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A. Chinea, P. Triverio, S. Grivet-Talocia

Dip. Elettronica, Politecnico di Torino, C. Duca degli Abruzzi 24, 10129 Torino, Italy

Ph. +39 011 5644104, Fax +39 011 5644099 (e-mail stefano.grivet@polito.it)

Abstract: This paper introduces a new macromodeling algorithm for electrically long interconnects. This technique starts from frequency-domain scattering data and produces compact macromodels based on multiple delay extraction and rational approximations. The proposed model structure significantly reduces model complexity and simulation time with respect to standard purely rational macromodels.

1 Introduction

Use of macromodels for electrical interconnect structures is now a standard approach in system-level simulations for Signal Integrity assessment. Macromodels are compact mathematical expressions that are derived from frequency- or time-domain responses via suitable identification or approximation algorithms [2]. Rational approximations in the Laplace domain are commonly used, leading to a lumped model structure that is compatible with standard circuit solvers of the SPICE class.

Interconnects with a small to medium electrical size are ideally suited to lumped rational macromodeling. Conversely, when the electrical size of the interconnect increases due to its physical length, the number of poles in the above rational approximation may grow very large, implying numerical problems in model identification and model inefficiency in the simulation phase. This problem is due to the presence of propagation delays in the interconnect responses, which a lumped macromodeling scheme can only approximate with high-order partial fraction expansions. The relevance of these issues grows with the interconnect length and the maximum operating frequency.

In this work, we extend standard purely rational macromodeling techniques to the case of electrically long interconnects. We consider a model structure that explicitly includes propagation delay terms, mixed with suitable rational coefficients [4–6]. The resulting delayed rational approximation is computed from raw tabulated frequency data using a generalization of the well-known Vector Fitting (VF) algorithm [2]. Hence, we denote the algorithm as Delayed Vector Fitting (DVF) [9]. The results show that this approach is able to approximate highly complex behaviors with very few poles, thus achieving excellent efficiency in model simulation via standard circuit solvers.

2 Delayed-rational macromodeling

We consider an arbitrary electrically long interconnect characterized by an unknown transfer function $H(s)$. Without loss of generality, we will consider a single (scalar) response in this presentation, since the multiport case is easily handled by applying the proposed scheme to all responses independently. The starting point is a set of frequency samples of the structure, here denoted as

$$H_k = H(j\omega_k)\,, \quad k = 1, \ldots, \bar{k} \tag{1}$$

These samples are usually available from direct measurement or from numerical simulations. A typical scenario is a long interconnect made of a chain of different blocks, such as transmission-line segments, via fields, connectors, discontinuities, etc. Each block is first characterized independently using an appropriate 2D or 3D simulation tool. Then, the terminal responses (1) are computed via a frequency-domain solution of the interconnected chain.

For the class of structures under investigation, it can be shown [4–6] that the transfer function can be written in closed form as

$$H(s) = \sum_m Q_m(s) e^{-sT_m}\,, \tag{2}$$

where T_m represent signal propagation delays, and $Q_m(s)$ are suitable coefficients representing other effects such as attenuation and dispersion. The model structure that we consider matches (2), with some approximations. First, the number of delays is truncated to a (small) finite number $\bar{m}$. Second, a rational approximation is applied to each coefficient $Q_m(s)$. The resulting delayed rational model is written as

$$H(s) \simeq \sum_{m=1}^{\bar{m}} \frac{R_{m0} + \sum_{n=1}^{\bar{n}} \frac{R_{mn}}{s-a_n}}{r_0 + \sum_{n=1}^{\bar{n}} \frac{r_n}{s-a_n}}\, e^{-s\tau_m}\,, \tag{3}$$

where $\tau_m \simeq T_m$ are suitable estimates of the dominant propagation delays, and a_n is a prescribed set of "basis" poles. The case with $\bar{m} = 1$, $T_{\bar{m}} = 0$ reduces to the standard purely rational case, with the same model representation adopted in the well-known Vector Fitting [2] scheme. The case with $\bar{m} = 1$, $T_{\bar{m}} \neq 0$ reduces to the case studied in [3]. In [5], a model identification technique from time-domain responses is suggested, while in [6], the use of time-freqency decompositions is advocated for the separation of single-delay atoms in (2) in order to simplify the modeling process. Here, we introduce a simpler global macromodeling scheme that directly processes frequency-domain responses.

The first stage for the identification of a delayed macromodel (3) is the estimation of the dominant delay terms τ_m from the raw data. For this task, we adopt the algorithm described in [6], based on the so-called Gabor transform [8]. Essentially, the dominant delays are computed from the local maxima of the time-frequency energy distribution of the data. This procedure allows for the discrimination of individual single-delay atoms in (2), leading to good delays estimates. Details can be found in [6] and are not repeated here.

Once the set of dominant delays is known, the fundamental task is the estimation of the coefficients R_{mn} and r_n in (3), such that the deviation between model response and raw data is minimized at the available frequency points. Main objective is the minimization of the cumulative model to data approximation error

$$\mathcal{E}^2 = \sum_{k=1}^{\bar{k}} \left| H_k - \sum_{m=1}^{\bar{m}} \frac{R_{m0} + \sum_{n=1}^{\bar{n}} \frac{R_{mn}}{j\omega_k - a_n}}{r_0 + \sum_{n=1}^{\bar{n}} \frac{r_n}{j\omega_k - a_n}} e^{-j\omega_k \tau_m} \right|^2 \tag{4}$$

Since this expression is nonlinear in the unknowns R_{mn} and r_n, we linearize it using a generalization of the Sanathanan-Koerner (SK) iteration [1]. An outer iteration loop is devised. The i-th pass of this loop minimizes a modified error metric

$$\mathcal{E}_i^2 = \sum_{k=1}^{\bar{k}} \left| \frac{H_k \left(r_0^{(i)} + \sum_{n=1}^{\bar{n}} \frac{r_n^{(i)}}{j\omega_k - a_n^{(i-1)}} \right) - \sum_{m=1}^{\bar{m}} \left(R_{m0}^{(i)} + \sum_{n=1}^{\bar{n}} \frac{R_{mn}^{(i)}}{j\omega_k - a_n^{(i-1)}} e^{-j\omega_k \tau_m} \right)}{r_0^{(i-1)} + \sum_{n=1}^{\bar{n}} \frac{r_n^{(i-1)}}{j\omega_k - a_n^{(i-1)}}} , \right|^2 \tag{5}$$

which is obtained by multiplying (4) by the ratio between the (unknown) denominator at current iteration and the known denominator at previous iteration $i - 1$. In order to setup the iterations, the initial denominator coefficients are set to $r_0^{(0)} = 1$ and $r_{n>0}^{(0)} = 0$. It can be easily recongnized that (5) represents a standard (weighted) linear least squares problem, which can be solved via standard techniques. Iterations stop when all the coefficients are stabilized. Note also that the two error metrics (4) and (5) become equivalent at convergence.

In (5), also the "basis" poles are modified at each iteration. The poles $a_n^{(i)}$ are computed as the zeros of the rational function

$$r_0^{(i)} + \sum_{n=1}^{\bar{n}} \frac{r_n^{(i)}}{s - a_n^{(i-1)}} = \frac{r_0^{(i)} \prod_{n=1}^{\bar{n}} \left(s - a_n^{(i)} \right)}{\prod_{n=1}^{\bar{n}} \left(s - a_n^{(i-1)} \right)} \tag{6}$$

In principle, there is no guarantee that these poles remain stable. However, standard techniques are available for enforcing stability, using suitable pole flipping schemes [2, 7]. It is clear that the above iterative scheme is a direct generalization of the classical VF algorithm to the case of delayed rational functions, therefore we denote this algorihm as Delayed Vector Fitting (DVF). As for model passivity, the delayed-rational model representation (3) is suitable for application of the general techniques of [10, 11], which allow both passivity characterization and enforcement via iterative perturbation of the model coefficients.

3 Examples

The performance of the DVF algorithm is first demonstrated via application to an IBM GX bus structure. The raw specification is a set of frequency-dependent scattering parameters (courtesy of IBM), obtained by cascading several different simpler models of lumped blocks and frequency-dependent transmission lines and performing a simple frequency-domain solution of the interconnected system. Main task is to compute a global model from the terminal responses of the entire bus, without using any information on the internal structure.

For this structure, only one delay of 2.73 ns was necessary for the insertion loss $S_{1,2}$, while four (0, 0.56, 5.44, 6.71 ns) and three (0, 1.29, 5.87 ns) delays were required for return losses $S_{1,1}$ and $S_{2,2}$, respectively, in order to account for the reflections from the various discontinuities along the bus. The model identification results are summarized in Table 1. These results demonstrate that a good accuracy is obtained with a reduced number of model poles, since the rapid phase variations due to signal reflections from the discontinuities of the propagation path are automatically accounted for by the explicit delay terms in the macromodel expression. Figure 1 reports a comparison between the $S_{1,1}$ and $S_{1,2}$ model responses and the raw data, showing excellent correlation. Similar results were obtained for $S_{2,2}$, not shown.

Table 1: Results of delayed macromodel identification for the GX bus

Parameter	k	$\bar{m}$	$\bar{n}$	RMS error	MAX error
$S_{1,1}$	1000	4	20	3.09×10^{-3}	1.04×10^{-2}
$S_{1,2}$	1000	1	15	6.13×10^{-3}	1.65×10^{-2}
$S_{2,2}$	1000	3	25	5.34×10^{-3}	1.22×10^{-2}

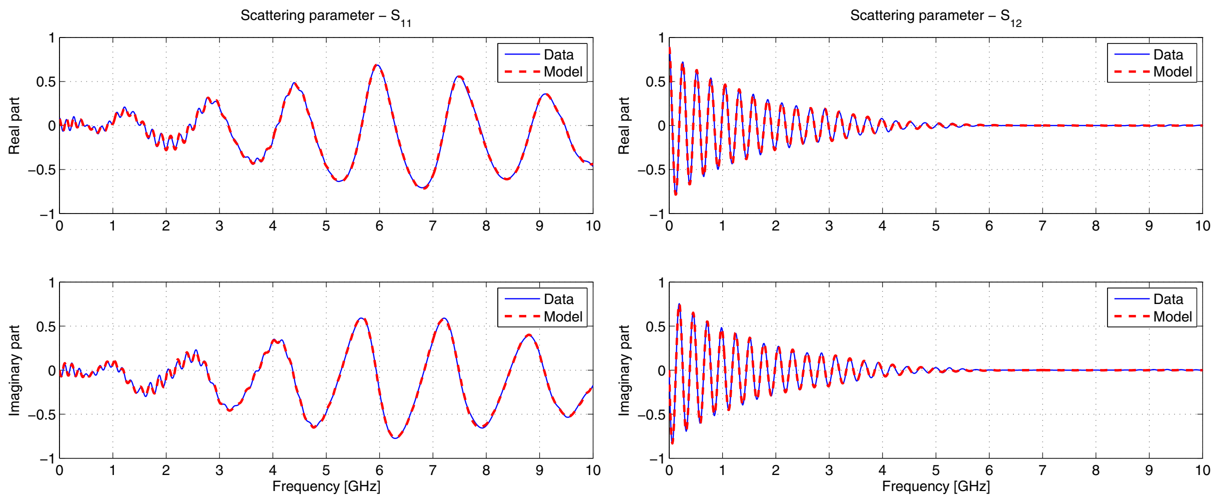

Figure 1: Comparison between model and data for S_{11} (left) and S_{12} (right) of the GX bus structure.

In order to compare these results with state-of-the-art modeling techniques, the classical Vector Fitting (VF) scheme was also applied, leading to a purely rational macromodel without delay terms. The number of poles that were required by VF for achieving the same level of accuracy of Table 1 was 80, 48, and 66 for $S_{1,1}$, $S_{1,2}$, and $S_{2,2}$, respectively. Therefore, the proposed DVF scheme allows for this case a reduction of the model complexity by a factor 3–4, depending on the response.

The advantages of delay extraction become evident when applied to interconnects with a significant electrical length. In order to show this, we consider a 50 cm long backplane interconnect, for which measured scattering parameters are available (courtesy of IBM) up to a maximum frequency of 40 GHz. Application of the DVF and VF schemes leads to the results of Fig. 2, where model and data insertion losses are compared over a restricted 5 GHz bandwidth (for readability of the plots). DVF only required 15 poles leading to a 4.11×10^{-3} RMS error, whereas VF required 145

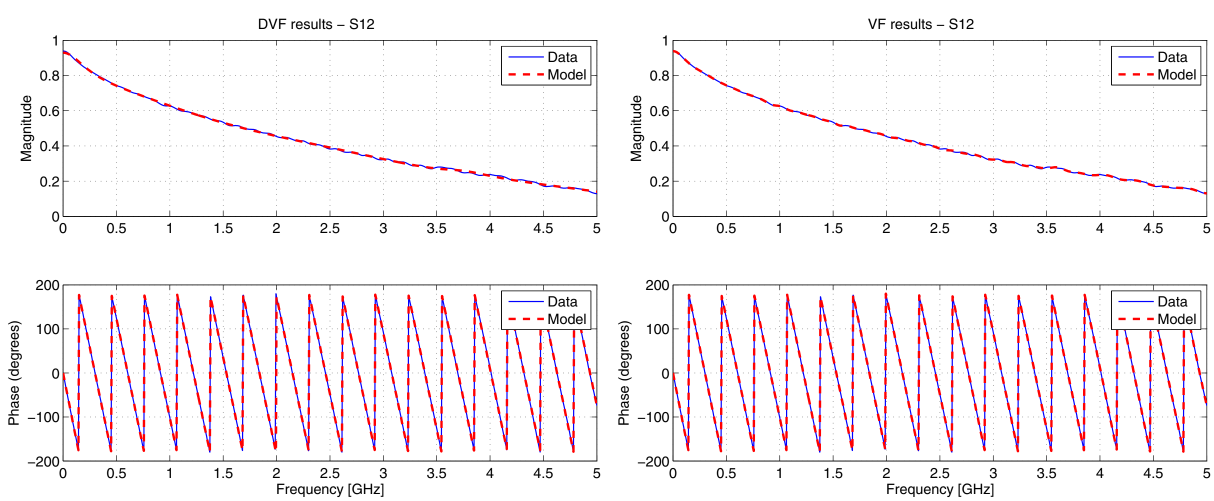

Figure 2: Comparison between S_{12} model and data responses for a 50-cm backplane interconnect. Results of DVF and VF algorithms are reported in the left and right panel, respectively.

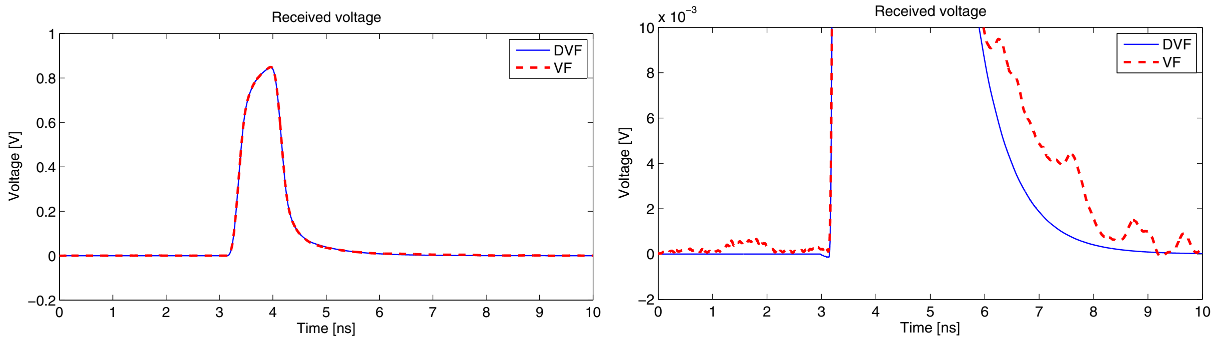

Figure 3: Results of a transient SPICE run on DVF and VF models of a 50 cm backplane interconnect. Right panel is a zoom of left panel.

poles with a RMS error of 8.40×10^{-3}. Model complexity is reduced by a factor of ten, and DVF still results more precise than VF.

We conclude this paper showing the advantages of delayed macromodeling in transient SPICE runs. The delayed macromodel (3) can be easily synthesized into a SPICE-compatible netlist using ideal (lossless) transmission line elements for the representation of the multiple propagation delays [9], while synthesis of the rational part is standard.

A SPICE netlist was generated for both VF and DVF models of the backplane interconnect. The two netlists were terminated by matched loads and excited on one end by a single pulse (1 ns width, 100 ps rise/fall times). The received far end voltage was computed using a transient SPICE run. The VF-based deck required 7.49 seconds, whereas the DVF-based deck only 0.81 seconds, with a speedup factor of 9.3. Figure 3 compares the transient voltage waveforms. On a large scale (left panel), the two waveforms appear identical. However, a zoom (right panel) reveals that the VF model presents, as expected, spurious oscillations before the propagation time has elapsed. These oscillations degrade the accuracy of the results for the entire extent of the simulation. We conclude that, for electrically long interconnects, the proposed DVF algorithm leads to more accurate and faster models with respect to the classical VF.

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