Direct data-driven filter design for automotive controlled suspensions

Direct Data-Driven Filter Design for Automotive Controlled Suspensions

F. Ruiz, M. Taragna, M. Milanese

Abstract— **This paper investigates the filter design problem for automotive controlled suspensions when no mathematical model of the system is available, but a set of initial experiments can be performed, where also the variable to be estimated is measured. The problem of designing suitable linear time-invariant filters is here investigated, focusing the attention on the estimation of the relative vertical speed between chassis and wheel, using the data provided by two accelerometers measuring the chassis and wheel accelerations. Disturbances and noises are supposed to be norm-bounded and optimality refers to the minimization of the induced norm from disturbances to the estimation error. A Set Membership formulation is followed and, for classes of filters with exponentially decaying impulse response, an approximating set is determined guaranteed to contain all the solutions to the optimal filtering problem. A method is proposed for designing almost-optimal filters with finite impulse response, whose worst-case estimation error is at most twice the lowest achievable one. Numerical simulations using standard "benchmark" road profiles illustrate the effectiveness of the proposed solutions.**

Keywords—**filter design from data, controlled suspensions, Set Membership estimation, automotive control.**

I. Introduction

The design of controlled suspension systems for road vehicles aims to enhance the vehicle performances with regard to comfort and road handling. Such performance requirements have received, in the last two decades, a growing interest witnessed by an intense research activity developed from both industrial and academic sides (see e.g. [1] and the references therein). Vehicles suspensions serve several conflicting purposes: in addition to counteracting the body forces resulting from cornering, acceleration or braking and changes in payload, suspensions must isolate the passenger compartment from road irregularities. For driving safety, a permanent contact between the tires and the road should be assured. Passive suspension systems built of springs and dampers have serious limitations. Their parameters have to be chosen to achieve a certain level of compromise between road holding, load carrying and comfort, under wide variety of road conditions. This motivated extensive researches on active and semiactive suspension systems.

In the case of semiactive suspensions, many different control algorithms have been proposed, such as the well established "two state" Sky-Hook (see e.g. [1]) and "clipped" strategies (see e.g. [2]), or Model Predictive Control techniques (see e.g. [3]). The computation of the control move requires to know, at each sampling time, the state of the suspension system. Assuming a rigid chassis, the most usual configuration of sensors for semiactive suspensions requires five accelerometers: three of them measure the vertical accelerations of the chassis (sprung mass) corners, while the other two measure the vertical accelerations of the front wheels (unsprung masses). Thus, an estimate of the system state has to be provided. Sprung and unsprung masses speeds can be obtained by suitable filtering actions of accelerometer signals. In particular, in order to remove DC offset effects, speeds are obtained by filtering the measured accelerations by means of suitable bandpass filters as described in [4]. Positions are then obtained via pseudo-integration of the estimated speed signals as proposed in [5]. A decoupled observer technique as described in [6] can be used to obtain an estimate of the tire deflection. However, the design of suitable observers for this specific application (see e.g. [6], [7]) is a still open research problem.

When the vehicle vertical dynamics are not completely known, a data-driven approach to the filtering problem can be followed, originating from an important practical consideration. In many applications, it is possible (if not mandatory) to perform a set of initial experiments, where also the variable to be estimated is measured. In the data-driven approach, the information provided by the initial set of experiments is fully exploited for the filter design and replaces the system description required by the model-based approaches. The usual way to deal with the measurements in the data-driven approach is to adopt a two-step procedure:

1) A model of the process is identified from prior information (physical laws,...), making use of measurements that include the variable to be estimated;
2) On the basis of the identified model, a filter is designed whose output is an estimate of the variable of interest.

Indeed, the filter in step 2 should be designed with some robust technique, that allows to account for the unavoidable discrepancies between the process and the model identified in step 1.

An alternative approach has been proposed in [8], [9], where the data needed in step 1 of the two-step procedure are used to directly design the filter, thus avoiding the model identification. The advantages of this direct design approach with respect to the two-step procedure have been put in evidence in [8] within an stochastic framework, while [9] deals with this direct design approach within a Set Membership framework. In [10], the behavior of filters designed from

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data, using both direct and two-step methodologies, were evaluated for a quarter-car suspension system following an stochastic approach.

In this paper, a new direct design approach is investigated, focusing the attention on the estimation of the relative vertical speed between chassis and wheel, using the data provided by two accelerometers measuring the chassis and wheel vertical accelerations. Assuming norm-bounded disturbances and following a Set Membership formulation, for classes of filters with exponentially decaying impulse response, an approximating set is determined that guarantees to contain all the solutions to the optimal filtering problem and a linear almost-optimal filter is designed, with guaranteed worst-case performances when applied to new data. Different simulations are carried out using standard "benchmark" road profiles employed in industrial tests, in order to evaluate the estimation quality and to verify the overall approach feasibility.

II. Direct data-driven filter design for uncertain LTI systems

Consider a discrete-time, linear, time-invariant, dynamic system S, initially at rest, described in state-space form as:

$$\begin{array}{rcl} x^{t+1} & = & Ax^t + Bw^t \\ y^t & = & C_1 x^t + Dw^t \\ z^t & = & C_2 x^t \end{array}$$

where, for a given time instant $t \in \mathbb{N}$: $x^t \in \mathbb{R}^n$ is the unknown system state, with $x^0 = 0$; $y^t \in \mathbb{R}^{n_y}$ is a known (measured) output; $w^t \in \mathbb{R}^{n_w}$ is an unknown one-sided input (i.e., $w^t = 0 \ \forall t < 0, w^0 \neq 0$), including process disturbances and measurement noises; $z^t \in \mathbb{R}$ is the variable to be estimated; A, B, C_1, C_2 and D are constant matrices of suitable dimensions.

In this paper, a deterministic description of disturbances and noises is adopted, considering that the input w is unknown but bounded in a given norm, and the aim is to design a filter that provides an estimate of z that minimizes the worst-case gain from w to the estimation error, measured in some norm. To this purpose, let us recall the definition of p-norm for a one-sided discrete-time signal $s = \{s^0, s^1, \ldots\}$, $s^t \in \mathbb{R}^{n_s}$ and $p \in \mathbb{N}$:

$$\|s\|_p \doteq \left[\sum_{t=0}^{\infty} \sum_{i=1}^{n_s} |s_i^t|^p\right]^{\frac{1}{p}}, \quad p < \infty$$

$$\|s\|_\infty \doteq \max_{t=0,..,\infty} \max_{i=1,..,n_s} |s_i^t|$$

and the (q,p)-induced norm of a linear operator T:

$$\|T\|_{q,p} = \sup_{\|s\|_p = 1} \|T(s)\|_q, \quad p, q \in \mathbb{N}$$

While the system S is supposed to be known in the literature on worst-case filtering, in most practical applications this is not the case and a model of S is typically identified from measurements y and $\tilde{z} = z + v$ collected during an initial experiment of finite length N, being v an additive noise on z. In the present paper, these initial data y and $\tilde{z}$ are used to directly design a filter that provides an estimate of z using new measurements y, with (possibly) minimal estimation error.

In the following, the system S is unknown, but its pair $[A, C_1]$ is supposed to be detectable. The further information on S, used for the direct filter design, is represented by the measured data, collected in the following column vectors:

$$\begin{array}{rcl} Y & = & [y^0;\ y^1;\ \ldots;\ y^{N-1}] \ \in \mathbb{R}^{N n_y} \\ \tilde{Z} & = & [\tilde{z}^0;\ \tilde{z}^1;\ \ldots;\ \tilde{z}^{N-1}] \ \in \mathbb{R}^{N} \end{array}$$

The disturbance column vector

$$W = [w^0;\ w^1;\ \ldots;\ w^{N-1}] \ \in \mathbb{R}^{N n_w}$$

and the measurement noise vector

$$V = [v^0;\ v^1;\ \ldots;\ v^{N-1}] \ \in \mathbb{R}^{N}$$

are unknown but with known bounds:

$$\|W\|_p \leq \delta$$

$$\|V\|_q \leq \epsilon$$

It has to be pointed out that, without loss of generality, $\|W\|_p \leq 1$ can be assumed if the matrices B and D of the dynamic system S are properly scaled. For this reason, $\delta = 1$ will be considered in the sequel of the paper.

In order to allow the user to suitably design the filter, let us consider the following $\mathcal{H}_\infty$ subset containing systems with bounded and exponentially decaying impulse response:

$$\mathcal{K}(L,\rho,\mu) = \{G \in \mathcal{H}_\infty : \|h_G^t\|_\infty \leq L \quad \forall t \in [0,\, \mu], \ \|h_G^t\|_\infty \leq L\rho^{t-\mu} \quad \forall t \geq \mu,\, t \in \mathbb{N}\}$$

where the triplet (L, ρ, μ) is a design parameter, with $L > 0$, $0 < \rho < 1$, $\mu \in \mathbb{N}$, $h_G = \{h_G^0, h_G^1, \ldots\}$ is the filter impulse response with $h_G^t \in \mathbb{R}^{n_y}$ and n_y is the dimension of the filter input. This set represents a filter design choice, allowing the user to require acceptable effects of the fast dynamics of the filter, occurring in the first instants of the impulse response, and an exponentially decaying bound on the slow dynamics due to the dominating poles.

Within the above context, the following filtering problem can be defined.

Optimal filtering problem: given scalars $L > 0$, $0 < \rho < 1$ and integers μ, p and q, find an optimal filter $G_o \in \mathcal{K}(L, \rho, \mu)$ such that the estimate $\hat{z}_{G_o} = G_o(y)$ achieves a finite gain

$$\gamma_o = \inf_{G_o \in \mathcal{K}(L,\rho,\mu)} \sup_{\|w\|_p = 1} \|z - \hat{z}_{G_o}\|_q$$

■

The set of all the solutions to this problem is given by:

$$\mathcal{G}_o(L,\rho,\mu) = \left\{ G \in \mathcal{K}(L,\rho,\mu) : \sup_{\|w\|_p=1} \|z - \hat{z}_G\|_q = \gamma_o \right\}$$

No results are available in literature about the construction of the set $\mathcal{G}_o(L, \rho, \mu)$, even when S is known. The purposes of the proposed methodology are to determine a tight approximation of this set considering finite experiment length and to select from it a filter with guaranteed worst-case performances, by suitably exploiting the information provided by the noisy dataset $(Y,\tilde{Z})$ and the noise bound ϵ.

A. Direct data-driven filtering

In this subsection, the optimal filtering problem is investigated and a tight approximation of the set $\mathcal{G}_o(L,\rho,\mu)$ is provided considering an initial experiment of finite length N.

Consider the following filter set:

Definition 1: Feasible Filter Set

$$FFS = \left\{ G \in \mathcal{K}(L,\rho,\mu) : \left\| \tilde{Z} - \hat{Z}_G \right\|_q \leq \gamma_o + \epsilon \right\}$$

where $\hat{Z}_G = [\hat{z}_G^0;\ \hat{z}_G^1;\ \ldots;\ \hat{z}_G^{N-1}] \in \mathbb{R}^N$ is the estimate vector provided by G when applied to data Y. ■

This set contains all the filters consistent with the bounds on disturbances and noises, the information coming from the dataset $(Y,\tilde{Z})$ and the design triplet (L,ρ,μ). Moreover, by choosing N sufficiently high, the FFS turns out to be the tightest set guaranteeing to contain $\mathcal{G}_o(L,\rho,\mu)$, as stated in the following lemma.

Lemma 1 ([11]): Let the dataset $(Y,\tilde{Z})$, the scalars $L,\ \rho,\ \epsilon$ and the integer μ be given. Then

$$\mathcal{G}_o(L,\rho,\mu) \subseteq FFS$$ ■

In the present data-driven approach, the worst-case gain γ_o is unknown since the system matrices are not known. In order to choose a suitable value of γ_o, an hypothesis validation problem is initially solved where one asks if, for given model class $\mathcal{K}(L,\rho,\mu)$ and finite data length N, the assumption on γ_o leads to a non-empty FFS. However, the only test that can be actually performed is if such an assumption is invalidated by the available data, checking if no filter consistent with the overall information exists. This leads to the following definition.

Definition 2: Let the dataset $(Y,\tilde{Z})$, the scalars $L, \rho,\ \epsilon$ and the integer μ be given. Prior assumption on γ_o is considered validated if $FFS \neq \emptyset$. ■

The fact that the prior assumption is consistent with the present dataset $(Y,\tilde{Z})$ does not exclude that it may be invalidated by future data. Indeed, values much lower than the true γ_o may be validated if the actual disturbance realization occurred during the initial experiment is far from the worst-case one. In the next subsection, a validation test is presented allowing one to determine an estimate of γ_o.

When a filter $F \in \mathcal{K}(L,\rho,\mu)$ has been obtained by means of a design algorithm, it is obviously of interest to evaluate, for any measured output y, the difference between the estimate $\hat{z}_F$ provided by F and the estimate $\hat{z}_G$ provided by an optimal filter $G \in \mathcal{G}_o(L,\rho,\mu)$. This can be measured by the term

$$\sup_{\|y\|_q=1} \|\hat{z}_G - \hat{z}_F\|_q = \|G-F\|_{q,q}$$

being $G-F$ the LTI dynamic system with input y and output $\hat{z}_G - \hat{z}_F$.

The induced norm $\|G-F\|_{q,q}$ depends on the particular G that is considered. However, it cannot be computed exactly, since on the basis of the available information it is only known from Lemma 1 that $G \in \mathcal{G}_o(L,\rho,\mu) \subseteq FFS$. For this reason, its tightest upper bound is given by:

$$\|G-F\|_{q,q} \leq \sup_{G\in\mathcal{G}_o(L,\rho,\mu)} \|G-F\|_{q,q} \leq \sup_{G\in FFS} \|G-F\|_{q,q}$$

thus leading to the following definition.

Definition 3: Worst-case filtering error of a given filter $F \in \mathcal{K}(L,\rho,\mu)$:

$$E(F) = \sup_{G\in FFS} \|G-F\|_{q,q}$$ ■

Then, the following optimality criterion can be defined.

Definition 4: A filter $F_o \in \mathcal{K}(L,\rho,\mu)$ is optimal if

$$E(F_o) = \inf_{F\in\mathcal{K}(L,\rho,\mu)} E(F) \doteq r(FFS)$$

where $r(FFS)$ is the so-called radius of information. ■

An optimal filter F_o is a Chebyshev center of FFS and it is the closest filter to any element of FFS. The radius of information is the smallest worst-case filtering error that can be guaranteed on the basis of the overall information and the design choice. It is well known in the Set Membership literature that optimal filters are hard to be determined. This motivated the interest in deriving algorithms having lower complexity, at the expense of some degradation in the accuracy of the designed filter. A good compromise is provided by the following family of filters.

Definition 5: A filter F_I is interpolatory if $F_I \in FFS$. ■

Any interpolatory filter is consistent with the overall information. An important well-known property of these filters is that $E(F_I) \leq 2\ r(FFS)$. Due to such a property, these filters are called 2-optimal or almost-optimal.

If the worst-case filtering error $E(F)$ can be computed, it is possible to evaluate a bound on the worst-case estimation error guaranteed by a filter F for any possible disturbance, according to the following result.

Lemma 2 ([11]): Let the system S be asymptotically stable. For any given filter $F \in \mathcal{K}(L,\rho,\mu)$, the estimate $\hat{z}_F = F(y)$ guarantees

$$\sup_{\|w\|_p=1} \|z-\hat{z}_F\|_q \leq \gamma_o + E(F)\,\|S_y\|_{q,p}$$

where S_y is the LTI dynamic subsystem of S such that $y = S_y(w)$. ■

The above lemma shows that the worst-case estimation error depends on $\|S_y\|_{q,p}$, that has to be somehow estimated from the available information.

B. Direct data-driven design of almost-optimal filters

As often happens in Set Membership approaches, optimal data-driven filter design appears to be difficult. In this subsection, an almost-optimal filter is designed as a suitable interpolatory one. This requires to look for filters in FFS, which is a difficult task because FFS is an infinite dimensional set. For this reason, FIR filters are used hereafter to approximate any filter $F \in \mathcal{K}(L,\rho,\mu)$ and the search in FFS is transformed into a search in a finite dimensional space.

Because of the well-known capabilities of FIRs to approximate asymptotically stable systems, these filters are looked for inside the following model class of finite fading memory systems:

$$\mathcal{K}^m(L,\rho,\mu) = \left\{ F \in \mathcal{K}(L,\rho,\mu) :\ h_F^t = 0,\ \forall t > m \right\}$$

where $L > 0$, $0 < \rho < 1$, $\mu \in \mathbb{N}$, $m \in \mathbb{N}$ such that $m \geq \mu$ and $h_F^t \in \mathbb{R}^{n_y}$. It is obvious that $\mathcal{K}^m(L,\rho,\mu) \subset \mathcal{K}(L,\rho,\mu)$, $\forall m \in \mathbb{N}$.

Given a system G, let G^m be its truncation, i.e., the FIR filter having the same first $m+1$ impulse response samples of G: $h_{G^m} = \left\{h_G^0, h_G^1, \ldots, h_G^m, 0, 0, \ldots\right\}$. Let $m \in \mathbb{N}$ be such that $m \geq \mu$, and $q = 2$ or $q = \infty$. For any given system $G \in \mathcal{K}(L,\rho,\mu)$, its truncation G^m guarantees

$$\|G - G^m\|_{q,q} \leq \eta_m$$

where $\eta_m = n_y \frac{L\rho^{m+1-\mu}}{1-\rho}$ is the so-called truncation error of G through G^m. The sequel of the paper will be focused on the cases $q = 2$ and $q = \infty$.

The problem of checking the prior assumption validity, i.e., to check if FFS is non-empty according to Definition 2, is now considered.

Theorem 1 ([11]): Let the dataset $(Y,\tilde{Z})$, the scalars L, ρ, ϵ, the integers μ and m be given. Let ν^* be the solution to the optimization problem:

$$\nu^* = \min_{F \in \mathcal{K}^m(L,\rho,\mu)} \left\|\tilde{Z} - T_y H_F\right\|_q \tag{1}$$

where $T_y H_F$ is the estimate of Z provided by a FIR filter F, with $H_F = [h_F^0; h_F^1; \ldots; h_F^m] \in \mathbb{R}^{(m+1)n_y}$ the column vector of the first $m+1$ coefficients of F and the matrix $T_y \in \mathbb{R}^{N \times (m+1)n_y}$ defined as follows:

- if $m < N$, then $T_y = T_y^m$ is the block-Toeplitz matrix formed by the samples y^t, $t = 0,\ 1, \ldots, N-1$.
- if $m \geq N$, then $T_y = [T_y^{N-1}\ 0_{N \times (m+1-N)n_y}]$.

Then:

i) A sufficient condition for prior assumption being validated is
$$\nu^* \leq \gamma_o + \epsilon$$

ii) A necessary condition for prior assumption being validated is
$$\nu^* \leq \gamma_o + \epsilon + \eta_m \|Y\|_q$$

iii) If $m \geq N-1$ is chosen, a necessary and sufficient condition for prior assumption being validated is
$$\nu^* \leq \gamma_o + \epsilon$$

■

Note that the gap between the necessary and sufficient conditions *i)* and *ii)* can be made as small as desired by increasing m and becomes negligible when $\eta_m \|Y\|_q \ll \nu^*$. Indeed, no gap exists just for $m = N-1$.

Theorem 1 can be used for choosing the model class $\mathcal{K}^m(L,\rho,\mu)$. In fact, if the gap between the conditions *i)* and *ii)* is negligible, the function

$$\nu^*(L,\rho,\mu) = \min_{F \in \mathcal{K}^m(L,\rho,\mu)} \left\|\tilde{Z} - T_y H_F\right\|_q$$

individuates, for a given value of $\mu \in \mathbb{N}$, a surface in the space $(L,\rho,\gamma_o+\epsilon)$ separating validated values of $(L,\rho,\gamma_o+\epsilon)$ from falsified ones. Clearly, the triplet $(L,\rho,\gamma_o+\epsilon)$ has to be chosen in the validated region with some "caution" (i.e., not too near the separation surface, as illustrated in the following section) and exploiting the information on the experimental setting. Useful information on L, ρ and μ values is provided by the impulse responses of filters designed by means of "untuned" algorithms which do not make use of prior assumptions, such as projection or standard prediction error algorithms. Moreover, the value of ϵ can be obtained by evaluating the instrumentation accuracy.

The aim of the design procedure here developed is to choose a filter in FFS that guarantees a small worst-case estimation error on future data. Let us consider the filter F^* given by the following algorithm:

$$F^* = \arg \min_{F \in \mathcal{K}^m(L,\rho,\mu)} \left\|\tilde{Z} - T_y H_F\right\|_q \tag{2}$$

that is, F^* is the model class element that achieves ν^* as solution to the optimization problem (1). The following theorem shows the properties of F^*.

Theorem 2 ([11]): If $\nu^* \leq \gamma_o + \epsilon$, the filter F^* is interpolatory (i.e., $F^* \in FFS$) and then almost-optimal. Moreover, if the system S is asymptotically stable, the estimate $\hat{z}_{F^*} = F^*(y)$ guarantees

$$\begin{aligned}\sup_{\|w\|_p=1} \|z - \hat{z}_{F^*}\|_q &\leq \gamma_o + E(F^*)\,\|S_y\|_{q,p} \\ &\leq \gamma_o + 2r(FFS)\,\|S_y\|_{q,p}\end{aligned}$$

■

Remark The algorithm (2) involves a convex optimization problem with linear constraints on the FIR filter coefficients due to the model class $\mathcal{K}^m(L,\rho,\mu)$. In the case $q = 2$, the cost function is quadratic and the problem can be efficiently solved using quadratic programming techniques. In the case $q = \infty$, the problem is a *minimax* and it can be solved efficiently using linear programming techniques (see, e.g., [12]). ■

III. EXPERIMENTAL SETUP

In this work, the direct data-driven filter design methodology is applied to the vertical dynamics of a road vehicle. The used model is a quarter-car semiactive suspension system, having the structure depicted in Figure 1. The chassis and the wheel are modeled as rigid bodies and static linear characteristics are assumed for the suspension.

The variables describing the system are the chassis vertical

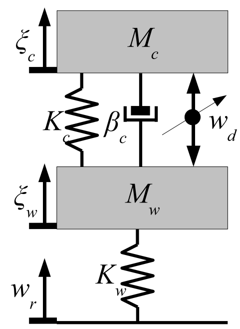

Fig. 1. Quarter-car suspension schematic

position ξ_c, the wheel vertical position ξ_w, the road profile w_r and the damping force w_d. The quarter-car model dynamics are given by the following set of differential equations:

$$\begin{aligned} M_c\ddot{\xi}_c &= -K_c(\xi_c - \xi_w) - \beta_c\left(\dot{\xi}_c - \dot{\xi}_w\right) + w_d \\ M_w\ddot{\xi}_w &= K_c(\xi_c - \xi_w) + \beta_c\left(\dot{\xi}_c - \dot{\xi}_w\right) - K_w(\xi_w - w_r) - w_d \end{aligned} \tag{3}$$

where M_c is the sprung mass (chassis), M_w is the unsprung mass (tire, wheel and other suspension components),K_c is the suspension spring constant, K_w is the tire stiffness coefficient and $\beta_c$is the suspension damping coefficient.The parameter values used in the simulations are $M_c = 432.82$ kg, $M_w = 40$ kg, $K_c = 17200$ N/m, $K_w = 200000$ N/m and $\beta_c = 3000$ Ns/m. These values have been taken from [3].
The damping force is assumed to be generated by a semi-active suspension system and can be written as $w_d(t) = -\beta(t)\left[\dot{\xi}_c(t) - \dot{\xi}_w(t)\right]$, where the damping coefficient $\beta(t)$ is variable. In this work, the widely used semiactive suspension control technique known as "On-Off Sky-Hook" control (see e.g. [13]) is used, where the damper is adjusted at maximum or minimum damping to provide the following force:

$$w_d = \begin{cases} \overline{w_d}\left(\dot{\xi}_c - \dot{\xi}_w\right) & \text{if } \dot{\xi}_c\left(\dot{\xi}_c - \dot{\xi}_w\right) \geq 0 \\ \underline{w_d}\left(\dot{\xi}_c - \dot{\xi}_w\right) & \text{if } \dot{\xi}_c\left(\dot{\xi}_c - \dot{\xi}_w\right) < 0 \end{cases}$$

The quarter-car model (3) has been implemented in Simulink, choosing a sample time $T_s = 1/512s$. Six experiments have been performed, all with a length of 13.7 seconds (7000 samples). Each experiment corresponds to the system response to a "benchmark" road profile, subject to zero initial conditions, as described in [3]. The considered road profiles are among those used for the on-road tuning of the CDC-Skyhook (continuous damping control) system. These road profiles allow to test different dynamic conditions of the vehicle, in terms of frequencies and amplitudes:

- Random (shortened as RR): random road,
- Motorway (shortened as MW): level road,
- Pavé (shortened as PV): road with small amplitude irregularities,
- English Track (shortened as ET): road with irregularly spaced sequences of bumps and holes,
- Short Back (shortened as SB): impulsive road,
- Drain Well (shortened as DW): negative impulsive road.

IV. Direct Data-Driven Filter Design for Active Suspensions

In this section, the direct data-driven filter design methodology is applied to estimate the vertical dynamics of a road vehicle. Assuming $p = q = 2$, an $\mathcal{H}_\infty$ filter is here designed to estimate the relative vertical speed between the chassis and the wheel, using the chassis and wheel vertical acceleration measurements.

For the filtering design problem the model is considered unknown with the road profile w_r and the damping force w_d as unknown but bounded inputs. The assumption about the damping force is motivated by the fact that in a semiactive suspension systems this force is no directly manipulable but is a consequence of an applied command, usually a current, and the relative speed between chassis and wheel (see e.g. [3]). Then, the damping force is not directly measurable and is strongly correlated with the system state, thus it can not be modeled as an stochastic process.

The accelerations $\ddot{\xi}_c$ and $\ddot{\xi}_w$ are the measured outputs, corrupted by additive noise. The relative vertical speed $\dot{\xi}_{cw} = (\dot{\xi}_c - \dot{\xi}_w)$ is the variable to be estimated, that can be measured only on an initial experiment where it is corrupted by an additive noise. This leads to the following variables of the data-driven filtering framework developed in the present paper:

$$y = \begin{bmatrix} \ddot{\xi}_c + w_1 \\ \ddot{\xi}_w + w_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \\ w_r \\ w_d \end{bmatrix}, \quad z = \dot{\xi}_{cw}, \quad \tilde{z} = \dot{\xi}_{cw} + v$$

The measurements are affected by noises w_1, w_2 and v, that are i.i.d. zero-mean normal sequences, such that the noise-to-signal ratios are equal to 2%. In particular, the noise vector V is bounded as $\|V\|_2 \leq \epsilon = 0.44$.

The complete dataset has been partitioned as follows:

- filter design dataset: data y and z corresponding to the experiment with a Pavé road profile;
- validation dataset: data y corresponding to the experiments with the other five road profiles. This set is used for testing the accuracy of the estimates on data not involved in the filter design.

The prior assumption validation on γ_o has been analyzed for different model classes $\mathcal{K}(L, \rho, \mu)$ with $p = q = 2$. In order to obtain a negligible gap between the necessary and the sufficient conditions of Theorem 1, the value of m has been chosen such that $\eta_m \|Y\|_2 \leq \nu^*/100$ is guaranteed.
The values of L, ρ, μ have been assessed according to the procedure discussed in the Section II. Standard prediction error algorithms have been applied to the filter design dataset, in order to obtain some information about the model class parameters. The impulse responses of filters of orders 2 to 7, provided by the MATLAB `pem` routine and plotted in Figure 2, suggest $L = 14 \cdot 10^{-3}$, $\rho = 0.92$ and $\mu = 5$ as possible initial choice.
Starting from these values, the surface $\nu^*(L, \rho, \mu)$ separating validated values of $(L, \rho, \gamma + \epsilon)$ from falsified ones has been computed using Theorem 1 for different values of μ. A reasonable choice of the model class parameters is $L = 0.007$, $\rho = 0.8$ and $\mu = 10$. Figure 3 shows some sections of the surface $\nu^*(L, \rho, \mu)$ for $\mu = 10$. Since the value $\nu^*(0.007, 0.8, 10) = 0.69$, the value 0.75 appears a reasonable choice for the term $\gamma + \epsilon$, that guarantees a bound $\gamma = 0.31$ on the worst-case estimation gain, since the bound $\epsilon = 0.44$ is assumed as prior information on the noise vector V. As a consequence of the selected values, $m = 54$ is the FIR order.

The interpolatory filter F^* has been obtained using the identification algorithm (2) on the filter design dataset. The quality of the estimates provided by F^* has been assessed by evaluating the actual ratios between the disturbances and the estimation errors on the different experiments of the validation dataset, as reported in Table I. These ratios are not higher than twice the estimated worst-case gain $\gamma = 0.31$ that would be obtained with a filter designed using the model information. For comparison, filters F_n^{ut}, of orders $n \in [2, \ldots, 7]$, have been obtained using "untuned" prediction error methods on the filter design dataset and their performances on the validation dataset are also reported.

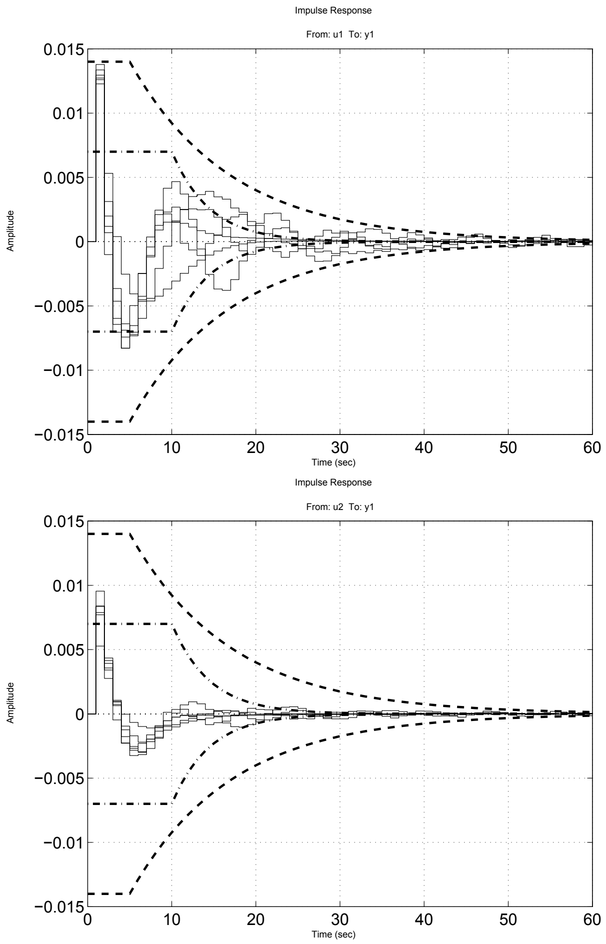

Fig. 2. Estimated filter impulse responses (solid lines) and possible bounds for the model class $\mathcal{K}(L,\rho,\mu)$ using: $L = 14\cdot 10^{-3}$, $\rho = 0.92$ and $\mu = 5$ (dashed lines); $L = 7\cdot 10^{-3}$, $\rho = 0.8$ and $\mu = 10$ (dash-dotted lines). Up, impulse response from measured chassis acceleration to relative speed. Down, impulse response from measured wheel acceleration to relative speed.

TABLE I

ESTIMATION ERROR TO DISTURBANCE RATIO ON THE VALIDATION DATASET

Filter	RR	MW	ET	SB	DW	mean
F^*	0.49	0.60	0.59	0.45	0.60	0.55
F_2^{ut}	3.35	1.86	2.16	3.52	2.75	2.73
F_3^{ut}	0.76	0.83	0.82	0.73	0.86	0.80
F_4^{ut}	1.07	0.96	0.98	1.14	1.10	1.05
F_5^{ut}	1.27	1.03	1.05	1.28	1.17	1.16
F_6^{ut}	0.80	0.83	0.82	0.75	0.85	0.81
F_7^{ut}	0.76	0.82	0.81	0.75	0.84	0.79

V. CONCLUSIONS

A direct filter design methodology has been presented for uncertain LTI dynamic systems, where the information provided by an initial set of experiments is exploited and the identification of a model of the data generating process is avoided. The filter design problem for automotive controlled suspensions has been considered, focusing the attention on the estimation of the relative vertical speed between chassis and wheel, using the data provided by two accelerometers measuring the chassis and wheel accelerations. A Set Membership formulation is followed and, for classes of filters with exponentially decaying impulse response, a method is proposed for designing almost-optimal filters with finite impulse response, whose worst-case estimation error is at most twice the lowest achievable one. Numerical simulations using standard "benchmark" road profiles illustrate the effectiveness of the proposed solutions.

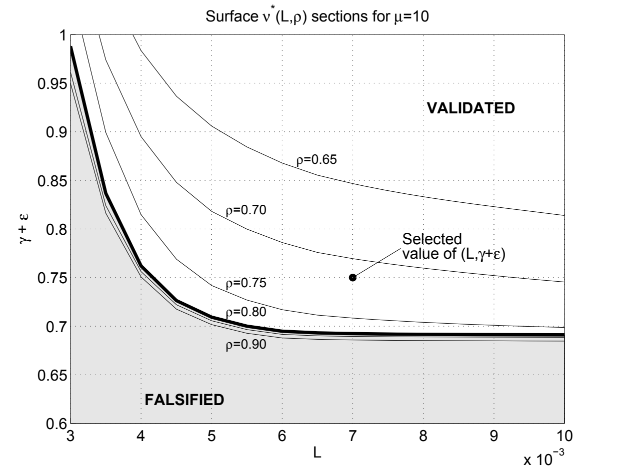

Fig. 3. Section of the surface $\nu^*(L,\rho,\mu)$ for $\mu = 10$, separating validated values of $(L,\rho,\gamma+\epsilon)$ from falsified ones

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