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# Analysis of Diffraction by three semi-infinite planes using Wiener-Hopf method

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## Abstract

The diffraction of an incident plane wave on three equally spaced half-planes is formulated in terms of Wiener-Hopf (W-H) equations using an equivalent circuit model. The order of matrix factorization is reduced due to the symmetry of the problem. A new solution of this classic Wiener-Hopf problem is proposed and the formulation of the problem is presented for the general case of skew incident plane wave.

## 1. Formulation

The well-known in literature diffraction problem of three equally spaced semi-infinite PEC planes [1] is formulated in terms of Wiener-Hopf (W-H) equations. We consider the general case of an arbitrary plane wave incident (1) on the structure:

$$\begin{cases} \mathbf{E}_z^i = \mathbf{E}_0 e^{j\tau_o \rho \cos(\varphi - \varphi_o)} e^{-j\alpha_o z} \\ \mathbf{H}_z^i = \mathbf{H}_0 e^{j\tau_o \rho \cos(\varphi - \varphi_o)} e^{-j\alpha_o z} \end{cases} \quad (1)$$

where  $k$  is the free-space wave number,  $\tau_o = k \sin \beta$ ,  $\alpha_o = k \cos \beta$ , and  $\beta$  and  $\varphi_o$  are respectively the zenithal and azimuthal angles of the incident wave. Figure 1 illustrates the geometry of the problem for an E-polarized normal incident plane wave ( $\beta = \pi / 2$ ).

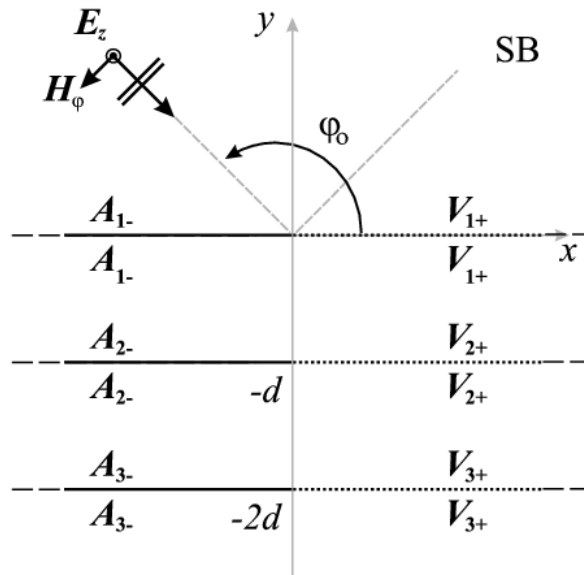


Figure 1: Diffraction on three equally spaced semi-infinite PEC planes: E-polarization. The figure reports the Wiener-Hopf unknowns.

In order to formulate the problem in the spectral domain we apply the Fourier transform to the transverse components of the electromagnetic fields (2):

$$\begin{aligned} \mathbf{V}(\eta, y) &= e^{j\alpha_0 z} \int_{-\infty}^{\infty} \hat{y} \times \mathbf{E}_t(x, y, z) e^{j\eta x} dx \\ \mathbf{I}(\eta, y) &= e^{j\alpha_0 z} \int_{-\infty}^{\infty} \mathbf{H}_t(x, y, z) e^{j\eta x} dx \end{aligned} \quad (2)$$

We define an equivalent circuit problem, see Figure 2, where the characteristic admittance of the free space is a matrix [2]:

$$\mathbf{Y}_c = \frac{Y_o}{k\sqrt{\tau_o^2 - \eta^2}} \begin{vmatrix} \tau_o^2 & -\eta \alpha_o \\ -\eta \alpha_o & k^2 - \eta^2 \end{vmatrix} \quad (3)$$

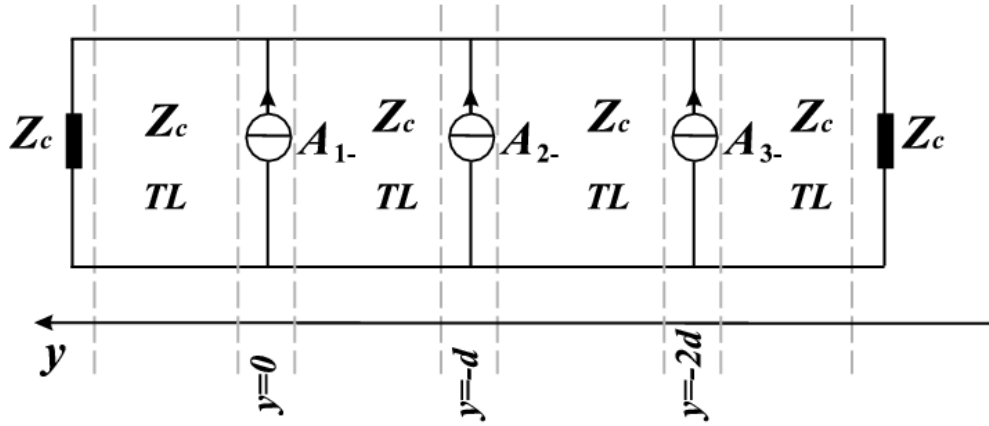


Figure 2: Equivalent Circuit model of the problem under study.

The equivalent circuit models the propagation inside the region defined by the half-planes (region 1, 2 and 3) using a  $\Pi$  model of matrix admittance:

$$\mathbf{Y}_1 = \mathbf{Y}_2 = j\mathbf{Y}_c \tan \frac{\tau d}{2}, \quad \mathbf{Y}_3 = -j\mathbf{Y}_c \frac{1}{\sin \tau d} \quad (4)$$

where  $\tau = \sqrt{\tau_o^2 - \eta^2}$ .

The circuit nodal analysis provide the three following equations

$$\begin{aligned} \text{node 1} &\Rightarrow (\mathbf{Y}_c + \mathbf{Y}_1 + \mathbf{Y}_3) \mathbf{V}_{1+} - \mathbf{Y}_3 \mathbf{V}_{2+} = \mathbf{A}_{1-} \\ \text{node 3} &\Rightarrow (\mathbf{Y}_c + \mathbf{Y}_1 + \mathbf{Y}_3) \mathbf{V}_{3+} - \mathbf{Y}_3 \mathbf{V}_{2+} = \mathbf{A}_{3-} \\ \text{node 2} &\Rightarrow 2(\mathbf{Y}_1 + \mathbf{Y}_3) \mathbf{V}_{2+} - \mathbf{Y}_3 \mathbf{V}_{1+} - \mathbf{Y}_3 \mathbf{V}_{3+} = \mathbf{A}_{2-} \end{aligned} \quad (5)$$

where  $\mathbf{V}_{1+}$ ,  $\mathbf{V}_{2+}$  and  $\mathbf{V}_{3+}$  and  $\hat{y} \times \mathbf{A}_{1-}$ ,  $\hat{y} \times \mathbf{A}_{2-}$  and  $\hat{y} \times \mathbf{A}_{3-}$  are respectively the Fourier transforms of the transverse electric field on the three apertures  $y = 0, -d, -2d$  and the total currents induced on the three half-planes. The previous equations constitute a W-H system of third order where the unknowns and the admittance are respectively vector and matrix of size 2, therefore the W-H problem is a scalar matrix problem of size 6.

The symmetry of the geometry allows to reduce the order of the vector W-H problem. In fact by summing the first two equations we get the following system of equations:

$$\begin{cases} (Y_c + Y_1 + Y_3)(V_{1+} + V_{3+}) - 2Y_3 V_{2+} = A_{1-} + A_{3-} \\ -Y_3(V_{1+} + V_{3+}) + 2(Y_1 + Y_3)V_{2+} = A_{2-} \end{cases} \quad (6)$$

Equation (6) is a homogeneous vector W-H problem of order 2 ( $\mathbf{G} \cdot \mathbf{V}_+ = \mathbf{A}_-$ ), where the plus unknowns are  $(V_{1+} + V_{3+})$  and  $2V_{2+}$ , the minus unknowns are  $(A_{1-} + A_{3-})$  and  $A_{2-}$  and the kernel takes the following form:

$$\mathbf{G} = \begin{vmatrix} (Y_c + Y_1 + Y_3) & -Y_3 \\ -Y_3 & (Y_1 + Y_3) \end{vmatrix} = Y_c \otimes \mathbf{H} \quad (7)$$

where the operator  $\otimes$  is the Kronecker matrix product and where

$$\mathbf{H} = \begin{vmatrix} 1 - j \cot(\tau d) & j \csc(\tau d) \\ j \csc(\tau d) & -j \cot(\tau d) \end{vmatrix} = \frac{2}{1 - e^{-2j\tau d}} \begin{vmatrix} 1 & -e^{-j\tau d} \\ -e^{-j\tau d} & \left( \frac{1 + e^{-j\tau d}}{2} \right) \end{vmatrix} \quad (8)$$

## 2. Solution

In order to obtain the solution of the Wiener-Hopf problem we need to factorize (7).

By applying the mixed-product property of the Kronecker product (9) to the plus and minus factorized matrices of  $Y_c$  and  $\mathbf{H}$  we obtain the factorization of  $\mathbf{G}$ , see (10).

$$A \cdot C \otimes B \cdot D = A \otimes B \cdot C \otimes D \quad (9)$$

$$\mathbf{G} = \mathbf{G}_- \cdot \mathbf{G}_+ = Y_{c-} \cdot Y_{c+} \otimes \mathbf{H}_- \cdot \mathbf{H}_+ = Y_{c-} \otimes \mathbf{H}_- \cdot Y_{c+} \otimes \mathbf{H}_+ \quad (10)$$

From the homogenous Wiener-Hopf equation (6) we obtain the normal form ( $G(\alpha)F_+(\alpha) = F_-(\alpha) + F_{o+}(\alpha)$ ) by extracting the source term related to the incident plane wave:

$$\mathbf{G} \cdot \mathbf{V}_+ = \mathbf{A}_-^d + \frac{\mathbf{R}_o}{\eta - \eta_o} \quad (11)$$

$\mathbf{R}_o$  is the source obtained as GO component from the incident field.

We remind the spectral quantities are functions of the spectral variable  $\eta$ . The solution takes the following form:

$$\begin{cases} V_+(\eta) = \mathbf{G}_+^{-1}(\eta) \cdot \mathbf{G}_+^{-1}(\eta_o) \cdot \frac{\mathbf{R}_o}{\eta - \eta_o} \\ \mathbf{A}_-(\eta) = \mathbf{G}_-(\eta) \cdot V_+(\eta) = \mathbf{G}_-(\eta) \cdot \mathbf{G}_+^{-1}(\eta_o) \cdot \frac{\mathbf{R}_o}{\eta - \eta_o} \end{cases} \quad (11)$$

We note that the matrix  $Y_c$  can be factorized in closed form. However no exact factorization has been obtained for matrices  $G$  and  $H$  up to now, therefore we can apply the approximate procedure presented in [3, 4] based on integral equations. Numerical results will be presented at the Conference.

### 3. Acknowledgments

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### 4. References

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