, 1969) and multiple signal classification (MuSiC; Schmidt, 1986). An overview of beam-forming techniques is provided by Johnson and Dudgeon (1993) and Zywicki (1999), and an overview of the SPAC method is presented in Okada (2003).

STATE OF THE ART WITH A GLANCE TOWARD THE FUTURE

In this section, we present the state of the art concerning surface-wave analysis, and we highlight the general research and application trends, considering methodological and practical issues. In this overview, we mainly refer to papers published in international peer-reviewed journals, even though growing interest in the use of surface-wave analysis for different applications has stimulated significant attention recently in international conferences, as testified by several special sessions and workshops. Blind tests organized by engineers and applied seismology should also be mentioned (Boore, 2006; Cornou et al., 2007). Different researchers have been called on to execute and/or interpret active and passive surface-wave data at these events. The results reported by Cornou et al. (2007) with respect to synthetic and experimental passive-source data sets show the relatively high scatter that can be caused by an improper assessment of the propagation modes. The issue of wavelength resolution and array aperture is also detailed in their paper.

A total of 235 papers published in 26 different journals are considered here. The papers were selected through a focused search of the main journals’ Web sites and a further wide search based on Google Scholar. The search was based on a broad series of keywords. Journals in which more than five papers are present are classified accord-
Surface-wave types

The first relevant aspect concerns the type of surface waves considered for the analysis (Figure 3). Rayleigh waves are analyzed the most frequently for site characterization because they are easily generated and detected with cheap and readily available equipment traditionally used for gathering seismic refraction data. Some applications related to Love waves are also reported in the literature (e.g., Mari, 1984; Winsborrow et al., 2003; Guzina and Madyarov, 2005), even though (a) their acquisition requires horizontally polarized sources and receivers and (b) the extraction of Love-wave dispersion curves from passive data requires sophisticated processing techniques based on combining the two horizontal components. The literature also reports some interesting marine applications based on the analysis of Scholte waves. Some of these applications are related to small-scale specific experiments (Wright et al., 1994; Luke and Stokoe, 1998), whereas others are related to the reinterpretation of larger data sets collected for oil and gas exploration (e.g., Bohlen et al., 2004; Kugler et al., 2005; Park et al., 2005a; Muyzert, 2007a). Even though they are relatively few, surface-wave analyses provide information about marine-sediment properties that are very difficult to obtain through alternative methods, and this kind of analysis appears to be very promising for ocean-bottom-cable (OBC) data sets. An application to time-lapse monitoring of a compacting field using Scholte waves is presented by Wills et al. (2008). In Figure 4, we show an example of sea-bottom sediment characterization obtained through the inversion of mud roll present in marine data by Bohlen et al. (2004).

Finally, some papers exist that deal with guided P-wave analysis

Table 1. Analyzed journals, classified according to main field of reference.

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<tr>
<th>Discipline</th>
<th>Title</th>
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<td>Geophysical Journal International</td>
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<td>Journal of Applied Geophysics</td>
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<td>Journal of Environmental &amp; Engineering Geophysics</td>
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<td>Near Surface Geophysics</td>
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<td>The Leading Edge</td>
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<td>Other</td>
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<td>Bulletin of the Seismological Society of America</td>
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Figure 2. Trend of the scientific literature about surface waves for building near-surface velocity models: (a) last 50 years; (b) detail of last 10 years.
(e.g., Shtivelman, 2004; Klein et al., 2005; Strobbia et al., 2005). Although not fully exploited at present, the analysis of such dispersive waves provides a very promising tool for evaluating P-wave static corrections. A few examples that deal with other types of guided waves are reported in literature, especially concerning Lamb waves used for assessing road pavements (Ryden and Park, 2006) and guided waves generated along boreholes (Stoneley waves) for S-wave velocity logging (Stevens and Day, 1986) or for monitoring permeability changes (acoustic waves) along deepwater completions (Bakulin et al., 2009). Guided waves in crosswell seismic data have also been used to assess the continuity of gas formations (Parra et al., 2002).

Retrieved models

The final model retrieved from the inversion of the dispersion curve can be classified according to investigation depth and geometry.

Regarding the maximum investigated depth (Figure 5), most applications focus on the shallower 30 m. This is partly because of the limitations that arise at low frequencies, which are often experienced in active data. It is also from regulatory requirements because seismic building codes associate site amplification with an averaged S-wave velocity of the first 30 m (the so-called $V_{S,30}$ (Moss, 2008)). Most of the applications related to deeper investigations are based on passive methods (Okada, 2003), in which low-frequency data can be obtained without the need for heavy and expensive seismic sources (Rosenblad and Li, 2009). Some interesting examples of deeper investigations obtained with active data are related to OBC data acquired with broadband sensors (Bohlen et al., 2004) and land data acquired with low-frequency vibros (Bagaini, 2008).

Another important issue related to the investigation targets is the reference geometry of the final $V_s$ models (Figure 6). Although the model used to solve the inverse problem in surface-wave methods is always one dimensional, composed of a stack of linear (visco-)elastic layers, the results are often interpreted as 2D or 3D geometries, which are obtained by merging several adjacent 1D models (Figure 6a). Such interpretations clearly are somewhat forced, but this strategy has proven to be effective and is often used. Figure 6b shows the distribution of the analyzed literature with respect to the dimensions of the reference framework. Considering the journal classifications in Table 1, we can observe that the main efforts to describe complex geometries and to retrieve lateral variations are related to geophysical papers. In engineering seismology, a 1D description of the site is considered sufficient for seismic-response studies. This topic is clearly increasing in interest and offers the possibility of promising developments for the near future.

In this context, one of the following sections is devoted to discussing, in more detail, the relationship between the 1D models usually assumed for the inversion and pseudo-2D-3D reference models obtained by combining several individual velocity profiles (Bohlen et al., 2004; Kugler et al., 2005; Ivanov et al., 2006a; Socco et al., 2009). We show the results obtained from two large marine data sets inverted to retrieve the lateral variability of sea-bottom properties by Park et al. (2005a), merging S-wave properties retrieved by surface-wave analysis and P-wave properties retrieved by first-arrival interpretation (Figure 7).

Different approaches to surface-wave analysis

Other very important issues that may be analyzed are the different approaches used for the surface-wave analysis steps: acquisition, processing, and inversion. Many different strategies can be adopted for these three main steps, according to the scale of the survey and the application targets. The choice of the approach is not always related to optimization criteria but is often determined by the methodologies accepted in different disciplines.

**Figure 3.** Different surface waves used for building near-surface velocity models in scientific literature. Rayleigh waves are by far the most widely used.

**Figure 4.** Example of seabed characterization through Scholte-wave dispersion inversion: (a) inverted 1D shear velocity models; (b) zoom into upper 35 m; (c) misfit between measured and modeled dispersion curves. (Courtesy of Bohlen et al., 2004).
Surface-wave near-surface velocity models

Acquisition

Surface-wave dispersion curves can be extracted from active and passive data (Figure 8). By active data, we mean acquisitions performed on purpose to extract surface-wave information and approaches based on processing the data gathered for other seismic methods, such as refraction or reflection surveys. The advantage of on-purpose data concerns the choice of optimal equipment and testing setup. Data sets acquired for body-wave analysis, even though not specifically designed for surface-wave analysis, often present a large amount of surface waves that can be processed along with body waves with significant synergy (Foti et al., 2003; Yilmaz et al., 2006; Socco et al., 2008). Moreover, the acquisition of deep exploration seismic data sets is often performed with larger budgets and more sophisticated equipment, which leads to very interesting, albeit challenging, data sets. To analyze surface waves in exploration data sets, one should perform a preliminary evaluation of the data to assess the presence and quality of the surface waves in the seismic records because the high-frequency sensors, sensor groups, and low-cut filters could significantly affect the surface-wave signals (Socco et al., 2009).

Passive data sets are often used in projects related to seismologic and earthquake engineering. They are usually gathered with 2D arrays to obtain a complete characterization of the wavefield, which is generated by unspecified natural and artificial sources. A few examples of passive tests with linear arrays have been reported, particularly with the so-called refraction microtremors (ReMi) method (Louie, 2001), which is actually a particular surface-wave test rather than a refraction survey. A homogeneous distribution of seismic noise sources, in space, is assumed when interpreting ReMi data, and the dispersion curve is estimated using the much simpler approach adopted for active surface-wave tests. Although the approach is very simple and fast, it can lead to overestimating the S-wave velocity profile if the background noise travels along a preferential direction that is not in line with the receiver spread. A few authors have suggested different strategies to handle this kind of data acquired in controlled conditions (e.g., Halliday et al., 2008; Park and Miller, 2008).

A significant number of data sets presented in the literature are acquired with the two-station SASW. The main drawback of this approach is related to the need to deploy different receiver layouts, which makes the acquisition more time consuming than multistation methods.

Processing

As far as processing experimental data to extract dispersion curves is concerned, the most common methods for active tests are reported in Figure 9a. It is worth noticing that Park et al.’s (1999a) MASW phase-difference method can be considered a particular implementation of the $f-k$ method proposed by McMechan and Yedlin (1981), which is based on the subsequent application of a slant-slab $r-p$ transform and a Fourier transform in time to the seismic data set. The difference is that, in the case of MASW, a normalization of the traces is applied in the frequency domain prior to the transforms.

The two-station procedure of the SASW test is more common in civil-engineering applications, and it accounts for about one-quarter of the existing literature (Figure 9). This method requires careful data evaluation to handle possible processing ambiguities caused by the phase unwrapping of the cross-power spectrum of the two receivers. Unwrapping can be problematic on some data sets, and there is the risk of introducing errors (Al-Hunaidi, 1992; Rosenblad and Bertel, 2008). For this reason, automation of the processing is not straightforward (Nazarian and Desai, 1993). To overcome the problem, the two-station procedure, based on the cross-power spectrum phase, can be considered as a special case of $f-k$ analysis performed with two receivers and an infinite zero padding (Foti et al., 2002).

The implementation of $f-k$ analysis is straightforward because the fast Fourier transform (FFT) algorithm can be applied. The adoption of different spectral estimators can be useful to obtain the $f-k$ spectrum using fewer receivers, even irregularly spaced (Trad et al., 2003; Zywicki and Rix, 2005). Other possible approaches that analyze unevenly spaced receiver data sets are based on the regression of the experimental transfer function (Rix et al., 2001) or the regres-

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**Figure 5.** Investigation depth of reference for the cases presented in literature. More than 60% of the analyzed papers refer to shallow targets above 30 m depth.

**Figure 6.** Reference geometry for the final $V_s$ models presented in literature. (a) Despite the 1D approach adopted for processing and inversion, almost one-quarter of the literature refers to 2D or 3D sites. (b) The interest toward complex sites is confined in geophysical journals to a few examples in geotechnics, whereas in engineering seismology the method is still considered strictly one dimensional.
sion of the phase difference as a function of the offset (Strobbia and Foti, 2006). The latter also provides information on possible lateral variations that occur below the receiver spread, whereas the former provides the experimental attenuation curve of the surface waves, which can be used to estimate the quality-factor (damping ratio) profile at the site (Lai et al., 2002). Alternatively, the attenuation dispersion curve can be obtained from amplitude-variation-with-offset regression (Malagnini et al., 1995; Rix et al., 2000; Xia et al., 2002). We report an example of S-wave velocity and damping-ratio estimation obtained by Foti (2003) using the procedure based on the regression of the complex-valued Rayleigh-wave transfer function and simultaneous inversion of phase velocity and attenuation curves. The results are compared with crosshole test and laboratory results in Figure 10.

Some authors (Grandjean and Bitri, 2006; Neducza, 2007) have shown that, regardless of the wavefield transform used to extract the dispersion curve, the quality of the experimental curve can be enhanced by stacking in the spectral domain when multifold data are available.

As far as passive data sets are concerned, f-k analysis and SPAC are used more or less with the same frequency in the analyzed literature, even though the latter is certainly more popular in the applied seismology community. SPAC and its generalization, extended autocorrelation (ESAC), can provide more stable results in the low-frequency band and can be used with fewer receivers. However, they suffer from limitations associated with assuming homogeneous distribution of the passive sources in space. Frequency-wavenumber analysis can be used profitably to assess the validity of such an assumption for a given data set; hence, the two techniques should be used in conjunction (Asten and Henstridge, 1984). Other approaches have also been proposed (Cho et al., 2004).

Active and passive data acquired at the same site provide different branches of dispersion curves pertaining to different frequency bands. The merged dispersion curves supply broadband data that correspond to increased information at the site (Park et al. 2005b; Foti et al., 2009).

Inversion

As far as the inversion of dispersion curves is concerned, we have classified the papers on the basis of two criteria: the way in which fundamental and higher modes are considered (Figure 11) and the approach adopted to solve the inverse problem (Figure 12).

The inversion is usually performed using a layered linear (visco-)elastic model. Rix and Lai (2007) discuss the consequence of this assumption, which is unrealistic for, say, homogeneous deposits of cohesionless soils in which a continuous variation of model parameters with depth is caused by the increase in confining pressures. They propose an inversion algorithm based on a soil model in which the model parameters are a continuous function of depth.

Another important simplification in most inversion approaches is a reduction in the number of unknowns, which is obtained by assuming some model parameters a priori. Soil density and Poisson’s ratio are usually fixed to presumptive values a priori on the basis of sensitivity analyses on numerical models that show their limited influence on the dispersion of surface waves (Nazarian, 1984). Nevertheless, it should be considered that full saturation can significantly affect the expected values of these parameters; hence, the presence and position of the water table should always be considered (Foti and Strobbia, 2002).

The debate on the importance of higher modes of propagation has always been very lively. It is now recognized that higher modes play a significant role in interpretation; failing to appropriately consider them in the inversion process can cause errors in several situations. However, fundamental-mode approaches are much easier to implement and can save computing costs. Moreover, in simple stratigraphic conditions, i.e., when the wave velocity gradually increases with depth, fundamental-mode inversion can lead to reliable results.
The outcome of this trade-off is given in Figure 11a, which shows that fundamental-mode inversion is the most commonly adopted method.

The relevance of higher modes in surface-wave testing for near-surface characterization has been studied widely with respect to the two-station SASW procedure, where a single dispersion curve is obtained. In this respect, numerical analyses (Gucunski and Woods, 1992) show that, in the case of regular stratigraphies, most of the energy is carried by the fundamental mode and the experimental dispersion curve is associated with it. However, in more complex stratigraphic conditions, an apparent dispersion curve is obtained (Tokimatsu et al., 1992). Approaches based on the preliminary application of multiple filters have been proposed to extract modal curves from two-station measurements (Al-Hunaidi, 1994; Karray and Lefebvre, 2009). Through multistation methods, in principle it is possible to extract multiple modes from the experimental data, as shown, among others, by Gabriels et al. (1987); however, the limitations concerning spatial resolution caused by the spread length again lead to an apparent dispersion curve, which is a combination of the modal curves (Foti et al., 2000; Zhang et al., 2003).

Considering the approaches used over the last 10 years (Figure 11b), it is clear that there is no trend to abandon fundamental-mode inversion. On the contrary, papers in which only the fundamental mode is considered have increased slightly over the years. This is also because it is often difficult to retrieve higher modes experimentally; they can be much less energetic than the fundamental mode. This topic is of great relevance with respect to current trends.

The other important issue related to inversion is the choice of algorithm. The most important distinction is between local search methods (LSMs) and global search methods (GSMs). The former minimize the misfit between the experimental and the synthetic dispersion curve, starting from an initial velocity model and searching in its vicinity; the latter perform a systematic exploration of the solution space.

An analysis of the literature shows that LSMs are used far more often than GSMs (Figure 12a), but the trend over the last 10 years clearly shows an increase in the use of the latter (Figure 12b), likely because of increasing computing power at lower costs. LSMs are undoubtedly faster because they require a limited number of forward-solver runs; however, because the solution is sought in the vicinity of a tentative profile, there is the risk of being trapped in local minima. GSMs are attractive because they avoid all assumptions of linearity between the observables and the unknowns, and they offer a way of handling the nonuniqueness problem and its consequences (Cercato, 2009; Foti et al., 2009). On the other hand, they require greater computing effort because many simulations must be performed to sample the model parameter space adequately.

Several optimization methods have been applied over the years to make GSMs affordable. These methods use random generation of the model parameters, but they can guide their search using a transition probability rule, i.e., simulated annealing.
annealing (SA), on the basis of the Metropolis algorithm (Metropoli et al., 1953), or they can apply genetic algorithms (GAs) or an importance sampling method (Sen and Stoffa, 1996). These approaches reduce the number of required simulations; the sampling is concentrated on the high-probability-density regions of the model parameter space.

Several examples of GSM applications can be found for surface-wave inversion. GAs have been applied at different scales by several authors. Yamanaka and Hishida (1996) use earthquake data to characterize sedimentary basins. Hunaidi (1998) uses GA to invert data for the nondestructive testing of pavements. Feng et al. (2005) introduce higher modes in surface-wave inversion performed through GA to retrieve velocity inversions. Nagai et al. (2005) use GA in a laterally varying medium to improve the final result. Pezeshk and Zarrabi (2005) and Dal Moro et al. (2007) apply them to synthetic and field data, obtaining very good matches with borehole logging at an engineering scale. Picozzi and Albarello (2007) use them in combination with linearized inversion and also include horizontal/vertical (H/V) spectral ratio curves. SA has been applied by Beaty et al. (2002), among others, for geotechnical characterization to a depth of 10 m, using fundamental and higher modes. Ryden and Park (2006) use it to invert surface-wave spectra to assess road pavements. Calderón-Macías and Luke (2007) use SA to invert surface-wave data with an improved parameterization method. The neighborhood algorithm (Sambridge, 1999a), which can be considered an importance sampling method, is adopted by Wathelet et al. (2004) to invert dispersion curves retrieved from noise measurements on a seismologic scale. Socco and Boiero (2008) propose an improved Monte Carlo approach that uses nondimensionalization of the forward problem of Rayleigh-wave propagation to optimize sampling of the model space and a statistical test to draw inferences on the final results. Figure 13 shows the effect of applying scale properties of modal curves on the random sampling of the model-parameter space demonstrated by Socco and Boiero (2008).

A technique that falls somewhere between the local and global search methods has been proposed by Degrande et al. (2008). In their algorithm, a local search method that includes a jump mechanism is used to mitigate the risk of ending up in a local minimum.

One of the drawbacks of GMSs is that the final result is not a single $V_S$ profile but is a set of acceptable $V_S$ models. This result, even though more rigorous and consistent with the nonuniqueness of the solution, is not easy to handle. The most desirable result for many engineering applications, which are inherently deterministic,
would be a unique $V_s$ model. For this reason, although many methods have been implemented to draw inference and to supply consistent results for engineering applications (Vasco et al., 1993; Lomax and Snieder, 1995; Sambridge, 1999b, 2001; Wathelet et al., 2004; Socco and Boiero, 2008), engineers often prefer LSMs. In some cases, GSMs are used as a preliminary inversion to guide the operator toward a proper model parameterization and then to conduct LSM refinement (Picozzi and Albarello, 2007; Socco et al., 2009).

Additional constraints and a priori information from borehole logs or other geophysical tests are useful elements in LSMs to mitigate the problem of solution nonuniqueness. Integrating surface-wave data with P-wave refraction data is a straightforward consequence of the similarity between the acquisition schemes of these surveying techniques and can lead to interesting synergies (Foti et al., 2003; Ivanov et al., 2006b). Joint or constrained inversion algorithms can also significantly improve the reliability and consistency of the final result. For example, joint inversion with vertical electrical sounding benefits from the similarity of the electrical and seismic models used in the interpretation, which, in both cases, is a 1D stack of homogeneous layers (Hering et al., 1995; Misiek et al., 1997; Comina et al., 2002; Wisen and Christiansen, 2005). Another interesting joint inversion scheme that combines surface-wave data and a microgravity survey is proposed by Hayashi et al. (2005) (Figure 14). Finally, integrating information from the horizontal/vertical (H/V) and vertical/total (V/T) spectral ratios can provide useful information that can help constrain the bedrock position in joint inversion with surface-wave dispersion curves (Arai and Tokimatsu, 2005; Muyzert, 2007b). Again, the two experimental curves to be inverted can be obtained from the same passive source data set with a consequent optimization of the acquisition.

The spatial integration of different surface-wave soundings with the additional contribution of downhole test results has been applied successfully to synthetic and real data (Socco et al., 2009). Although the improvement obtained by the joint or constrained inversion of different data has been proven, the use of these approaches remains limited. When other data are available at the same site, they are usually used for an a posteriori comparison of the results (Socco et al., 2010) rather than introducing them as a priori information in the inversion process. In geotechnical engineering and engineering seismology applications, it is common practice to compare the surface-wave analysis results with S-wave downhole or crosshole test results at the same site (e.g., Malovichko et al., 2005), and the agreement between the two results is taken as proof of the reliability of the surface-wave analysis results. In general, borehole tests are considered more reliable than surface-based seismic surveys, even though the uncertainties of the final results are often similar.

A successful example of site characterization with Rayleigh-wave inversion is reported by Monaco et al. (2009). The results of two opposite shot-dispersion-curve inversions are compared with S-wave velocity profiles obtained from two different sets of downhole measurements (Figure 15). The first was obtained with the classical approach by inserting a couple of geophones in a borehole. The other was obtained with a seismic dilatometer (SDMT); the receivers were inserted inside the rods used to drive the flat dilatometer (DMT), a probe developed in technical engineering to measure an index of soil deformability and in situ stress, into the soil. Integrating active and passive surface-wave data and including higher modes in the inversion provided greater investigation depths than borehole measurements.

### CURRENT RESEARCH TOPICS

In this section, we discuss two issues that are particularly relevant for the development of surface-wave analysis and its applications. The first is the inclusion of higher modes and other dispersive events in the inversion. The second is the effect of lateral variations and the possibility of retrieving them despite the 1D approach used for processing and forward modeling.

### Higher modes

In most cases, the inversion of the surface-wave dispersion curve is still performed assuming that the experimental dispersion curve coincides with the fundamental mode of propagation (Figure 11a). In several cases, even though the authors recognize their data are likely to be influenced by higher modes, fundamental-mode inversion is adopted (e.g., Donohue and Long, 2008).

In spite of this trend (Figure 11b), there are many reasons why higher modes should be included in the inversion. Higher modes are sensitive to some model parameters to which the fundamental mode...
is poorly sensitive (Socco and Strobbia, 2004) and may improve the accuracy of the result (Ernst, 2008; Maraschini et al., 2010), especially in the presence of a velocity decrease with depth (Gucunski and Woods, 1992; Xia et al., 2003). Higher modes may increase the investigation depth (Gabriels et al., 1987; Socco et al., 2010), stabilize the inversion process (Xu et al., 2006), and enhance the resolution of the inverted model (Xia et al., 2003).

Several authors propose different ways of including higher modes in the inversion. Gabriels et al. (1987) invert several experimental dispersion-curve branches, minimizing the distance with theoretical modal curves and adopting a least-squares approach. Tokimatsu et al. (1992) and Tokimatsu (1997) invert Rayleigh-wave higher modes and show the errors that can be made by fundamental-mode inversion in the case of complex velocity profiles. Park et al. (1999b) perform a preliminary fundamental-mode inversion with a successive multimodal refinement. Xu et al. (2006) theoretically describe a method to recognize and integrate fundamental and higher Rayleigh modes. Song and Gu (2007) use a multimodal genetic algorithm that minimizes a weighted sum of the least-squares error for each mode in sites presenting velocity decreases with depth. All of these authors demonstrate the importance of higher modes but also point out the difficulties involved in including them in the inversion.

The first problem is to separate and identify different modes in experimental data, which can be obtained only if dense spatial sampling and long acquisition spreads are used (Foti et al., 2000; Socco and Strobbia, 2004). In this respect, deep exploration data are often more favorable than on-purpose acquired data at an engineering scale. In this context, we show an example of fundamental and several higher modes in Figure 16, where one can observe that the higher modes have been retrieved very well in a broad frequency band from marine data by Klein et al. (2005). A strategy to increase the spread length artificially, which involves moving the source instead of the receivers to identify different modes, is shown by Gabriels et al. (1987); however, this method can be adopted only if the site does not present lateral variations within the maximum offset. If short spreads are used, the retrieved dispersion curves are always apparent dispersion curves (mixing modes) in which the contribution of different modes is hard to distinguish. (Tokimatsu et al., 1992; Tokimatsu, 1997). This is also the case of the two-station approach, in which only a single dispersion curve can be retrieved (Gucunski and Woods, 1992).

When acquisition and processing schemes allow different modes to be retrieved, it is necessary to compare the experimental-curve branches with a specific theoretical mode; hence, the data points need to be attributed to a specific propagation mode. This task is not straightforward because some modes may not be present in the experimental data and very smooth passages from one mode to another may occur. The misidentification of modes may produce significant errors, as shown by several authors (e.g., Zhang and Chan, 2003; Maraschini et al., 2010). We show an example of inversion pitfalls from mode misidentification at low frequencies reported by Maraschini et al. (2010) in Figure 17a and b.

Many strategies have been implemented to solve this problem. Lu and Zhang (2006) invert multimodal dispersion curves and select the theoretical mode that must be compared with the experimental data points, considering the modal curve associated with the maximum displacement. Several authors have shown that mode misidentification can be avoided if the inversion is performed using a forward al-

![Figure 15](image)  
**Figure 15.** Successful example of site characterization with Rayleigh-wave inversion at L’Aquila-Roio Piano, Italy. The results of two opposite shots’ dispersion-curve inversions (SW1 and SW2) are compared with S-wave velocity profiles obtained by downhole and seismic dilatometer (SDMT) tests. The integration of active and passive data and the inclusion of higher modes in inversion provide deeper investigation with respect to borehole measurements. (Courtesy of Monaco et al., 2009).

![Figure 16](image)  
**Figure 16.** Data set gathered with an ocean-bottom hydrophone streamer. (a) Common receiver gather; (b) wavefield transform obtained using the slant-stack method. The fundamental and several higher modes are very well retrieved in a broad frequency band. (Courtesy of Klein et al., 2005).
algorithm that computes the apparent dispersion curve and accounts for modal superposition on the basis of modal displacements (e.g., Ganji et al., 1998; Lai and Rix, 1999). One promising approach is proposed by Forbriger (2003a, 2003b), who implements a multimodal inversion procedure based on computing the full waveform; this approach includes higher and leaky modes with their amplitudes. The method has proven to be robust and is used by other authors (O’Neill and Matsuoka, 2005). Ryden and Park (2006) propose an interesting example of simulated annealing inversion of phase-velocity spectra for pavement assessment. The theoretical spectra are calculated using the stiffness-matrix method (Kausel and Roesset, 1981), and extraction of the experimental dispersion curves is not required.

Maraschini et al. (2010) propose an alternative approach using a new misfit function based on the implicit function (equation 2) whose zeroes are modal curves. The minimization of the determinant of the stiffness-matrix approach was suggested by Ernst (2007). This approach accounts for modal superposition without the need to calculate the apparent dispersion curve and allows several dispersion-curve branches to be inverted efficiently and simultaneously without associating them with a specific mode number. An example of multimodal inversion using this approach is reported in Figure 17 for a site in which the smooth passage from the fundamental to the first higher mode at low frequency makes proper mode identification very difficult. The main drawback of the method concerns the increased nonlinearity of the inversion process, which requires great care in selecting the initial model in linearized inversion. It is instead suitable for GSMs (Maraschini and Foti, 2010).

Lateral variations

In 2D environments, the traditional 1D approach usually neglects the presence of lateral variations (Semblat et al., 2005). Because the surface-wave path crosses different materials, the resulting model is a simplified description of the site.

Several authors propose methods to preliminarily identify the possible occurrence of lateral variations below the acquisition gathers. Shitivelman (2003) proposes a method based on singular value decomposition (SVD) of seismic sections to separate events with different horizontal coherence; it allows shallow heterogeneities to be located. Strobbia and Foti (2006) use the regression of the phase difference as a function of the offset to locate lateral variations and to evaluate the frequency band affected by them. Ernst et al. (2002) and Campman et al. (2004, 2005) study scattered surface waves with the aim of removing them from seismic data, but they also provide promising tools for recognizing and locating shallow heterogeneity. Nasseri-Moghaddam et al. (2005) propose a method based on analyzing attenuation and amplification of Rayleigh-wave amplitude to detect the presence and embedment of voids. An interesting application, devoted to nondestructive testing of concrete members, is proposed by Zerwer et al. (2005). This approach is based on calculating autospectra to highlight reflections caused by cracks. A very simple but appealing approach is proposed by Woelz and Rabbel (2005), who plot several time slices of 3D data, acquired with a dense spatial grid over an archaeological site, and highlight Love-wavefront deformations that coincide with buried structures.

Kennett and Yoshizawa (2002) and Strobbia and Foti (2006) show that if the wave path is horizontally heterogeneous, it can cause perturbations on the observed phase (velocity) of the surface waves. As a consequence, Lin and Lin (2007) point out that artifacts may be introduced in spatially 2D $V_s$ imaging if the effect of lateral heterogeneity is not accounted for. Their results demonstrate that lateral heterogeneities induce a nonstationary property in the space domain, resulting in false depth-related dispersion or higher modes if a conventional approach, based on a stationary assumption, is used for the dispersion analysis. Bodet et al. (2005) use a laser-Doppler physical modeling of surface-wave propagation in the presence of dipping layers to assess the limitations of conventional 1D surface-wave inversion. They show that the estimated interface depth depends on shot position.

Hence, lateral variations in surface-wave analysis represent a problem but also an important target, particularly for exploration applications (e.g., static corrections). The 1D approach is still adopted to explore lateral variations for processing and inversion, and 1D velocity profiles are eventually merged to display lateral variations (Tian et al., 2003; Bohnel et al., 2004; Yilmaz et al., 2006; Neducza, 2007). In other words, data are processed and inverted, disregarding the effect of lateral variations, but the lateral variations are then retrieved and considered in the final interpretation. In this context, it is very important to assess the errors that could be introduced because

![Figure 17. Example of multimodal inversion to avoid a typical pitfall: the experimental dispersion curve passes smoothly from the fundamental to the first higher mode at low frequency. The misidentification of modes produces significant errors that are avoided with a multimodal approach. (a) $V_s$ profiles compared with the first branch of the real data ($A/2.5 V_s$ domain), a downhole test result, and a refraction result. (b) Dispersion curves. (c) Real dispersion curve compared with the misfit surface of the determinant approach. (courtesy of Maraschini et al., 2010).](image-url)
of the presence of unknown lateral variations, but it is also important to improve the possibility of retrieving lateral variations with good spatial resolution as much as possible.

As far as the retrieval of a laterally varying \( V_g \) model is concerned, Al-Eqabi and Herrmann (1993) propose a procedure based on the visual examination of data, stacking, waveform inversion of selected traces, phase-velocity adjustment by crosscorrelation, and phase-velocity inversion. Similarly, Hayashi and Hikima (2003) show that if lateral variations are identified in the data, one possibility is to analyze a subset of the traces to identify a 2D distribution using appropriate modeling tools. Hayashi and Suzuki (2004) propose an approach where common-midpoint crosscorrelation gathers of multichannel and multishot surface waves give phase-velocity curves that enable 2D velocity structures to be reconstructed. Grandjean and Bitri (2006) extend this method by applying the summation principle as a technique to increase the signal-to-noise ratio (S/N) of the local dispersion image. This is achieved with a multifold processing procedure. Lin and Lin (2007) suggest using a walkaway survey and a phase-seamining procedure when synthesizing seisograms with different nearest source-to-receiver offsets, which allows dispersion analysis within a small spatial range.

Socco et al. (2009) propose a strategy to retrieve smooth lateral variations through a multifold processing approach and a laterally constrained inversion of surface-wave dispersion (Figure 18). They use surface waves contained in seismic reflection or refraction data, similar to the approach proposed by Grandjean and Bitri (2006), through an automatic processing procedure that stacks the dispersion curves obtained from different records in the \( f-k \) domain and retrieves the experimental uncertainties. The processing is performed in a moving window along the seismic line. The data set to be inverted is therefore a set of dispersion curves evenly spaced along the seismic line. The set of dispersion curves is then inverted simultaneously with a laterally constrained inversion algorithm (Auken and Christiansen, 2004; Wisén and Christiansen, 2005) that reconstructs smooth lateral variations in spite of the 1D model assumed for the forward problem solution (Boiero and Socco, 2010).

One of the main issues in lateral variation reconstruction is assessing the achievable lateral resolution. O’Neill et al. (2008) highlight that spatial resolution is related to the ratio between the width of the heterogeneity and the spread length. To improve the lateral resolution, the data can be windowed in the offset domain to maintain a good spectral resolution and at the same time to enhance the contribution of the central part of the spread (Bohlen et al., 2004). The windowing of the data deteriorates the spectral resolution, so a compromise between spectral resolution and lateral resolution is required. The use of high-resolution spectral estimators is suggested by a few authors to preserve wavenumber resolution when reducing spread length (Trad et al., 2003; Winsborrow et al., 2003; Tillman, 2005; Luo et al., 2008).

All of the aforementioned methods have been developed for shallow and very shallow applications, but a great amount of literature concerning the retrieving of lateral variations using surface-wave traveltime tomography is available regarding regional or global scales (Ritzwoller and Levshin, 1998; Kennett and Yoshizawa, 2002; Yoshizawa and Kennett, 2002; Yao et al., 2006; Yao et al., 2008).

**DISCUSSION**

On the basis of our general overview, we devote this section to a discussion of the best practices in surface-wave analysis. When inferring subsoil properties through a geophysical investigation, the purpose is to obtain a final result that ideally is (1) representative of the subsoil, (2) unique, (3) accurate and reliable, and (4) useful. Each of these features can be back-projected to obtain the requirements for the different steps of surface-wave analysis: inversion, processing, and acquisition.

To obtain a representation of the subsoil, parameterization is necessary. The number of layers of the reference model for the inversion should be large enough to describe the system properly but small enough to avoid an overparameterized system for which the sensitivity to model parameters is too low. If no a priori information is available to define a proper conceptual model of the site, the reference model should be selected on the basis of the data (dispersion curve). A very common approach is to plot the dispersion curve in \( \lambda/d \) versus 1. \( V_g \) where \( d \) is usually in the 2–4 range and \( V_g \) is the phase velocity of the Rayleigh-wave fundamental mode, which can be considered an approximate solution in normally dispersive sites, and to discretize this approximated velocity profile in a layered system with a minimum parameterization criterion (e.g., Ismail and Anderson, 2007). To define a proper parameterization, preliminary inversion with overparameterized models using GSMSs may be performed to supply a general trend of the solution, which is then used as the basis for the choice of the reference model (Socco et al., 2009). Sensitivity analysis may also be applied to assess the sensitivity to different model parameters (e.g., Tarantola, 2005). In the case of a gradual velocity variation with depth, a vertically smoothed reference model can be adopted in which only the velocity is unknown and the system is discretized with thin layers (Socco et al., 2008).

The second issue concerns the uniqueness of the result. The result of the inversion process is by definition nonunique. In particular, the surface-wave inverse problem is strongly nonlinear, ill posed, and mix determined; hence, it suffers from a strong solution nonuniqueness (e.g., Sambridge, 2001; Wathelet et al., 2004). Global search inversion methods provide insight into the solution nonuniqueness because they show local minima and regions of the model-parameter space where the solution may fall according to available information (data quality and quantity) (Vasco et al., 1993; Lomax and Snieder, 1995; Sambridge, 1999b, 2001; Socco and Boiero, 2008). In this respect, data uncertainties play an important role, and acquisition and

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**Figure 18.** An example of laterally constrained inversion of a set of dispersion curves along a seismic line. The introduction of lateral constraints improves the internal consistency of the final pseudo-2D model. (a) Individual inversions versus (b) laterally constrained inversion. For the two inversion results, the velocity profiles are in the top plots and the normalized residuals (N.Res) at the last iteration are in the bottom plots. (courtesy of Socco et al., 2009).
processing procedures that enable a rigorous uncertainty evaluation are recommended (Lai et al., 2005; Socco et al., 2009).

To reduce the experimental uncertainty, we should improve the data quality by stacking in the frequency domain to improve the S/N (Grandjean and Bitri, 2006; Socco et al., 2009), adopting a dense spatial sampling (Socco and Strobbia, 2004), and applying high-resolution spectral estimators to retrieve the dispersion curve (Zywicki and Rix, 2005). To broaden the bandwidth of the dispersion curve, we should design our acquisition properly, using broadband sensors and sources, avoiding band-pass filters, and acquiring the data with long, dense sampling arrays and a long acquisition window. This may not be economical or practical, so passive data acquired with 2D arrays could be used to increase the information at low frequency.

To mitigate the solution nonuniqueness, after gathering high-quality data, we should adopt a proper inversion strategy. If we use deterministic inversion, the first problem is model parameterization. Another important issue is the introduction of constraints from a priori information and other available data. Even though other data are often available, only 5% of the scientific literature presents examples of joint or constrained inversion of surface-wave data. If boreholes are available at the site, we should include stratigraphic information in the inversion process and not use them only for an a posteriori assessment of the results. If the data are recorded with a high sampling rate, body waves can also be processed to supply stratigraphic information, which is then introduced as constraint in the inversion. If the water-table level is known from boreholes or from graphic information, which is then introduced as constraint in the inversion, the first problem is model parameterization.

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**Best-practices guidelines**

The main issues that have been dealt with in the previous discussion can be used to define general guidelines and recommendations for surface-wave surveys.

Starting from acquisition, we have seen that gathered surface-wave data should have a high S/N over a wide frequency band, allow different modes to be separated and recognized, and include associated experimental uncertainties. It is also useful to acquire several dispersion curves along the same line or over the same area. These characteristics of the data can be guaranteed using the following setup:

- broadband single sensors [possibly three component (3-C)] and powerful broadband sources
- no filters during acquisition
- properly designed acquisition spreads — long acquisition spreads with dense spatial sampling for active data, wide 2D arrays for passive data
- adequate time window to sample the whole wave train of surface waves over the whole seismic line
- multifold data to apply stacking and retrieve experimental uncertainties

One of the most attractive aspects of surface-wave analysis is that it can be performed on seismic data that have been acquired for other purposes. When a multifold seismic data set is available, a series of experimental surface-wave dispersion curves and their related uncertainties can be extracted along the seismic line. Because, in these cases, the acquisition is not optimized for surface-wave data analysis, we must assess whether the data fulfill some requirements. The source type, sensor frequency, and sampling in time and space may not be adequate.

As for the sensors, if 3-C sensors are used, only the vertical component is commonly used for surface-wave analysis, but very useful information can be retrieved analyzing the horizontal components. The horizontal inline component supplies additional information on the Rayleigh (or Scholte) waves that can be added to the information.
retrieved from the vertical component, whereas the horizontal crossline component supplies information on Love-wave propagation.

The seismic sources used in seismic reflection and refraction acquisitions are often powerful enough to supply very high S/N surface-wave signals. The time sampling and trace length are usually adequate to retrieve dispersion curves over a wide frequency band. If the acquisition window is not long enough to contain the whole surface-wave signal, the data should be muted properly to keep just the traces in which the surface waves are not truncated. The offset is usually sufficiently long to guarantee high wavenumber resolution and, hence, good modal separation; on the other hand, spatial sampling is sometimes too coarse to retrieve dispersion curves without spatial aliasing.

In seismic reflection surveys, surface waves are considered as coherent noise; therefore, several countermeasures can be taken to filter them out: high-frequency sensors, sensor and source arrays, and/or low-cut filters in acquisition. In these cases, the data should be evaluated carefully before using them for surface-wave analysis. The actual trend of land acquisition using dense spatial sampling with single 3-C broadband accelerometers and low-frequency vibroseis sources is particularly favorable for surface-wave data acquisition. A general recommendation for seismic exploration acquisition could be to avoid using sensor and source arrays to preserve surface waves.

As for any seismic method, careful preliminary data evaluation is also necessary for surface-wave analysis before extracting the dispersion curves. Regardless of which processing method is adopted, we should perform some preliminary evaluation in the spectral domain with the aim of defining (1) the spectral region (frequency band and velocity range) of interest, (2) the presence of different branches of the dispersion curve related to the fundamental and higher modes, (3) lateral variations along the seismic line or within the seismic volume, and (4) the optimum length of the processing window. After selecting the optimum processing parameters, assessing the lateral variations, and defining the proper windowing to make the retrieved dispersion curve local, the dispersion curves may be extracted. Several wavefield transforms may be used; in general, no technique can be defined as being better than others.

The dispersion curves and related uncertainties should be evaluated carefully along with any other available information to set up the inversion process. Global search methods are gaining popularity, thanks to the evolution of computing facilities that make computing costs reasonable. These methods are recommended, particularly when no a priori information is available. The initial model for deterministic methods should be carefully selected, and sensitivity analysis should be performed. Uncertainties should be included in the inversion, and other available data should be used to constrain the process. Higher modes should be considered with appropriate forward modeling, and different dispersion curves at the same site should be linked through spatial constraints.

CONCLUSION

Surface-wave analysis is extensively adopted to retrieve near-surface S-wave velocity models for different applications.

Despite its wide use, the method is still under continuous and significant methodological evolution and improvement and several open issues remain. The introduction of higher modes and P-guided waves in the inversion process is a challenging task; even though many methods have been proposed, no approach is recognized as standard by the scientific and professional community. Another important issue is represented by lateral variations. The combination of a processing approach that can locate subsurface heterogeneities and lateral variation (tomography), the enhancement of the spatial resolution by adopting proper spectral estimators and windowing, and the simultaneous inversion of several local models could represent a good compromise between the 2D/3D nature of the subsol and the 1D assumption that still governs surface-wave analysis.

Continuous improvement of surface-wave analysis is increasing the number of successful applications, and interest toward this technique is growing, even in the field of hydrocarbon exploration, making it a promising innovation for near-surface velocity model-building.

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APPENDIX A

INTERDISCIPLINARY GLOSSARY OF SURFACE-WAVE ANALYSIS

active test: Measurement performed recording the motion caused by a seismic source activated on purpose at the site. Also known as active source test.

apparent velocity: The phase velocity of surface waves as determined from the analysis of field measurements in which it is impossible to isolate the contribution of the different modes of propagation because of the limited resolution of finite recording arrays. Also referred to as effective velocity.

array measurements: Although very general, a term commonly used in applied seismology to address microtremor measurements with 2D arrays of low-frequency geophones aimed at surface-wave analysis.

body waves: Waves that travel within a medium in the form of compressional (primary) P-waves or shear (secondary) S-waves.

Capon method: See maximum-likelihood methods.

continuous-surface-wave (CSW) method: A test in which the surface-wave phase velocity at different frequencies is determined on the basis of the associated wavelength, identified using a controlled harmonic source (Jones, 1958; Matthews at al., 1996). Also referred to as steady-state Rayleigh method or SSRM.

dispersion: Variation of velocity with frequency. See also geometric dispersion, inverse dispersion, material dispersion, normal dispersion.

effective velocity: See apparent velocity.

epicenter: Projection on the ground surface of the focal point of an earthquake (i.e., the point from which energy is released).
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**equivalence**: Combinations of model parameters that would produce practically indistinguishable model responses with respect to a set of experimental data in inverse problem solution.

**extended spatial autocorrelation (ESAC) method**: Processing algorithm derived from the SPAC algorithm for analyzing passive source data sets with complex acquisition geometries.

**frequency-domain beam former (FDBF)**: Processing algorithm used to obtain an estimate of a spectral parameter from data scattered in space.

**full-waveform inversion**: Inversion based on the synthesis of the complete wavefield, comprised of body waves and surface waves with their modes of propagation.

**fundamental mode**: The mode that shows lowest phase velocity.

**geometric dispersion**: Dispersion caused by heterogeneity of a medium.

**ground roll**: Term adopted in geophysical exploration to indicate surface-wave energy that travels near the surface of the ground and that can mask the reflection signals in seismic surveys. For this reason, the term has a negative connotation. Several techniques have been developed to suppress it during processing. In a certain sense, the term can be considered a synonym of Rayleigh waves because, in most cases, ground roll is caused by their propagation.

**guided waves**: Waves propagating in a layer where most of the energy is trapped or along an interface.

**higher modes**: Modes propagating with higher phase velocity than that associated with the fundamental mode.

**horizontal/vertical (H/V) spectral ratio (HVSR)**: Method based on the spectral ratio between the horizontal and vertical components of a motion to assess the natural frequencies of a soil deposit, which are associated with resonance phenomena with respect to vertically propagating shear waves and to surface waves. Also referred to as the Nakamura method.

**inverse dispersion**: When phase velocity increases with frequency.

**Lamb waves**: Guided waves that travel along a plate or a thin layer (thickness < wavelength).

**lateral variations**: Changes in a horizontal direction that render a 1D model no longer representative.

**leaky modes**: A seismic guided wave that is imperfectly trapped reflecting boundaries with a loss of energy resulting from the excitation of other waves.

**Love waves**: Guided waves given by a horizontally polarized shear wave trapped in a softer upper layer. They are associated with horizontal motion perpendicular to the direction of propagation. Also referred to as the CSW method.

**material dispersion**: Dispersion caused by the dissipative properties of a medium.

**maximum-likelihood methods (MLM)**: High-resolution processing algorithm used to obtain an estimate of a spectral parameter from scattered data in space (Capon, 1969). Also referred to as the Nakamura method.

**microtremors**: Vibrations caused by natural phenomena (wind, ocean waves) or anthropic activities. They can be recorded and processed to extract information related to surface-wave propagation; hence, the term is sometimes used on its own to indicate passive tests.

**MML**: See maximum-likelihood methods.

**mud roll**: The equivalent of ground roll on the seabed in a shallow marine environment. In a certain sense, the term can be considered a synonym of Scholte waves because, in most cases, mud roll is caused by their propagation.

**multichannel analysis of surface waves (MASW)**: Active source test in which several receivers are used to collect data along a linear array. It is often associated with use of the slant-stack transform proposed by Park et al. (1999), who introduced the acronym.

**multiple signal classification (MuSiC, MUSIC)**: An advanced processing algorithm used to obtain an estimate of a spectral parameter from data scattered in space.

**near-surface geophysics**: Surveys dealing with the characterization of the uppermost 50–100 m from the ground surface.

**normal dispersion**: When the phase velocity decreases with frequency.

**ocean-bottom-cable (OBC) data sets**: Data collected along the seabed for seismic surveys.

**passive MASW**: See refraction microtremors.

**passive test**: Measurements performed recording the motion caused by microtremors. Also known as passive source test.

**Rayleigh waves**: A type of seismic surface wave propagating along the boundary of a semi-infinite medium.

**refraction microtremors (ReMi)**: Surface-wave method based on analyzing passive source data collected with linear arrays of receivers. Also known as passive MASW.

**SASW**: See spectral analysis of surface waves.

**Scholte waves**: A type of seismic surface wave propagating along the interface between a fluid layer and an underlying semi-infinite solid.

**slant-stack transform**: Processing tool based on the time-shifting and stacking of traces; the effect is to emphasize some events. It can be used in conjunction with a Fourier transform to obtain a slowness/frequency panel in which dispersive events can be identified easily.

**spatial autocorrelation (SPAC)**: Processing tool for passive source data sets based on the stochastic regression of theoretical functionals of wave propagation (Aki, 1957).

**spectral analysis of surface waves (SASW)**: Although the term is very general and could be used to designate any surface-wave method, it is commonly associated with the test based on the use of a couple of receivers deployed at increasingly larger spacings in multiple acquisitions at a site, as proposed by Nazarian and Stokoe (1984).

**static corrections**: Corrections applied to seismic data to compensate for near-surface low-velocity layers in seismic reflection surveys. Also known as statics.

**steady-state Rayleigh method (SSRM)**: See continuous-surface-wave (CSW) method.

**Stoneley waves**: An interface wave in a borehole.

**surface-wave method (SWM)**: Any surveying (characterization) technique based on the analysis of surface-wave propagation.

**surface waves**: Energy that travels close to the free boundary of a medium; its associated motion decays rapidly with depth.

**teleseismic**: Earthquake records collected at very large distances (thousands of kilometers) from the epicenter.

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