Reconciling Statistical Models with Practical Experience of Reverberation Chambers

Reconciling statistical models with practical experience of reverberation chambers

R. Serra and F.G. Canavero

Presented are novel theoretical probability density functions (PDF) for the magnitude and phase of electromagnetic fields inside reverberation chambers (RC) operating in a 'good-but-imperfect regime'. The derivation is based on considering two Gaussian random variables with mean values, variances and correlation between them that depart from the ideal assumptions. A multivariate approach using a complete joint Gaussian distribution of these variables is defined. Marginal distributions obtained by integration of this two-dimensional joint PDF are compared with theoretical PDFs for ideal situations, and significantly lower rejection rates are experienced for field data measured in real RCs. Additionally, these novel marginal PDFs are highly general since they are able to describe both ideal and non-ideal stirred fields.

Introduction: Reverberation chambers (RCs) are being used with significantly greater confidence for electromagnetic compatibility measurements and multipath environment characterisation. The correct interpretation of measurement results and the performance optimisation of RCs requires a full understanding of their working principles.

Statistical models represent the preferred approach when analysing the behaviour of the fields in an overmoded, 'well-stirred' RC [1]. A fundamental universally accepted result is that both the real and the imaginary part of each Cartesian field component at any arbitrary location within the working volume follow a Gauss-normal probability density function (PDF). This, in turn, implies that, under certain conditions, the pdf of the field magnitude and phase follow a Rayleigh and a uniform distribution, respectively.

Experimental characterisations of RCs are mainly focused on assessing whether a particular chamber is operating in a good reverberation regime. This assessment is often done by comparison of the empirical histograms of the measured fields with the expected theoretical PDFs. For a systematic and rigorous comparison, powerful goodness-of-fit (GoF) tests are employed, since they must provide reliable results even for a reduced number of measured samples, as is typically the case in RCs [2].

It has been reported in [2] and confirmed by our own measurements that powerful GoF tests often result in considerably higher and nearly unacceptable rejection rates of the measured data when compared to the theoretical PDF. These deviations from ideality are clearly a consequence of uncertainties present in every RC measurement setup (i.e. direct illumination, cross-polarisation, antennas and equipment-under-test size and type, chamber loading, etc.) that are not considered in the theoretical models. Very often, an ad hoc PDF (i.e. Rice, Weibull, Log Normal, etc.) is heuristically proposed and shown to better represent the field magnitude than the theoretical Rayleigh distribution (a recent example is [2], where a Weibull pdf is advocated to optimally represent the measured data). However, such heuristic approaches, apart from raising the question of their generality with respect to the chamber dimensions and characteristics, do not provide understanding of the RC physical mechanisms. Therefore, a need arises to build a reconciliation between ideal statistical models and empirical experience. In the following, we discuss a novel PDF derivation for the field components, releasing some ideality assumptions on which the theoretical PDF of the literature are based.

Theoretical development: The PDFs of the field magnitude ($|E_\alpha|$) and phase (arg $E_\alpha = \phi$) are found to be a Rayleigh and a uniform one, respectively, [1], where $\alpha = x, y, z$ is an index of the Cartesian field component. This is true, provided that some basic assumptions on the first- and second-order statistical properties of the real ($E_{\alpha r}$) and imaginary ($E_{\alpha i}$) part of the field components are held. Such assumptions require (a) zero mean ($\mu E_{\alpha r} = \mu E_{\alpha i} = 0$); (b) equal variances ($\sigma^2_{E_{\alpha r}} = \sigma^2_{E_{\alpha i}}$); and (c) statistical independence ($\rho_{E_{\alpha r}E_{\alpha i}} = 0$). In a non-ideal situation, the above assumptions should be relaxed, and a new expression of the PDFs for $|E_\alpha|$ and ϕ needs to be derived.

Let us define ξ and η as the real and imaginary part of an electric field component in a point of the test volume, namely $\xi = E_{\alpha r}$ and $\eta = E_{\alpha i}$, for convenience in notation. We start by considering the joint PDF of two Gauss-normal random variables ξ and η [3]:

$$f_{\xi\eta}(\xi, \eta) = \frac{1}{2\pi\sigma_\xi\sigma_\eta(1-\rho^2)} e^{(-1/2(1-\rho^2))((\xi-\mu_\xi)^2/\sigma_\xi^2+(\eta-\mu_\eta)^2/\sigma_\eta^2-2\rho(\xi-\mu_\xi)(\eta-\mu_\eta)/\sigma_\xi\sigma_\eta)} \quad (1)$$

where μ_ξ and μ_η are the mean values for ξ and η, respectively, σ^2_ξ and σ^2_η are the corresponding variances and ρ is the correlation coefficient. The function $f_{\xi\eta}(\xi, \eta)$ will be denoted by $N(\mu_\xi, \mu_\eta, \sigma_\xi, \sigma_\eta, \rho)$. A change of variables representing each field component in terms of its magnitude and phase, leads us to consider a new couple of random variables, i.e.

$$r = |E_\alpha| = \sqrt{\xi^2 + \eta^2} \quad \phi = \arg E_\alpha = \arctan\frac{\eta}{\xi} \quad (2)$$

where $r \geq 0$ and $-\pi < \phi \leq \pi$. The joint PDF of the new set of variables in (2) is

$$f_{r\phi}(r, \phi) = r f_{\xi\eta}(r\cos\phi, r\sin\phi) \quad (3)$$

This result is readily obtained extending the procedure illustrated in [3], where the Jacobian of the variable transformation ($\xi = r\cos\phi$; $\eta = r\sin\phi$) is simply

$$J_{\xi\eta r\phi}(\xi, \eta) = \frac{1}{r} \quad (4)$$

The probability distributions of the individual magnitude and phase of the field components are expressed by the marginal distributions of (3):

$$f_r(r) = \int_{-\pi}^{\pi} f_{r\phi}(r, \phi)d\phi \qquad f_\phi(\phi) = \int_0^\infty f_{r\phi}(r, \phi)dr \quad (5)$$

Explicit, closed-form expressions of the marginal probabilities can be found only for specific cases. As an example, if $f_{\xi\eta}(\xi, \eta) \sim N(0, 0; \sigma, \sigma; \rho)$, it can be proved that the phase PDF becomes

$$f_\phi(\phi) = \frac{\sqrt{1-p^2}}{2\pi(1-\rho\sin 2\phi)} \qquad -\pi < \phi \leq \pi \quad (6)$$

which reduces to the uniform distribution when $\rho = 0$, as expected. Whenever an explicit form cannot be found, numerical integration has to be performed. It is worth noticing that for $f_{\xi\eta}(\xi, \eta) \sim N(0, 0; \sigma, \sigma; 0)$, the random variables $|E_\alpha|$ and ϕ are Rayleigh and uniformly distributed, respectively. Therefore, the results of the theoretical PDFs in [1] assuming ideal reverberation conditions, are found as special cases of (5).

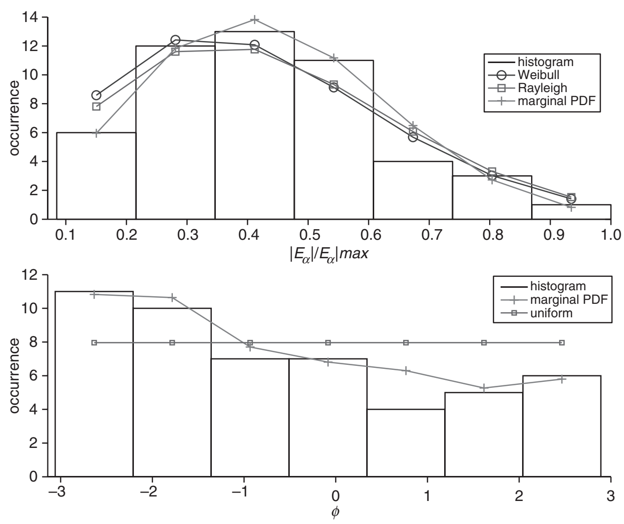

Fig. 1 *Experimental histograms, heuristic PDF and model-based PDFs of electromagnetic field magnitude (upper panel) and phase (bottom panel) at 1 GHz in Alenia RC*

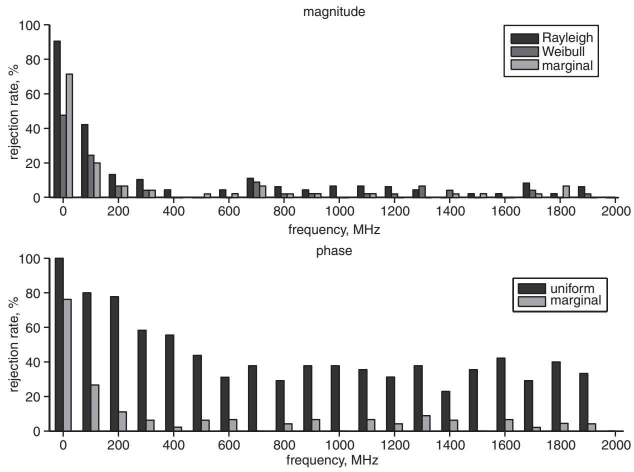

Fig. 2 *Rejection rate of different PDFs (Rayleigh, Weibull, marginal) for magnitude (upper panel) and phase (bottom panel) of electric field. (data refer to measurements in Alenia RC)*

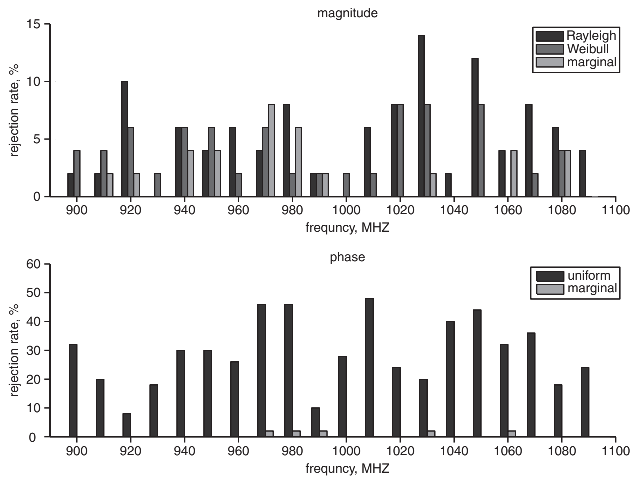

Fig. 3 *Rejection rate of different PDFs (Rayleigh, Weibull, marginal) for magnitude (upper panel) and phase (bottom panel) of electric field (data refer to measurements in IETR RC)*

Validation: Some experimental evidence is here provided to support the fact that the PDFs found in (5) give good reason for the deviations of the field statistics from ideal reverberation. Experiments were carried out in the Alenia Aeronautica RC (Torino, Italy), and in the IETR RC (Rennes, France) using a vector network analyser to measure the complex field components E_α. For each frequency, 50 uniformly spaced angular positions of the stirrer were set. Fig. 1 shows the experimental histograms and model-based PDFs of the normalised electric field magnitude and phase measured in the Alenia RC at 1 GHz. The normalised magnitude ($|E_\alpha|/|E_\alpha|_{max}$) histogram is compared with a Rayleigh PDF (statistical models [1]), a Weibull PDF (heuristically proposed in [2]) and the marginal PDF of (5). The phase (ϕ) is compared with the uniform PDF (statistical models [1]) and the marginal PDF of (5). For the case shown in Fig. 1, $f_{E_{\alpha r}E\alpha i}(E_{\alpha r}, E_{\alpha i}) \sim N(0.017, -0.162; 0.128; 0.075; -0.27)$. Qualitatively, the marginal PDF seems to fit the histogram of the field magnitude measurements better than the empirical Weibull and the theoretical Rayleigh PDF. Also, the marginal PDF for the phase seems to behave better than the uniform pdf. To perform a rigorous comparison, GoF tests were applied to the magnitude and phase distributions; the Anderson-Darling (AD) GoF test was chosen for its selection power and in coherence with the literature [2]. Figs. 2 and 3 represent the rejection rate of the AD test over a wide frequency span; the rejection rate is the ratio between the number of AD tests rejecting the null hypothesis H_0 (meaning that data follow the expected PDF) and the total number of tests performed within each frequency bin. From Figs. 2 and 3 we conclude that, for all measurements we analysed, the Rayleigh distribution for the magnitude is largely rejected, justifying the statement that real experimental measurements depart from ideal reverberation conditions. Measurements in the Alenia RC cover also the frequency band of the undermoded regime (low frequencies up to 250 MHz, approximately), where obviously none of the studied PDFs is able to represent the actual data samples. The Weibull distribution already has a good performance, but the proposed marginal distribution undoubtedly provides the best fit. This fact is even more evident if we look at the rejection rate of the phase, which is rarely considered.

Conclusions: Theoretically-justified statistical models for the magnitude and phase of electromagnetic fields in a RC have been presented and validated by means of experimental results conducted at two different RCs. This study extends the theoretical models (Rayleigh and uniform PDF for magnitude and phase, respectively) by means of marginal PDFs integrated from a complete bivariate joint Gaussian function, the parameters of which are related to imperfect reverberation conditions in real RCs. The found marginal PDFs are able to also embody the empirical PDFs.

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