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Parametric Macromodeling of Multiport Networks from Tabulated Data

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Abstract: We propose a numerical technique to compute, from tabulated frequency data, compact macromodels parameterized by design variables, to be used for efficient optimization, Montecarlo analysis and design centering of complex systems. Important theoretical results on the stability of parameter-dependent models are also presented.

1 Introduction and motivation

A proper interconnect design is crucial, in modern digital systems, to achieve high performance and reliable operation. This task is made difficult because of the ever increasing clock frequencies and integration levels, that enhance the role played by electromagnetic phenomena. Only if these effects are accurately predicted in all system level simulations, the design can be carried out successfully, and Signal Integrity constraints satisfied.

Nowadays, the most common approach to derive interconnects models for mixed signal simulations is macromodeling. Based on the tabulated frequency response of the interconnect parts, obtained via full-wave simulations or direct measurements, macromodeling algorithms derive efficient and accurate models compatible with standard CAD and EDA programs.

This approach, very effective for simulation purposes, becomes time consuming for performance optimization, Montecarlo analysis and design centering procedures, i.e. when simulations must be repeated for different system parameters. Indeed, each time a design parameter changes, a new measurement or full-wave simulation must be done, and a new macromodel built "from scratch". To overcome this drawback, it is highly desirable to devise macromodels *parameterized* by the design variables, that reproduce the system behavior over the required range of configurations. In this way, the model identification and the necessary measurements/simulations have to be done only once, before the optimization or statistical analysis. Then, just a new evaluation of the parametric model coefficients is necessary to update the macromodel as required by the optimization routine. Motivated by this fact, we propose in this paper an efficient technique to create parametric models starting from tabulated frequency data.

The paper is organized as follows. In Section 2, a parametric expression suitable for electrical macromodels is introduced, followed in Section 3 by a numerical algorithm for its identification from tabulated data. Section 4 tackles the challenging problem of stability, providing the key mathematical tools to ensure this property over the whole parameters range, as well as an algorithm to check the stability of parametric models. Finally, in Section 5, an application example is reported to highlight the usefulness of these techniques. For clarity, we will focus on a scalar transfer function dependent on one parameter. However, all results hold for multiport systems dependent on several parameters.

2 Parameter-dependent macromodels

Problem statement. We consider a single-input single-output system dependent on a parameter λ, which varies[1] in the interval $[\underline{\lambda}, \bar{\lambda}]$. Its response is known at the frequency points $\omega_1, \ldots, \omega_{\bar{k}}$ and for several values $\lambda_1, \ldots, \lambda_{\bar{l}}$ of the parameter, spread in the interval $[\underline{\lambda}, \bar{\lambda}]$. We denote the available samples as

$$H_{kl} \qquad k = 1, \ldots, \bar{k} \quad l = 1, \ldots, \bar{l} \tag{1}$$

with H_{kl} being the value of the system response for $\omega = \omega_k$ and $\lambda = \lambda_l$. We aim at constructing a rational macromodel $H(s, \lambda)$ that well reproduces the original system response (1) uniformly in the frequency band $[\omega_1, \omega_{\bar{k}}]$ and in the parameter range $[\underline{\lambda}, \bar{\lambda}]$.

Model formulation. Our first step is to identify a good expression for the parametric model that can accurately follow the parameter variation with a small number of coefficients. Since macromodels are often written in poles-residues form, it would be tempting to parameterize these quantities. However, the variation of the poles induced by the parameter may be very odd and cumbersome to fit. We argue a better approach by recalling a general result on lumped networks. If we denote a generic element (either a capacitor, inductor or resistor) as λ we know [1] that

[1] As a general notation, we denote the minimum and maximum values of a quantity x as $\underline{x}$ and $\bar{x}$, respectively.

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$H(s,\lambda)$ is expressed as the ratio of two polynomials

$$H(s,\lambda) = \frac{\sum_{n=0}^{\bar{n}} Q_n(\lambda) s^n}{\sum_{n=0}^{\bar{n}} q_n(\lambda) s^n}, \tag{2}$$

where the coefficients $Q_n(\lambda)$ and $q_n(\lambda)$ are always linear functions of λ. Therefore, if (2) is used instead of the poles-residues form, a large class of systems can be described, *independently* of the network order and complexity, even with a simple linear parameterization. Although a similar result cannot be claimed for a generic parameter (e.g. geometrical dimensions, temperature), it is clear that the coefficients of the form (2) are preferred candidates for the parameterization scheme. We therefore adopt this form as the expression for our parametric model, with $Q_n(\lambda)$ and $q_n(\lambda)$ being polynomials of degree $\bar{m}$. The macromodel can thus be expressed as

$$H(s,\lambda) = \frac{\sum_n (Q_{n0} + Q_{n1}\lambda + \ldots Q_{n\bar{m}}\lambda^{\bar{m}}) s^n}{\sum_n (q_{n0} + q_{n1}\lambda + \ldots q_{n\bar{m}}\lambda^{\bar{m}}) s^n} = \frac{\sum_n \sum_m Q_{nm}\psi_{nm}(s,\lambda)}{\sum_n \sum_m q_{nm}\psi_{nm}(s,\lambda)} \qquad \psi_{nm}(s,\lambda) = \lambda^m s^n \tag{3}$$

where $\{\psi_{nm}(s,\lambda)\}$ denotes the polynomial basis used to represent the numerator and the denominator. Throughout this work, the range of variation for n and m is assumed to be, respectively, $n = 0,\ldots,\bar{n}$ and $m = 0,\ldots,\bar{m}$.

3 The parametric Sanathanan-Koerner iteration

We now develop a realible algorithm to estimate the coefficients Q_{nm} and q_{nm} of the model (3) in order to fit the given data (1), i.e., to numerically minimize the modeling error $\mathcal{E}$ defined as

$$\mathcal{E}^2 = \|H(j\omega_k,\lambda_l) - H_{kl}\|^2 = \sum_{k=1}^{\bar{k}} \sum_{l=1}^{\bar{l}} |H(j\omega_k,\lambda_l) - H_{kl}|^2 . \tag{4}$$

This task presents two main difficulties: *(i)* the strong ill-conditioning due to the high order powers of s in (3), and *(ii)* the nonlinear dependence of the error in the unknowns q_{nm} of the model denominator. In the identification of standard, non-parametric models, Vector Fitting (VF) [2] can be used to overcome these problems. Unfortunately, it cannot be easily extended to the parametric case, because a parameterization of the model poles would be required. However, VF has been recently recognized as a smart reformulation of the classical Sanathanan-Koerner iteration (SK) [3], which instead can be extended to the parametric case as follows.

First, we avoid the high powers of s by expressing the model (3) with a new basis $\phi_{nm}(s,\lambda)$

$$H(s,\lambda) = \frac{N(s,\lambda)}{D(s,\lambda)} = \frac{\sum_n \sum_m R_{nm}\phi_{nm}(s,\lambda)}{\sum_n \sum_m r_{nm}\phi_{nm}(s,\lambda)} \qquad \phi_{nm}(s,\lambda) = \begin{cases} \lambda^m & \text{for } n = 0 \\ \frac{\lambda^m}{s-a_n} & \text{for } n \neq 0 \end{cases} \tag{5}$$

The obtained expression is equivalent to (3), but much more suitable for numerical computations. The new basis $\phi_{nm}(s,\lambda)$ is essentially the combination of a polynomial basis $\{1,\lambda,\ldots,\lambda^{\bar{m}}\}$ for the parameter and the Vector Fitting basis $\{1,(s-a_1)^{-1},\ldots,(s-a_{\bar{n}})^{-1}\}$ for frequency. The quantities a_n will be referred as *basis poles* and are fixed a priori. Then, we exploit the SK iteration to minimize the modeling error

$$\mathcal{E}^2 = \left\| D^{-1}(j\omega_k,\lambda_l)\left[N(j\omega_k,\lambda_l) - D(j\omega_k,\lambda_l)H_{kl}\right]\right\|^2 , \tag{6}$$

which is non-linear in the unknowns of $D^{-1}(j\omega_k,\lambda_l)$. To circumvent this problem the SK method estimates $D(s,\lambda)$ iteratively by minimizing the linearized error

$$\mathcal{E}_i^2 = \left\| w_{kl}^{(i)}\left[N^{(i)}(j\omega_k,\lambda_l) - D^{(i)}(j\omega_k,\lambda_l)H_{kl}\right]\right\|^2 \qquad w_{kl}^{(i)} = [D^{(i-1)}(j\omega_k,\lambda_l)]^{-1}, \tag{7}$$

where the weight $w_{kl}^{(i)}$ plays the role of the factor $D^{-1}(j\omega_k,\lambda_l)$ in (6), and is obtained from the previous estimate of the denominator. As the iterations go, the weight makes (7) progressively close to the original error (6). Since this new minimization problem is linear in the coefficients of $N^{(i)}(j\omega_k,\lambda_l)$ and $D^{(i)}(j\omega_k,\lambda_l)$, it can be solved with the efficient algorithms for linear least-squares equations. At the first iteration ($i = 1$) the weight $w_{kl}^{(1)}$ is set to unity for all k and l values. To enhance the accuracy and convergence speed of the iteration, the basis poles a_n are chosen close to the poles of the real system by applying VF to one of the available responses.

Multiparameter case. We now briefly discuss how the proposed technique can be extended to the multiple parameters case. For clarity, let us consider two parameters λ and μ only. If they are component values in a lumped

network, the coefficients of (2) are known to be multiaffine[2] functions of λ and μ, i.e. in the form $q(\lambda,\mu) = q_{00} + q_{10}\lambda + q_{01}\mu + q_{11}\lambda\mu$. In the light of this property, a suitable basis for the generalization of (5) to the multivariate case is

$$\phi_{nm_1m_2}(s,\lambda,\mu) = \begin{cases} \lambda^{m_1}\mu^{m_2} & \text{for } n = 0 \\ \frac{\lambda^{m_1}\mu^{m_2}}{s-a_n} & \text{for } n \neq 0 \end{cases} \qquad \begin{array}{l} m_1 = 0,\ldots,\bar{m}_1 \\ m_2 = 0,\ldots,\bar{m}_2 \end{array} \tag{8}$$

4 Stability of parametric models

The parametric SK iteration developed in Section 3 does not guarantee the model stability. The main difficulty towards this goal is that, for parametric models, stability must be uniformly ensured over a continuous set $[\underline{\lambda},\bar{\lambda}]$. Computationally tractable conditions must therefore be devised. Without them, fitting techniques with stability constraints cannot be derived, and only a rough check of the model stability can be performed with a sweep of the parameter.

Let us consider the model (5): its poles are given by the zeros of $D(s,\lambda)$, since the basis poles a_n are common between the numerator and the denominator, and so cancel out. By direct substitution of the basis $\phi_{nm}(s,\lambda)$, $D(s,\lambda)$ can be written as

$$D(s,\lambda) = r_0(\lambda) + \boldsymbol{c}(\lambda)\,(s\mathbf{I} - \boldsymbol{\Delta})^{-1}\,\boldsymbol{b} \qquad \begin{array}{ll} r_n(\lambda) = \sum_m r_{nm}\lambda^m, & \boldsymbol{b} = [1,\ldots,1]^T \text{ (size } \bar{n}\times 1), \\ \boldsymbol{c}(\lambda) = [r_1(\lambda),\ldots,r_{\bar{n}}(\lambda)], & \boldsymbol{\Delta} = \operatorname{diag}(a_n) \end{array} \tag{9}$$

and its zeros computed as the eigenvalues of the matrix [4]

$$\mathbf{A}(\lambda) = [r_0(\lambda)\boldsymbol{\Delta} - \boldsymbol{b}\boldsymbol{c}(\lambda)]/r_0(\lambda)\,. \tag{10}$$

Therefore, (5) is stable if and only if $\mathbf{A}(\lambda)$ is a stable matrix, having all eigenvalues in the strict half plane $\mathbb{R}\mathrm{e}\{s\} < 0$. The stability of parametric matrices like (10) has been analyzed in the robust control literature. For the case of degree $\bar{m} \leq 1$, the following *necessary and sufficient* stability condition exists [5].

Theorem 1 *If $r_0(\lambda)$ is not vanishing[3] in $[\underline{\lambda},\bar{\lambda}]$, $\mathbf{A}(\lambda)$ is stable for all $\lambda \in [\underline{\lambda},\bar{\lambda}]$* if and only if *a symmetric and positive definite matrix $\mathbf{P}$ exists such that*

$$\mathbf{A}^T(\underline{\lambda})\mathbf{P} + \mathbf{P}\mathbf{A}(\underline{\lambda}) < 0\,, \qquad\qquad \mathbf{A}^T(\bar{\lambda})\mathbf{P} + \mathbf{P}\mathbf{A}(\bar{\lambda}) < 0\,. \tag{11}$$

The linear matrix inequalities (LMIs) in (11) are basically the Lyapunov equation for $\mathbf{A}(\lambda)$ at the extremes of the interval $[\underline{\lambda},\bar{\lambda}]$. Since LMIs can be efficiently solved with convex optimization techniques, this theorem directly allows to numerically ascertain the uniform stability of (5). The scope of this crucial result is however much broader, since it reduces the complexity of the parametric stability condition to a computationally tractable level. In this paper Theorem 1 has been restricted to one parameter. Its extension to the multiparameter case can be found in [5].

For models with degree higher than one ($\bar{m} > 1$) a similar result involving only the extremes of the parameter range cannot exist [6]. Anyway, several sufficient conditions for stability have been proposed, like those derived in [5]. The analysis of these results will be discussed in a future work.

5 Illustrative example

A two-port circuit composed by two microstrip lines and a resistor-capacitor pair is considered, depicted in Figure 1. This configuration idealizes a 5 cm interconnect link loaded by a device, here represented by the resistor-capacitor group. The nominal features of the components are: $R = 4\,\mathrm{k}\Omega$, $C = 0.2\,\mathrm{pF}$, microstrips width $w = 80\,\mu\mathrm{m}$, lengths $l_1 = 3\,\mathrm{cm}$ and $l_2 = 2\,\mathrm{cm}$, dielectric height $h = 0.3\,\mathrm{mm}$, dielectric permittivity $\epsilon_r = 4$. The proposed technique was applied to derive two parameterized models suitable to analyze the effects of some components parameters on the 2-ports link.

The microstrips width w was first taken as design parameter, variable between 60 and 130 μm. The 2×2 scattering matrix of the circuit was computed for 100 points from 10 MHz to 10 GHz, and for 15 different values of w between 60 and 130 μm, at steps of 5 μm. For the sake of simplicity the per-unit-length parameters of the lines were computed with standard Wheeler's formulas, while in a real design scenario a 2D electromagnetic simulator would have been used. Among the 15 responses, those for $w = 60, 70, \ldots, 130\,\mu\mathrm{m}$ were used to construct the parametric model, and the others to verify, a posteriori, the approximation quality for intermediate values of w. In the left panel of Figure 2, the magnitude of S_{21} for a parametric model of order 16 and degree $\bar{m} = 3$ is depicted, together with the exact system

[2] A function of λ and μ is multiaffine if it is linear in λ with μ constant, and viceversa.

[3] This hypothesis is verified if and only if $r_0(\underline{\lambda})$ and $r_0(\bar{\lambda})$ have the same sign [6].

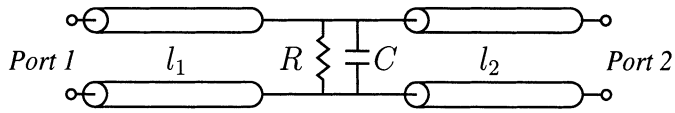

Figure 1: Test circuit analyzed in the example.

response. A very good agreement can be observed: in fact, the maximum modeling error among the S parameters was found to be 1.3×10^{-3} (absolute) and 0.8% (relative). Two SK iterations were required to identify the model, each one taking just 2 s. Since the model degree is higher than one, the model stability could not be ascertained with Theorem 1, so a parameter sweep was used. The model turned out to be stable.

The second test case considered R and C as parameters. Since they are values of lumped components, a model of degree one for both of them was expected to be sufficient, even for large parameters variations. The ranges for R and C were chosen as $[0.1, 10]$ kΩ and $[0.1, 0.9]$ pF, respectively. The circuit S matrix was computed for nine values of R and nine of C. Out of the 81 obtained datasets, only 9 corresponding to $R = 0.1, 1, 10\,\text{k}\Omega$ and $C = 0.1, 0.5, 0.9\,\text{pF}$ were used for the model identification. The right panel of Figure 2 shows the very good match of the model response with the data, for the real part of the S_{11} coefficient. The maximum absolute and relative errors between the model and the true system response are 8×10^{-5} and 0.1% respectively. The estimated model, of order 18 and unitary degree $\bar{m}$ in both R and C, was computed in only 11 s with two SK iterations. The application of the uniform stability test devised in Section 4 ascertained that the computed model is stable for all parameter values.

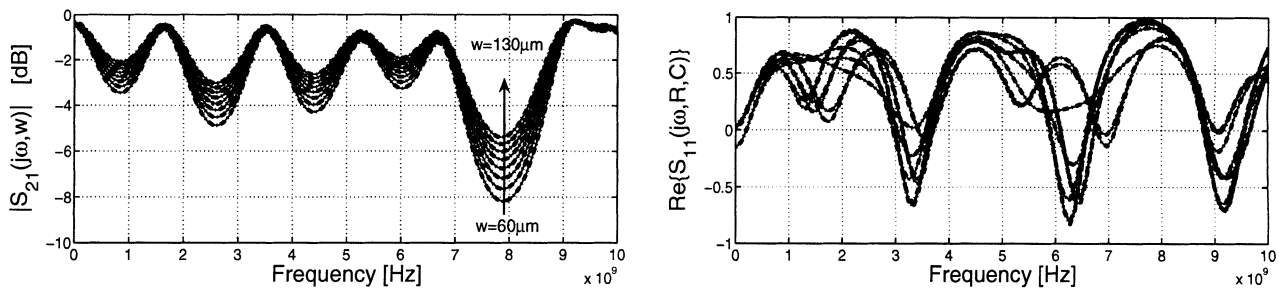

Figure 2: Left: Magnitude of the S_{21} response of the test circuit (solid line) and of the macromodel (dash-dot line) for different values of the microstrip width w. Right: Real part of the S_{11} coefficient of the true system (solid line) and of the model (dash-dot line) for different values of R and C (for clarity, only some validation responses are shown).

6 Conclusion

A numerical method has been proposed to compute parameterized macromodels of multiport systems characterized by tabulated frequency data. The algorithm provides, with low computational cost, accurate multiparameter models for fast optimizations and statistical analysis. While the model stability is not always guaranteed, the mathematical tools to address this tough point have been provided. The theoretical framework of this paper will constitute a good basis for future research on parameterized macromodeling.

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