# Crosstalk Statistics via Collocation Method

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***Abstract*—A probabilistic model for the evaluation of transmission lines crosstalk is proposed. The geometrical parameters are assumed to be unknown and the exact solution is decomposed into two functions, one depending solely on the random parameters and the other on the frequency. The stochastic collocation method is used to estimate the crosstalk statistical moments. The results are obtained from a limited number of carefully-chosen values of the random geometrical parameters. The estimated statistical moments are then used to build the probability density function of the crosstalk parameters. A Monte Carlo validation demonstrates the accuracy and efficiency of the advocated method.**

## I. Introduction

Cables in modern electronic systems such as in aircrafts, ships, and vehicles are employed to transmit power as well as signals throughout the system. These cables consist of a large number of individual wires that are packed into bundles for neatness and space conservation. The electromagnetic fields surrounding these closely spaced individual wires interact with each other and induce signals in all adjacent circuits. This crosstalk can cause functional degradation of the circuits at the ends of the cable. Thus, the prediction of crosstalk is one of the major objectives in EMC.

This widely investigated aspect has contributed to the development of prediction models for a wide variety of wiring structures and for several frequency ranges (e.g. [1], [2]). One result is gained from analyzing the near and the far end energy coupled into the victim wires. The magnitude of the electromagnetic interference varies significantly as a function of a number of factors including the wires geometries. The sensitivity of crosstalk to random wires position in the cable has led to several analytical probabilistic models for the crosstalk according to the frequency ranges (e.g. [3], [4], and [5]). However, analytical expressions of the probability density functions (PDF) can be fastidious or impossible to determine. Furthermore, the seemingly obvious approach to this problem is the use of uniform, multiconductor transmission line (MTL) theory to model the cable bundle (e.g. [1]). Another difficulty inherent in the application of the MTL model is the computation time required to obtain the response at each frequency.

Based on the study carried out in [3], where an analytical model of the crosstalk is proposed at low frequencies, this work proposes a complete probabilistic model (low frequencies and high frequencies) of the crosstalk in two-conductor transmission lines above a ground plane avoiding tedious calculations. The main feature of this model is that the ratios between the end voltages and the source voltage are factorized in two functions: one depending solely on the random parameters and the other on the frequency. As a consequence, the knowledge of the random variables distributions allows us to estimate the crosstalk statistical moments through a collocation method or unscented transforms (e.g. [6], [7]) independently of the frequency. The PDFs capable of predicting the variations of the crosstalk are then constructed from their statistical moments.

## II. Uncoupling between the Frequency and the Geometrical Parameters

The analysis is based on the literal frequency-domain solution for a weakly coupled lossless transmission line in a homogeneous medium (e.g. [1]). This model is valid over a wide bandwidth, the only limitation being that, quasi-TEM propagation can be assumed along the line.

### *A. The deterministic general solution*

The configuration under analysis is illustrated in Fig.1a. The line consists of two perfect conductors with a common length $\mathbf{L}$ immersed in a lossless medium. The generator circuit is composed of a generator wire with the reference conductor i.e. the ground plane; It is driven with a source voltage $\mathbf{V_S}$ and source resistance $\mathbf{R_S}$ and is terminated in a load resistance $\mathbf{R_L}$. The receptor circuit is made of a receptor wire and the reference conductor. It is terminated in a "near-end" resistance $\mathbf{R_{NE}}$ and a "far-end" resistance $\mathbf{R_{FE}}$. The generator wire radius and the receptor wire radius are $\mathbf{r_G}$ and $\mathbf{r_R}$ respectively. Their distance from the origin are $\mathbf{d_G}$ and $\mathbf{d_R}$, whereas $\mathbf{d} = \mathbf{d_G} - \mathbf{d_R}$ is the distance between the wires and $\mathbf{h_G}$, $\mathbf{h_R}$ denote their heights above the ground plane. The per-unit-length equivalent circuit is shown in Fig.1b where $\mathbf{l_m}$ and $\mathbf{c_m}$ are respectively the per-unit-length mutual inductance and capacitance between the two circuits. They satisfy:

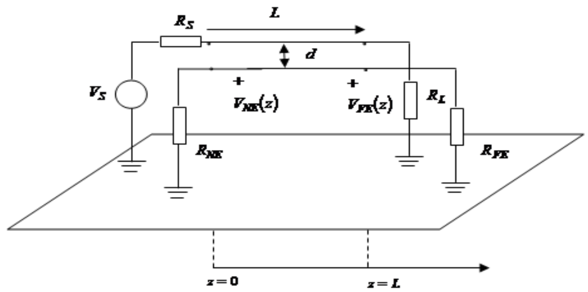
$$\mathbf{l_m} = \frac{\mu}{4\pi} \ln \frac{(\mathbf{h_G} + \mathbf{h_R})^2 + \mathbf{d}^2}{(\mathbf{h_G} - \mathbf{h_R})^2 + \mathbf{d}^2} \tag{1a}$$

$$\mathbf{c_m} = \frac{1}{\varepsilon\mu} \frac{\mathbf{l_m}}{\mathbf{l_G} \mathbf{l_R} - \mathbf{l_m^2}} \tag{1b}$$

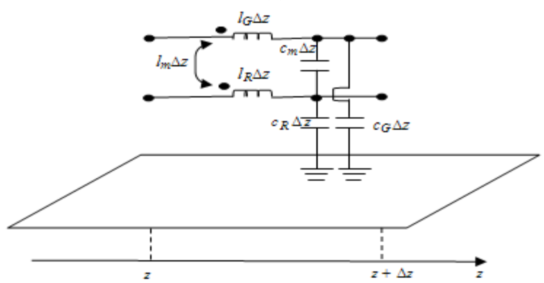
The quantities $l_G$ and $l_R$ are the per-unit-length inductances of the generator and receptor circuits, whereas $c_G$ and $c_R$ are the per-unit-length capacitances of the same circuits given by:

$$l_G = \frac{\mu}{2\pi}\ln\left(2\frac{h_G}{r_G}\right);\ l_R = \frac{\mu}{2\pi}\ln\left(2\frac{h_R}{r_R}\right) \tag{2a}$$

$$c_G = c_m\frac{(l_R - l_m)}{l_m};\ c_R = c_m\frac{(l_G - l_m)}{l_m} \tag{2b}$$




(a)




(b)

Fig.1. Configuration under analysis: (a) line dimensions and terminal characterization and (b) the per-unit-length equivalent circuit

The exact literal solutions for the near-end and far-end crosstalk voltages, $\hat{V}_{NE}$ and $\hat{V}_{FE}$ respectively, are (e.g. [1]):

$$\hat{V}_{NE} = \left[\frac{j\omega M_{NE}S}{C^2 + (j\omega)^2 S^2\tau_R\tau_G P + j\omega CS(\tau_r + \tau_g)}\left(C + j\omega\frac{L}{v}K_{NE}S\right)\right]\hat{V}_S \tag{3a}$$

$$\hat{V}_{FE} = \left[\frac{j\omega M_{FE}S}{C^2 + (j\omega)^2 S^2\tau_R\tau_G P + j\omega CS(\tau_R + \tau_G)}\right]\hat{V}_S \tag{3b}$$

where:

$$S = \frac{\sin\beta L}{\beta L} \tag{4a}$$

$$C = \cos\beta L \tag{4b}$$

$$\beta = \omega / v \tag{4c}$$

$$v = 1/\sqrt{\mu\varepsilon} \tag{4d}$$

$$M_{NE} = M_{NE}^{IND} + M_{NE}^{CAP} \tag{4e}$$

$$M_{FE} = M_{FE}^{IND} + M_{FE}^{CAP} \tag{4f}$$

$$M_{NE}^{IND} = \frac{R_{NE}}{R_{NE} + R_{FE}}l_m L\frac{1}{R_S + R_L} \tag{4g}$$

$$M_{FE}^{IND} = -\frac{R_{FE}}{R_{NE} + R_{FE}}l_m L\frac{1}{R_S + R_L} \tag{4h}$$

$$M_{FE}^{CAP} = M_{NE}^{CAP} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}}c_m L\frac{R_S}{R_S + R_L} \tag{4i}$$

The circuit time constants are defined as:

$$\tau_G = \frac{L}{v\sqrt{1-k^2}}\left\{\frac{1+\alpha_{SG}\alpha_{LG}}{\alpha_{SG}+\alpha_{LG}}\right\};\quad \tau_R = \frac{L}{v\sqrt{1-k^2}}\left\{\frac{1+\alpha_{SR}\alpha_{LR}}{\alpha_{SR}+\alpha_{LR}}\right\} \tag{5}$$

with:

$$\alpha_{SG} = \frac{R_S}{Z_{CG}}\ ;\ \alpha_{LG} = \frac{R_L}{Z_{CG}}\ ;\ \alpha_{SR} = \frac{R_{NE}}{Z_{CR}}\ ;\ \alpha_{LR} = \frac{R_{FE}}{Z_{CR}} \tag{6}$$

and:

$$Z_{CG} = \frac{1}{v(c_G + c_m)\sqrt{1-k^2}}\quad ;\quad Z_{CR} = \frac{1}{v(c_R + c_m)\sqrt{1-k^2}} \tag{7}$$

The coupling coefficient between the two-circuits is defined by $k = l_m / \sqrt{l_G l_R}$ and the coefficient $K_{NE}$ is defined by:

$$K_{NE} = \frac{1}{\sqrt{1-k^2}}\left(\frac{\alpha_{LG}+\alpha_{LR}}{1+\alpha_{LG}\alpha_{LR}}\right) \tag{8}$$

Finally, the factor $P$ in given by:

$$P = \left[1 - k^2\frac{(1-\alpha_{SG}\alpha_{LR})(1-\alpha_{LG}\alpha_{SR})}{(1+\alpha_{SG}\alpha_{LR})(1+\alpha_{LR}\alpha_{SR})}\right] \tag{9}$$

*B. Introduction of statistical variations*

The transmission lines geometrical parameters are supposed to be partially unknown, like in the case presented in [3], and the generator and receptor wires are considered to have the same radius $r = r_G = r_R$. We assume that the wires are placed at random heights $h_G$, $h_R$ and at random positions described by the geometrical distance $d$. However these three parameters are treated as independent Gaussian variables with mean values $\mu_{h_G}, \mu_{h_R}, \mu_d$ and variances $\sigma^2_{h_G}, \sigma^2_{h_R}, \sigma^2_d$, respectively. The main objective is to avoid intensive calculations and to find a quick way to determine the statistical moments of the crosstalk parameters for all frequencies. This can be done by separating (3a) and (3b) into two terms: one containing the random variables (generically represented by $\theta$ from now on), the other one mainly depending on the frequency $f$. The common denominator in (3),

$$Den = \cos^2\beta L + \left(jv\frac{\sin\beta L}{L}\right)^2 \tau_r\tau_g P + \left(jv\frac{\sin\beta L}{L}\right)\cos\beta L(\tau_r+\tau_g) \quad (10)$$

can be factorized by means of its roots, and expressions (3a) and (3b) are then casted in the following form:

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = H_{NE}(\theta)G_{NE}(f,\theta) \quad (11a)$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = H_{FE}(\theta)G_{FE}(f,\theta) \quad (11b)$$

where:

$$H_{NE}(\theta) = \frac{M_{NE}K_{NE}L}{\tau_r\tau_g Pv} \quad (12a)$$

$$H_{FE}(\theta) = \frac{M_{FE}L}{j\tau_r\tau_g Pv} \quad (12b)$$

$$G_{NE}(f,\theta) = \frac{[\sin(\beta L) + jx_3\cos\beta L]\sin\beta L}{[\sin(\beta L) + jx_1\cos\beta L][\sin(\beta L) + jx_2\cos\beta L]} \quad (12c)$$

$$G_{FE}(f,\theta) = \frac{\sin\beta L}{[\sin(\beta L) + jx_1\cos\beta L][\sin(\beta L) + jx_2\cos\beta L]} \quad (12d)$$

and:

$$x_1 = \frac{L}{v}\frac{-(\tau_r+\tau_g) - \sqrt{(\tau_r+\tau_g)^2 - 4\tau_r\tau_g P}}{2\tau_r\tau_g P} \quad (13a)$$

$$x_2 = \frac{L}{v}\frac{-(\tau_r+\tau_g) + \sqrt{(\tau_r+\tau_g)^2 - 4\tau_r\tau_g P}}{2\tau_r\tau_g P} \quad (13b)$$

$$x_3 = \frac{-1}{K_{NE}} \quad (13c)$$

A further simplification of the above expressions can be readily introduced, if we adopt two reasonable assumptions. The first one, related to weak coupling between the generator and the receptor circuits, implies $k \ll 1$, thus $P \cong 1$.
As a second assumption, the variations of $x_1, x_2, x_3$ with respect to their mean values are considered to be negligible.

With these assumptions, the factors $G_{NE}(f,\theta)$ and $G_{FE}(f,\theta)$ are approximated as follows:

$$G_{NE}(f) \cong \frac{[\sin(\beta L) + j<x_3>\cos\beta L]\sin\beta L}{[\sin(\beta L) + j<x_1>\cos\beta L][\sin(\beta L) + j<x_2>\cos\beta L]} \quad (14a)$$

$$G_{FE}(f) \cong \frac{\sin\beta L}{[\sin(\beta L) + j<x_1>\cos\beta L][\sin(\beta L) + j<x_2>\cos\beta L]} \quad (14b)$$

where $<\bullet>$ denotes the mean value. Equations (14) above are only function of the frequency.

The result of the above assumptions is that we are allowed to factorize Equations (11) in two parts, of which the $H_{NE}$ and $H_{FE}$ terms are independent of frequency and are only dependent on the random variables. Therefore, the statistical variations of crosstalk are provided by the $H_\bullet$ terms and the frequency behaviour is described by the $G_\bullet$ terms.

The most natural approach for determining the statistical characteristics of crosstalk is by means of Monte-Carlo method, which is easy to implement, but it is well known for its slow convergence rate. Other procedures as stochastic collocation method (e.g. [6]) can readily produce a statistical description of crosstalk by means of higher order statistical moments. This procedure is computationally very efficient, since it is performed with a limited number of well-chosen points related to the distribution of the random variables.

## III. Determining the Second Order Statistical Moments by a Stochastic Collocation Method

### A. *Determining the collocation points*

Higher order statistical moments of $|H_{NE}|$ and $|H_{FE}|$ are obtained as follows:

$$<|H_\bullet|^p(\theta_1,\theta_2,\theta_3)> = \iiint \begin{bmatrix} |H_\bullet|^p(\theta_1,\theta_2,\theta_3)\times \\ P_{\theta_1\theta_2\theta_3}(\theta_1,\theta_2,\theta_3) \end{bmatrix} d\theta_1 d\theta_2 d\theta_3 \quad (15)$$

where:

- $p$ denotes the moment order
- $(\theta_1,\theta_2,\theta_3)$ represent the independent variables [ $(h_G, h_R, d)$, in our case],
- $P_{\theta_1\theta_2\theta_3}(\theta_1,\theta_2,\theta_3) = P_{\theta_1}(\theta_1)P_{\theta_2}(\theta_2)P_{\theta_3}(\theta_3)$ is the PDF of the uncorrelated variables [in our case, a Gaussian distribution is adopted for each $P_{\theta_\bullet}(\theta_\bullet)$ ].

The first step of the stochastic collocation method consists in projecting the function containing the random variables on a Lagrange basis, and in truncating the obtained expansion to a given order to be defined. Therefore,

$$\begin{aligned}|\mathbf{H}_\bullet|^p &= \sum_{ijk=0}^{nmq}\left(|\mathbf{H}_\bullet|^p\right)_{ijk}\mathbf{L}_{ijk}\left(\theta_1^i,\theta_2^j,\theta_3^k\right)\\ &= \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{q}\left(|\mathbf{H}_\bullet|^p\right)_{ijk}\mathbf{L}_{ijk}\left(\theta_1^i,\theta_2^j,\theta_3^k\right)\end{aligned} \tag{16}$$

where $\{\mathbf{L}_{ijk}(\theta_1,\theta_2,\theta_3)\}_{(0\le i\le n)(0\le j\le m)(0\le k\le q)}$ forms a Lagrange polynomial basis of degree $(n-1)\times(m-1)\times(q-1)$.

The following approximation is obtained by substituting (16) into (15):

$$<|\mathbf{H}_\bullet|^p(\theta_1,\theta_2,\theta_3)>\approx \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{q}\iiint\begin{bmatrix}\mathbf{L}_i(\theta_1^i)\mathbf{L}_j(\theta_2^j)\mathbf{L}_k(\theta_3^k)\times\\ \mathbf{P}_{\theta_1}(\theta_1)\mathbf{P}_{\theta_2}(\theta_2)\mathbf{P}_{\theta_3}(\theta_3)\times\\ \left(|\mathbf{H}_\bullet|^p\right)_{ijk}d\theta_1 d\theta_2 d\theta_3\end{bmatrix} \tag{17}$$

where $\{\mathbf{L}_r(\theta)\}_{0\le r\le\eta}$ is defined as :

$$\mathbf{L}_r(\theta)=\prod_{s=0}^{\eta}\frac{\theta-\theta_s}{\theta_r-\theta_s} \tag{18}$$

The main property of the Lagrange polynomials is:

$$\mathbf{L}_r(\theta_s)=\delta_{rs}\quad\forall\ 0\le r,s\le\eta \tag{19}$$

where $\delta_{rs}$ denotes the Kronecker delta.

The Gauss-Hermite quadrature applied to (17), with the help of the above property, produces

$$<|\mathbf{H}_\bullet|^p>\approx\left(\frac{1}{\sqrt{\pi}}\right)^3\sum_{ijk=0}^{nmp}\mathbf{w}_i\mathbf{w}_j\mathbf{w}_k\left(|\mathbf{H}_\bullet|^p\right)_{ijk}\left(\theta_1^i,\theta_2^j,\theta_3^k\right) \tag{20}$$

where $(\mathbf{w}_i,\mathbf{w}_j,\mathbf{w}_k)$ are the weights associated to the collocation points $\left(\theta_1^i,\theta_2^j,\theta_3^k\right)$.

## B. *Numerical experiment*

In order to provide an example illustrating the performance of the collocation approach in comparison to the Monte-Carlo approach, we compute the moments of near and far end crosstalk for the circuit of Fig. 1, in which all loads are set to $\mathbf{50\Omega}$. The random variables $\mathbf{h_G}$ and $\mathbf{h_R}$ follow random distributions with the following parameters: $\mu_{h_G}=\mu_{h_R}=\mathbf{3cm}$ and $\sigma_{h_G}=\sigma_{h_R}=\mathbf{0.5cm}$. For $\mathbf{d}$, the parameters are $\mu_d=\mathbf{3cm}$ and $\sigma_d=\mathbf{0.7cm}$. The wires radius is chosen to be $\mathbf{r=0.4mm}$ and the length $\mathbf{L=10m}$.

Numerical computation of the first two moments of the near and far end crosstalk functions are compared in Fig.2 and Fig.3. The results of the Monte-Carlo procedure are shown for an increasing size of the sample set. As expected, the Monte Carlo method is largely inaccurate for a small number of samples, but its result stabilizes for a large number of samples. Figs. 2 and 3 indicate that the collocation method with $\mathbf{n=m=p=3}$ collocations points provides the same results as 20000 Monte-Carlo simulations, at a much lower computational cost.

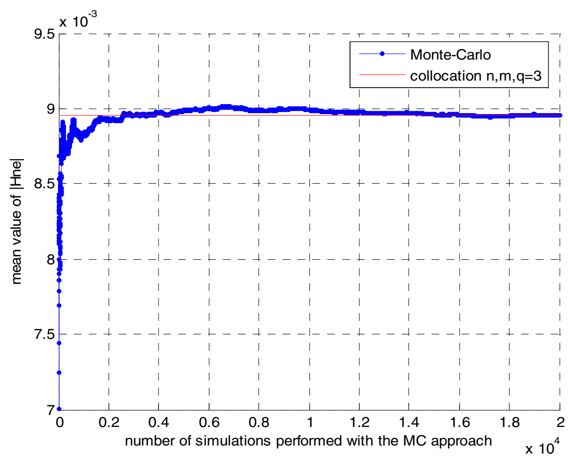



Fig. 2. Mean value of $|H_{NE}|$ obtained by stochastic collocation method and the Monte-Carlo approach at 150MHz

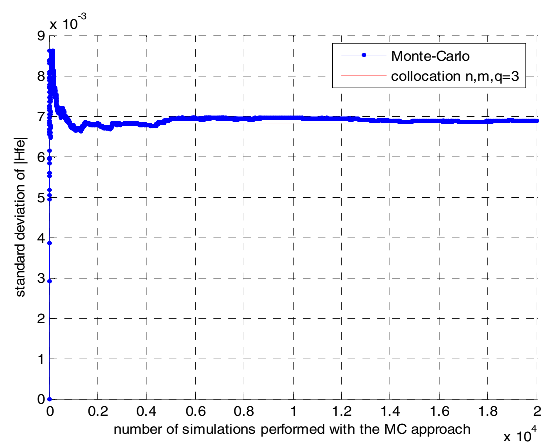



Fig. 3. Standard deviation of $|H_{FE}|$ obtained by stochastic collocation method and the Monte-Carlo approach at 150MHz

Moreover, statistical moments of the complete crosstalk transfer functions $\left|\hat{V}_{NE}/\hat{V}_S\right|$ and $\left|\hat{V}_{FE}/\hat{V}_S\right|$ can be obtained by means of the results derived in Sect. II. It is worth noticing that the mean values $< x_1 >, < x_2 >, < x_3 >$ required by the frequency factors in (14) are also estimated by means of the collocation approach.
The advocated approach is compared with Monte-Carlo direct evaluation of mean values of $\left|\hat{V}_{NE}/\hat{V}_S\right|$, and the results are represented in Fig.4; The standard deviations of $\left|\hat{V}_{FE}/\hat{V}_S\right|$ are compared in Fig.5. The good agreement between the estimation of the collocation method and the Monte Carlo simulation is evident from Figs.4 and 5.

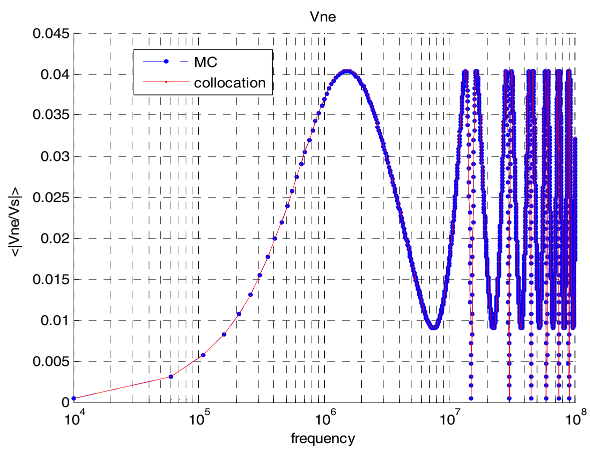



Fig. 4. Mean value of $\left|\hat{V}_{NE}/\hat{V}_S\right|$ vs. frequency, obtained by the stochastic collocation approach and a Monte-Carlo simulation.

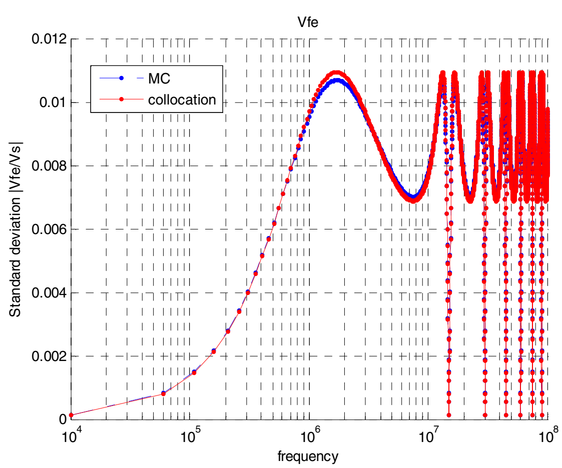



Fig. 5. Standard deviation of $\left|\hat{V}_{FE}/\hat{V}_S\right|$ vs. frequency, obtained by the stochastic collocation approach and a Monte-Carlo simulation.

## IV. Construction of the Crosstalk Parameters Distribution from Their Statistical Moments

Given the complexity of analytically deriving the crosstalk parameters distributions, we propose to estimate the PDF from the statistical moments.
The first step is to assume a theoretical distribution, which should be flexible enough to mimic the behaviour of a broad range of statistical distributions, and its parameters should be amenable to direct estimation from the statistical moments evaluated by the stochastic collocation method.
Assuming that the ratios $\left|\hat{V}_{NE}/\hat{V}_S\right|$ and $\left|\hat{V}_{FE}/\hat{V}_S\right|$ are restricted between 0 and 1 (which is certainly true for the problem at hand), we can direct our choice towards the generalized Beta distribution which is a continuous probability distribution family defined on the interval [0, 1]. This distribution is parameterized by two positive shape parameters denoted by $\alpha$ and $\gamma$ and defined by:

$$\alpha_{|\bullet|} = E[|\bullet|]\left(\frac{E[|\bullet|](1 - E[|\bullet|])}{\sigma^2_{|\bullet|}} - 1\right) \tag{21}$$

$$\gamma_{|\bullet|} = (1 - E[|\bullet|])\left(\frac{E[|\bullet|](1 - E[|\bullet|])}{\sigma^2_{|\bullet|}} - 1\right) \tag{22}$$

where $|\bullet|$ denotes either $\left|\hat{V}_{NE}/\hat{V}_S\right|$ or $\left|\hat{V}_{FE}/\hat{V}_S\right|$, $E[|\bullet|]$ and $\sigma^2_{|\bullet|}$ represent the average and variance, respectively.
The Beta distribution is defined by the following analytical distribution:

$$P_{|\bullet|}(u) = \frac{\Gamma(\alpha_{|\bullet|} + \gamma_{|\bullet|})}{\Gamma(\alpha_{|\bullet|})\Gamma(\gamma_{|\bullet|})} u^{\alpha_{|\bullet|}-1}(1-u)^{\gamma_{|\bullet|}-1} 1_{[0;1]}(u) \tag{23}$$

In order to cover a wide spectrum of theoretical distributions whose supports are $[0;+\infty[$ , the Gamma distribution is also tested. Its PDF is given by:

$$P_{|\bullet|}(u) = \frac{u^{\kappa-1}}{\vartheta^{\kappa}\Gamma(\kappa)} \exp\left(-\frac{u}{\vartheta}\right) 1_{[0;+\infty[}(u) \tag{24}$$

where:

$$\vartheta = \frac{\sigma^2_{|\bullet|}}{E[|\bullet|]} \tag{25}$$

and:

$$\kappa = \frac{(E[|\bullet|])^2}{\sigma^2_{|\bullet|}} \tag{26}$$

Figures 6–9 show a comparison of the two theoretical distributions defined above with the empirical one obtained by

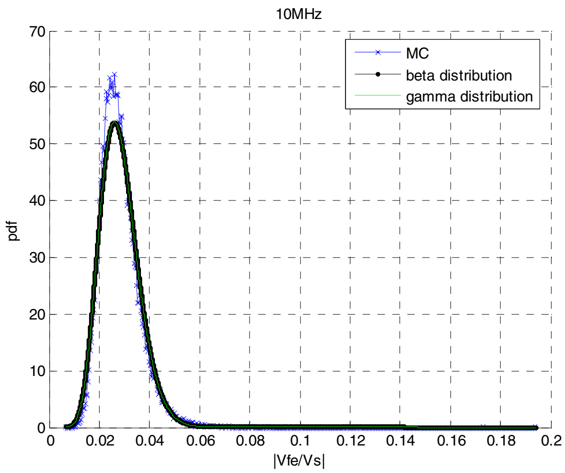



Fig. 6. Distribution of $\left|\hat{V}_{FE}/\hat{V}_S\right|$ at 10 MHz with $R_S = R_L = R_{NE} = R_{FE} = 50\Omega$

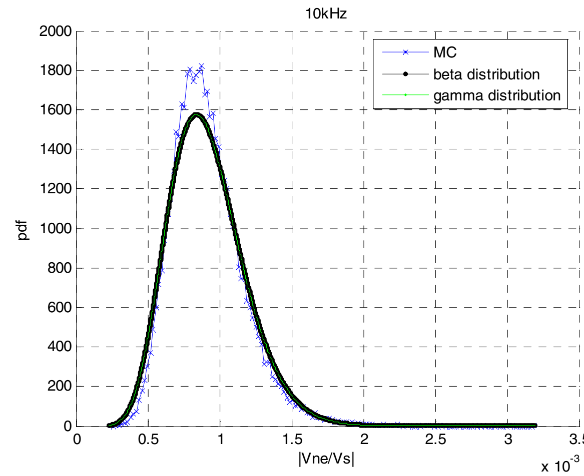



Fig. 8. Distribution of $\left|\hat{V}_{NE}/\hat{V}_S\right|$ at 10 kHz with $R_S = 0\Omega$ $R_L = 10\Omega$ $R_{NE} = 1\Omega$ $R_{FE} = 10\Omega$

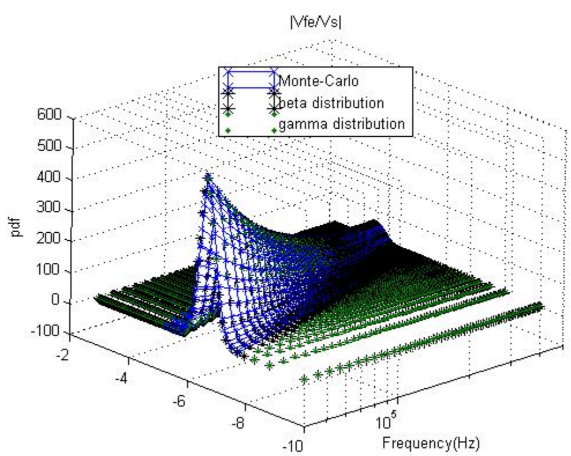



Fig. 7. Distribution of $\left|\hat{V}_{FE}/\hat{V}_S\right|$ between 50 kHz and 500 kHz with a frequency step of 7 kHz, and $R_S = 50\Omega$ $R_L = 100\Omega$ $R_{NE} = 70\Omega$ $R_{FE} = 10^9\,\Omega$

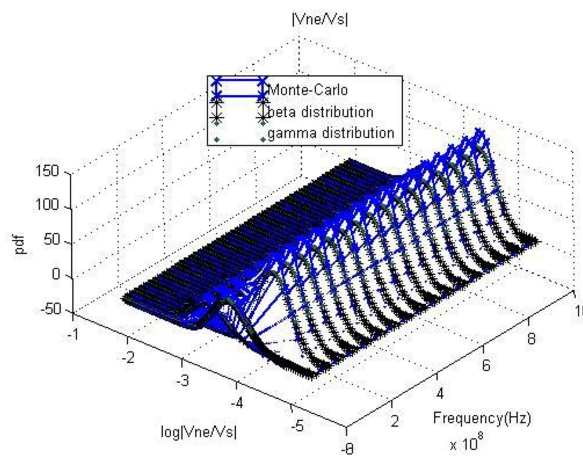



Fig. 9. Distribution of $\left|\hat{V}_{NE}/\hat{V}_S\right|$ between 11MHz and 1 GHz with a frequency step of 10 MHz and $R_S = R_L = R_{NE} = R_{FE} = 50\Omega$

means of the Monte-Carlo method applied to the general crosstalk model given in (3).

## V. CONCLUSION

A fast calculation of the statistical parameters of crosstalk between a pair of wires randomly distributed above a ground plane has been proposed. The crosstalk statistical moments are estimated by means of the collocation method, requiring a limited number of computations. Also, a probabilistic model can be derived from these statistical parameters, and all the results are valid for a wide frequency range (as far as the transmission line model holds). The results are validated by means of extensive Monte Carlo simulations that, however, are not affordable for practical purposes, due to their very high computational cost.

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