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Modeling and testing of plate structures using self-sensing piezoelectric transducers

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ABSTRACT

Electromechanical modeling of a structure is used to position piezoelectric elements and to devise readout networks for their use as self-sensing transducers. The positioning is aimed to act selectively on given vibration modes and is carried on by means of simple spatial filtering techniques. The piezoelectric readout network is implemented using active components to avoid the coupling between mechanical and electrical states usually found with passive circuits. The proposed layout is well suited for both the testing and the active control of smart structures.

Keywords: Piezoelectric transducers, Self-sensing devices, Transducer positioning

1. INTRODUCTION

The modeling techniques usually proposed for the study of integrated electromechanical structures typically do not take into account the dynamics of the added electric circuits, which determine the effectiveness of the total system. Even if the literature provides many detailed procedures for the design of the subsystems of a smart structure, a general systematic procedure dealing with the complete design is seldom presented due to the variety of materials and to the differences of the physical phenomena involved.

Several approaches dealing with the design of smart structures have been proposed in literature. Some practical aspects can be considered fundamental in the actual choice of the layout of the plant. For example, the design of the transducers can be optimized towards the goal of the maximum interaction with the structure. Transducer positioning is also a critical issue and it has been theoretically and experimentally investigated in order to achieve a desired controllability and observability. The use of modal sensors and actuators is proposed by the reversibility of the piezoelectric interaction is exploited in the so called self-sensing operation by

In this paper some aspects of the design of a structure with self-sensing piezoelectric transducers for the active control of vibrations are discussed and experimentally verified in the case of a plate. The aim is to outline a simple methodology for a consistent design, modelling and testing. This methodology starts from the positioning of the transducers and their connection to common electrical nodes. It exploits then the electromechanical modelling of the piezoelectric interaction for the design of self-sensing readout circuits which are then tuned following an experimental procedure. The response of the tuned system can be used for feedback control, system monitoring and fault detection.

The positioning of the transducers has been performed with the objective to maximize the observability and controllability of the modes to be controlled. In the case of plate and beam structures the modal interaction with a surface bond piezoelectric is maximum if the piezoelectric is installed on a region of maximum curvature of the mode shape. The issue of the positioning has been coped with by maximizing the product of the curvatures of the modes which must be controlled. The electric connection between the transducers is devised to approximate spatial modal filters by means of a small number of rectangular piezoelectric elements. The effect of different connection patterns is investigated by means of a FEM based electromechanical model. Its main feature is to provide a zero pole cancellation of some of the modal states reducing the risk of spillover.

The self-sensing operation has been studied in the case the transducers are included in bridge readout circuits based on active current-to-voltage converters. The proposed readout network has proved to substantially simplify the balancing procedure in the presence of the unavoidable nonidealities.

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2. PLANT SETUP

The test rig of figure 1 is a rectangular plate with 400 x 450 mm sides and 1 mm thickness made of a AISI 2024 aluminum alloy. Each side of the plate is tightly sandwiched between two 50 x 25 mm aluminum constraining bars bolted to a steel basement. To approximate as closely as possible clamping conditions the constraining bars have a cross section shaped so as the contact with the plate is isostatic and as uniform as possible. The constraining bars are connected to a 20 x 500 x 5.50 mm, 45 kg steel basement, which behaves as seismic mass, supported by four polymeric silent-blocks which isolate the plate-basement system from the outside vibrations.

Six Physik Instrumente PIC155 piezoceramic rectangular plates are installed on the surface of the aluminum plate with the same polarization direction. They have 30 x 20 mm sides and 0.5 mm thickness, their surface is coated with silver electrodes. The piezoelectric constants of the PIC155 material are $d_{31} = 140 \times 10^{-12}$ m/V, $d_{33} = 310 \times 10^{-12}$ m/V, $d_{45} = 450 \times 10^{-12}$ m/V, its dielectric constants are $\varepsilon_{33}^T/\varepsilon_0 = 1700, \varepsilon_{11}^T/\varepsilon_0 = 1500$. The piezoelectric elements are surface bonded by means of a layer of AE-10 epoxy adhesive. A thin film of kapton is bonded between the piezoelectric and the aluminum plate to avoid the risk of short circuits.

Each transducer is connected to a couple of dedicated electric terminals to be driven individually. These terminals are left electrically floating to make possible the inclusion of the transducers in self-sensing bridge readout circuits.

Several piezoelectric elements can be connected to the same electrical node thus behaving as a single device that is referred to as “piezoelectric transducer” in the following. Figure 2 shows four of the piezoelectric elements connected as a single transducer to the self-sensing bridge readout network. The parallel of the capacitance $C_i$ and the tunable resistance $R_{G_i}$ of figure 2 constitutes a reference impedance connected to the output of the high voltage operational amplifier PA41 in parallel to the piezoelectric. Although generic impedances $Z_{PE_i}$ and $Z_{RF}$, are represented in figure 2, in practice simple resistances have been included in the circuit. The operational amplifiers are then configured as two active current-to-voltage transresistance converters. Their output is a measure of the current flowing through the piezoelectric and the reference impedance. Two FET input TL081 operational amplifiers have
been used to implement the current-to-voltage converters due to the small currents flowing through them during operation. The output of the bridge is obtained as the voltage difference between the outputs of its two arms using a INA118 instrumentation amplifier.

3. POSITIONING OF THE TRANSDUCERS

The location of the piezoelectric elements has been chosen as a trade-off solution between the objectives of sensing the first four flexural modes of the plate and that of limiting the number of piezoelectric elements.

A good electromechanical coupling between a piezoelectric transducer and a structural mode is reached when the piezoelectric is located on a point of maximum curvature of the mode shape. To reduce the number of piezoelectric elements, they are positioned where the product of the curvatures corresponding to a given set of mode shapes is maximum. The analysis of the modal curvatures of the plate has been performed by neglecting the influence of the piezoelectric elements. This is justified in the case the dimensions of the piezoelectrics are small relative to the plate and their number is limited. The deflections of the plate have been described in terms of the $w$ displacements along the $z$ axis of a reference frame $Oxyz$. The origin $O$ is set at the centre of the plate and the $x$, $y$ axes lie on the midplane and are parallel to the 450 mm and to the 400 mm sides respectively.

With reference to figure 1 the six piezoelectric elements can be distinguished in two subsets: set 1-2, made of elements 2 and 5, is addressed to the measurement and control of the two bending modes of lower frequency. Set 3-4, made of elements 1, 3, 4, 6, is addressed to the third and fourth modes.

The transducers of the set $i-j$, meant for the $i^{th}$ and $j^{th}$ modes, have been positioned in the region of the plate where the product $p_{ij}$ of the curvatures along the $x$ and $y$ directions is maximum:

$$ p_{ij} = w_{ix}, w_{iy} w_{jx}, w_{jy} $$

with $ij = 12, 34$

Figures 3 show the contour plot of the curvature index $p_{12}$. Piezoelectrics 2 and 5 of figure 1 have been located where the function $p_{12}$ is maximum. The same has been done in the case of transducers 1,3,4,6 for modes 3 and 4 as shown in figure 4.
Due to the symmetry of the structure, mode 1 has no nodal lines while mode 2 has one along the $y$ axis. As all the piezoelectric elements are polarized in the same direction $z$, the elongation or the contraction of each depends on the sign of the applied voltage. Mode 1 is then observable and controllable by the transducer obtained by acting in phase on piezoelectrics 2 and 5. By converse, mode 2 is observed and controlled by acting 180 degrees out of phase on piezoelectrics 2 and 5. Since mode 3 has one nodal line along the $x$ axis, the transducer controlling it must act in phase on piezoelectrics 1 and 6 and out of phase on 3 and 4. Mode 4 has two nodal lines along $x$ and $y$ axes, the related transducer acts in phase on piezoelectrics 1 and 4 and out of phase on piezoelectrics 3 and 6. Due to the symmetry of the system, the above principle can be implemented as in figure 2 by connecting the piezoelectric elements in parallel and choosing their polarity.

The positioning of the transducers and the connection patterns here adopted are an example of the spatial filtering techniques described in\textsuperscript{19,18,23} they make possible to cope with modes having the same frequency but different mode shapes as in the case of the second and third flexural modes of a square plate. The same methodology can be adopted in a more general case by acting on arrays of discrete transducers installed on the structure by means of weight matrices.

4. PLATE MODEL

The structure has been modeled by means of the finite element method using an electromechanical plate element. The geometry of the element is rectangular with four nodes, one at each of its corners, the mechanical degrees of freedom are the two rotations about the $x, y$ directions, and the $w$ displacement of each node. The thickness is assumed to be constant through the element and a plane stress regime is supposed. The transducers are assumed to be perfectly bonded and the shear deformations within the thin adhesive layer are not taken into account. A Kirchhoff plate formulation has been introduced to describe the displacement field within the element, neglecting the shear effect on the flexural behavior of the plate. The electromechanical coupling between the piezoelectric and the plate is described in terms of an additional node whose degree of freedom is the electric potential across the piezoelectric part. A Lagrangian approach has been followed to determine the element matrices starting from the definition of a Lagrangian function which includes electric and magnetic energies and coenergies. Equilibrium and compatibility conditions at the mechanical and the electrical nodes are then adopted to obtain the mass, stiffness and coupling matrices of the structure.

Figure 3. Contour plot of the curvature index $p_{12}$ associated to modes 1 and 2, and location of piezoelectrics 2 and 5.
The plate of figure 1 has been modelled using a grid of equally spaced rectangular elements with 10 elements along the 400 mm side and 15 elements along the 450 mm side. This results in 176 nodes which corresponds to 278 degrees of freedom.

5. BRIDGE CIRCUIT DESIGN

Due to the reversibility of the piezoelectric effect, in principle the self-sensing operation of a piezoelectric transducer can be obtained by the simultaneous measure of the current and the voltage at its electric terminals.

A better observability of the mechanical states is obtained by means of bridge readout networks. Their purpose is to eliminate from the output signal the contribution due to the current flowing through the piezoelectric because of its capacitive nature. Usually the piezoelectric transducer in series to a passive shunt impedance forms the measure arm of the bridge, the reference arm is the series of a lossy capacitance, and a shunt impedance similar to that included in the measure arm. The lossy capacitance should replicate the electric behaviour of the piezoelectric except for what the electromechanical interaction is concerned. In the case the electric time constants of the two arms of the bridge are made equal, the bridge is balanced and an output signal just depending on the mechanical states is obtained as output. The slightly different lossy behaviour of the piezoelectric and of the reference impedance constitutes the main difficulty to obtain in practice the results expected in the case of perfect balancing. The key feature to approximate these results is that the bridge must be balanced both for what the ideal and the lossy behaviour of the components is concerned. This leads to the elaborate balancing procedures described in.

In the present study active readout circuits are implemented in the measure and in the reference arms of the bridge instead of the passive shunt impedances. The aim is to decouple the electrical and mechanical dynamics which occur in the measure arm when passive shunts are used, and to increase the achievable bandwidth of the readout network.

To avoid excessive complexities in the analytical expressions in the following just one piezoelectric transducer is assumed to be connected to the readout circuit as shown in figure 2. This assumption does not detract too much from the conclusions, which can be extended in the general case of multiple excitation and readout networks.

Assuming that the operational amplifiers of figure 2 are ideal components, the voltage at the inverting input of the operational amplifier is virtually grounded. The piezoelectric and the reference capacitance are then subject
to the same voltage $V_{in}$. The Lagrangian variables describing the configuration of the system are the vector $X$ of the displacements and rotations within the structure and the charges $q_p$, $q_r$ on the electrodes of the piezoelectric transducer and of the reference capacitance.

The dynamic equation of the structure including the piezoelectric is expressed in terms of the state variables as:

$$M \ddot{X} + C \dot{X} + K_{oc}X + \frac{\Theta}{C_p}q_p = F$$

$$\frac{q_p}{C_p} + \frac{\Theta^T}{C_p}X = V_{in}$$

(1)

$M$ and $C$ are the mass and damping matrices. Matrix $K_{oc}$ is the stiffness matrix of the structure when the piezoelectric is open circuited ($q_p = 0$). Indicating with $I_p$ the current flowing through it, the impedance $Z_p$ of the transducer is:

$$Z_p = \frac{V_{in}}{I_p} = \frac{1}{sC_p} \left[ 1 - \frac{1}{C_p} \Theta^T (M s^2 + C s + K_{oc})^{-1} \Theta \right]$$

(2)

A resistance $R_{lp}$ in parallel to an ideal piezoelectric is used to model the losses within its dielectric while the hysteretic behaviour is accounted for as a complex capacitance $C_p = |C_p| (1 + i \eta)$. The impedance $Z_{lp}$ of the lossy piezoelectric becomes then:

$$Z_{lp} = \frac{Z_p R_{lp}}{Z_p + R_{lp}}$$

(3)

With reference to figure 2 the input-output transfer function $V_{outp}/V_{in}$ from the measure arm of the bridge is:

$$\frac{V_{outp}}{V_{in}} = -\frac{Z_{Fp}}{Z_{lp}} = -Z_{Fp} \left( \frac{C_p s}{1 - \Theta^T (M s^2 + C s + K_{oc})^{-1} \Theta/C_p} + \frac{1}{R_{lp}} \right)$$

(4)

while the transfer function $V_{outr}/V_{in}$ of the reference arm:

$$\frac{V_{outr}}{V_{in}} = -\frac{Z_{Fr}}{Z_{lr}} = -Z_{Fr} \left( C_r s + \frac{1}{R_{lr}} \right)$$

(5)

the loss resistance $R_{lr}$ includes the contributions of the tuning resistance connected to the reference capacitor and that of the dielectric losses occurring within it. The transfer functions of the measure and of the reference arm have the same form, except for the electromechanical coupling.

In the limit case of purely resistive feedback impedances: $Z_{Fp} = R_{Fp}$, $Z_{Fr} = R_{Fr}$, the transfer functions of the two arms of the bridge are the sum of a derivative contribution and of a proportional contribution due to the resistive losses. This gives way to a transfer function with a low frequency real and negative zero. In the other limit case of a purely capacitive feedback impedance ($Z_{Fp} = 1/(sC_{Fp})$, $Z_{Fr} = 1/(sC_{Fr})$) the transfer functions of equations 4 and 5 are the sum of a proportional and of an integrative contribution. This gives way to a transfer function with a pole at zero frequency followed by a high frequency zero.

In the case of non ideal operational amplifiers, the open loop low frequency pole of the amplifier can lead to instability of both arms of the readout network. This is usually avoided by including a compensating network in parallel to the feedback impedances. Taking this into account, the transfer functions of equations 4 and 5 become approximations of the actual transfer functions which can be accepted for low frequencies.

The output voltage from the bridge is the difference between the voltages output from its two arms $V_{out} = V_{outp} - V_{outr}$. The expression of the input-output transfer function is then:

$$\frac{V_{out}}{V_{in}} = \left( \frac{Z_{Fp} C_p s}{1 - \Theta^T (M s^2 + C s + K_{oc})^{-1} \Theta/C_p} - Z_{Fr} C_r s \right) + \frac{Z_{Fp}}{R_{lp}} - \frac{Z_{Fp}}{R_{lr}}$$

(6)

The first term of the output accounts for the electrical and the mechanical dynamics, it is due to the lossless impedances connected to the input of the operational amplifiers. The second term is due to the resistive losses affecting the piezoelectric and the reference capacitor. This last contribution determines a direct input output link.
The poles of the system are given as the solution of the determinant equation:

$$C_p \det (Ms^2 + Cs + K_0) - \Theta^T \text{adj} (Ms^2 + Cs + K_0)^{-1} \Theta = 0$$  \hspace{1cm} (10)$$

The poles given by equation 10 are the eigenvalues of a purely mechanical system whose stiffness $K_{sc}$ is the stiffness of the structure with the piezoelectric short circuited:

$$\det (Ms^2 + Cs + K_0) = 0$$  \hspace{1cm} (11)$$

Open and short circuit stiffnesses are related as follows:

$$K_0 = K_{sc} + \frac{\Theta \Theta^T}{C_p}$$  \hspace{1cm} (12)$$

From equations 10 and 11 it follows that the poles of the bridge with active readout circuits are decoupled from the electrical dynamic. This is the main feature distinguishing circuits with active readouts relative to circuits based on passive shunts.

In the case of the structure of figure 1 the short circuit natural frequencies are reported in table 1 as evaluated from the finite elements model as the solutions of equation 11 and from experimental modal analysis.

6. BRIDGE BALANCING

The experimental tests on the system of figure 2 have been performed in the case of purely resistive feedback impedances $Z_{FP} = R_{FP}$ and $Z_{Fr} = R_{Fr}$. In this case the balancing condition of equation 7 lets the derivative gains of the measure and of the reference arms of the bridge equal.

$$R_{FP}C_p = R_{Fr}C_r$$

The connection pattern of the piezoelectric elements shown in figure 2 is such that they act as a single transducer with a capacitance $C_p = 50.4$ nF. The reference capacitance $C_r = 47$ nF has been implemented with a low loss plastic capacitor set to the closest available value which approximates the nominal value of $C_p$.

The balancing procedure was based on the experimental evidence that the loss resistor $R_{fr}$ mainly influences the sharpness of the phase shift associated to each couple of zeros of the $V_{in}/V_{out}$ transfer function while feedback resistor...
As the connection scheme of figure 2 is meant to deal with the fourth mode, the analysis has been addressed to the frequency range between 100 and 350 Hz, which includes the fourth and the fifth modes.

In a first step the loss resistance \( R_{ir} \) has been tuned so that the antiresonance associated to the fourth mode became evident, with a sharp 180 degrees phase shift. In a second step the reference resistance \( R_{Fr} \) was tuned so that the frequency of the zeros was shifted to the frequency predicted by the lossless theoretical model. Finally a further fine tuning of \( R_{fr} \) was done to sharpen the phase shift of the zeros.

The curve showing a deep antiresonance in figure 5 is the result of the tuning procedure with the active readout network. The other curve was obtained with a passive readout network implemented using the parallel of the same capacitor \( C_r \) and the same loss resistor \( R_{ir} \), as reference impedance. The shunt resistors connected in series to the piezoelectric and to the reference impedance were physically the same resistance \( R_{Fp} \) and the trimmer \( R_{Fr} \) which were previously tuned on the active circuits. Even if this procedure should have given the same result, it actually did not. This is probably due to the coupling between electrical and mechanical dynamics occurring within the measure arm of the bridge in the case of passive shunts.

Results similar to those obtained in the case of the bridge with active readout circuits were obtained also in the case of passive resistive shunts but with a much more elaborate tuning procedure, as described in 26.

It is worthwhile to note that once the bridge is balanced, the unknown parameters of the piezoelectric, i.e. \( C_p \) and \( R_{ip} \) can be identified using equations 7, 8. Taking the implemented values of \( C_r = 47 \, \text{nF}, R_{ir} = 2.2 \, \text{M\Omega}, R_{Fr} = 10.5 \, \text{k\Omega}, R_{Fp} = 10 \, \text{k\Omega} \) into account, it follows: \( C_p = 49 \, \text{nF} \) and \( R_{ip} = 2.1 \, \text{M\Omega} \).

### 7. CONCLUSIONS

A procedure for the design of structure with surface bonded piezoelectric transducers has been developed with the aim of sensing and acting on the structural modes. Spatial filtering concepts are adopted for the positioning of the
piezoelectric elements and their connection to the electrical circuitry. The procedure is effective to cope with closely spaced modal frequencies and allows to limit the number of piezoelectric devices.

Bridge networks based on the use of active readout circuits have been adopted to obtain the self-sensing operation of the transducers. They are shown to minimize the coupling occurring in the reference arm of the bridge between electrical and mechanical dynamic behaviour.

The tuning of the bridge network is shown experimentally to be simplified. The frequency of the zeros associated by colocation to a given mode and the sharpness of the related phase shift can be tuned individually. This suggests the possibility of feasible self-tuning procedures.

REFERENCES