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(Article begins on next page)

# Domain-wall dynamics and Barkhausen effect in metallic ferromagnetic materials. I. Theory

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The Barkhausen effect (BE) in metallic ferromagnetic systems is theoretically investigated by a Langevin description of the stochastic motion of a domain wall in a randomly perturbed medium. BE statistical properties are calculated from approximate analytical solutions of the Fokker-Planck equation associated with the Langevin model, and from computer simulations of domain-wall motion. It is predicted that the amplitude probability distribution  $P_a(\Phi)$  of the  $B$  flux rate  $\Phi$  should obey the equation  $P_a(\Phi) \propto \Phi^{\tilde{c}-1} \exp(-\tilde{c}\Phi/\langle\Phi\rangle)$ , with  $\tilde{c} > 0$ . This result implies scaling properties in the intermittent behavior of BE at low magnetization rates, which are described in terms of a fractal structure of fractal dimension  $D < 1$ . Analytical expressions for the  $B$  power spectrum are also derived. Finally, the extension of the theory to the case where many domain walls participate in the magnetization process is discussed.

## I. INTRODUCTION

Although the Barkhausen effect<sup>1-4</sup> (BE) plays, in principle, a fundamental role in the phenomenology of ferromagnetic hysteresis, since it reveals the microscopic behavior of Weiss domains, a comprehensive quantitative treatment of its connection with domain-wall (DW) dynamics is yet to be developed. Some general, well-known facts, like the intermittent character of BE when a ferromagnetic specimen is magnetized at a slow rate or the strong dependence of BE on demagnetizing fields, are usually justified by intuitive considerations. There is an old tradition of statistical models,<sup>1,2</sup> describing the intermittent character of BE in terms of clustering of elementary DW jumps, but only rough descriptions of the dynamic mechanism responsible for this clustering effect have been given. Detailed BE studies have been performed in systems containing a single active DW,<sup>5-7</sup> but the connection between these special cases and the general phenomenology of BE has remained essentially unexplored.

In this paper, we try to fill up this interpretative gap between BE phenomenology and DW dynamics by studying, in the frame of the theory of stationary Markov processes, the problem of a DW moving in a randomly perturbed medium. Basic results on this subject have been known for a long time. Williams, Shockley, and Kittel<sup>8</sup> showed in 1950 that DW motion in metallic systems is governed by eddy current damping, which gives rise to a linear dependence of DW velocity  $v$  on magnetic field  $H$  of the form

$$kv = H - H_c, \quad (1)$$

where  $H_c$  is the coercive field experienced by the moving DW and the constant  $k$  can be calculated from Maxwell's equations. On the other hand, several authors, following the basic results obtained by Néel,<sup>9</sup> have stressed the intrinsic random nature of the coercive field  $H_c$ , originating from the fine structure of the perturbations encountered by a DW in

its motion. The experimental studies reported in Refs. 5-7 show that  $H_c$  is a random function of DW position, which can be approximately described by a Wiener-Lévy<sup>10</sup> (WL) stochastic process.<sup>7</sup> Nobody seems to have paid much attention so far to the joint physical meaning of these results. Actually, Eq. (1), or rather its time derivative, plays the role of a Langevin equation,<sup>10,11</sup> where the properties of the internal field  $H$  can be inferred from the macroscopic magnetization law of the system, whereas  $H_c$  describes the stochastic disturbance induced by microscopic DW interaction processes. In the present paper, this Langevin equation and the associated Fokker-Planck equation are solved to obtain the statistical properties of the process  $v(t)$ . Since  $v(t)$  is basically proportional to the voltage detected in a BE experiment, definite predictions on BE properties and on its connection with DW dynamics will be obtained.<sup>12,13</sup>

When this conceptual frame is analyzed in detail, it appears to contain several physically interesting complications, closely connected with specific aspects of BE phenomenology. A first point concerns the evaluation of the internal field  $H$ . The applied field  $H_a$  and local magnetostatic fields are the main contributions to  $H$ . Under convenient approximations, the rate of change  $dH/dt$  can be expressed directly in terms of the magnetization rate  $\dot{I}$  imposed to the system and the differential permeability  $\mu$  associated with that part of the hysteresis loop where DW motion is assumed to take place. This leads to stringent predictions about the BE dependence on  $\dot{I}$  and  $\mu$ . On the other hand, the interactions giving rise to  $H_c$  fluctuations are basically a function of DW position and not of time. We will see that this fact is intimately related to the onset of intermittency in BE at low magnetization rates, and implies that this intermittent behavior is associated with a fractal structure<sup>14</sup> characterized by a fractal dimension  $D < 1$ . Finally, there may be additional terms in  $H_c$ , depending explicitly on time, which should be considered in more refined models. In particular, a viscosity field  $H_v$  should be included when magnetic after effects are present. By using the classical results obtained by Néel on the properties of  $H_v$ ,<sup>15</sup> the theory should be able to

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predict the BE behavior in the presence of magnetic aftereffects. However, this generalization lies outside the scope of the present article.

As a concluding remark, we wish to point out that, owing to the intrinsic complexity of BE phenomenology, carrying out a proper experimental characterization of BE and a meaningful comparison with theoretical predictions is by no means a trivial task. In particular, careful consideration should be given to the fact that BE is a nonstationary process, associated with different dynamic conditions (i.e., different values of  $I$  and  $\mu$ ) and different magnetization processes (DW motion, domain creation or annihilation) when different points of the hysteresis loop are considered. This means that many literature results, implying averages of BE properties over the entire hysteresis loop, are not easily amenable to a clear physical interpretation. In particular, they are not directly comparable with the theory discussed in this paper, whose basic assumptions are that the mechanism giving rise to BE is DW motion and that DW motion can be described as a stationary Markov process, associated with well-defined, fixed values of  $I$  and  $\mu$ . The problem of a satisfactory experimental characterization of BE and a detailed comparison of experiments with the present theory are discussed in a companion paper.<sup>10</sup>

## II. THE MODEL

We consider a planar, 180° DW moving across a metallic slab of thickness  $d$  and cross-sectional area  $S$ . If the internal degrees of freedom related to the DW flexibility are neglected, the DW motion is fully described by the DW velocity  $v$  or, equivalently, by the magnetic flux rate of change  $\dot{\Phi} = 2I_s dv$ , where  $I_s$  is the saturation magnetization of the material. We shall assume that the DW dynamics is controlled by Eq. (1), which can be written as<sup>17</sup>

$$\sigma G \dot{\Phi} = H - H_c, \quad (2)$$

where  $\sigma$  is the electrical conductivity and the dimensionless coefficient  $G = (4/\pi^2) \sum_{\text{odd}} (1/k^3) = 0.1356$ , if a wide slab ( $S \gg d^2$ ) is considered.<sup>18</sup> Equations (1) or (2) have been widely and successfully applied to the description of large-scale DW motion, in connection, for example, with the prediction of excess eddy current losses.<sup>18,19</sup> Here, however, we are assuming that Eq. (2) can be applied even to the description of fine-scale DW dynamics, where stochastic fluctuations of  $\dot{\Phi}$  are induced by corresponding fluctuations of  $H_c$ , originating from the fine structure of DW interactions with lattice defects or other perturbations.

The leading features of the stochastic behavior of  $H_c$  can be inferred from the results of the experiments<sup>3-7</sup> with systems containing a single active DW. The basic conclusions are that  $H_c$  is a random function of DW position, i.e., of  $\Phi$ , and that its fluctuations as a function of  $\Phi$  can be described approximately by a WL process.<sup>7</sup> However, when large DW displacements are considered, the presence of a finite correlation length  $\xi$  is inevitably expected, corresponding to the finite interaction range of the DW with a given perturbation.<sup>5</sup> A convenient description of these features is obtained by assuming that  $H_c$  obeys the Langevin equation

$$\frac{dH_c}{d\Phi} + \frac{H_c - \langle H_c \rangle}{\xi} = \frac{dW}{d\Phi}, \quad (3)$$

where the WL process  $W(\Phi)$  is characterized by

$$\langle dW \rangle = 0, \quad \langle |dW|^2 \rangle = 2A d\Phi, \quad (4)$$

$A$  being an unknown constant.

Some specific considerations are needed to determine the behavior of the magnetic field  $H$  of Eq. (2). In Ref. 8, this field is simply identified with the applied field  $H_a$ . However, the picture-frame single crystal considered in that case is a very special system, characterized by an extremely high permeability and negligible magnetostatic effects. In a generic system, we expect magnetostatic fields to contribute significantly to  $H$ , so that  $H = H_a - H_m$ , where  $H_m$  is the magnetostatic contribution. A detailed description of  $H_m$  is by no means a trivial task. Yet, in line with the Langevin approach anticipated in the Introduction, we can attempt an evaluation of  $H_m$  inferred from that of its mean value. We shall assume that the DW motion takes place in a portion of the hysteresis loop characterized by a constant differential permeability<sup>17</sup>  $\mu = dI/dH_a$ , and that the average magnetization of the system increases at a constant rate  $\dot{I}$ . This is obtained by applying an external field also increasing at a constant rate  $\dot{H}_a = \dot{I}/\mu$ . Under these conditions,  $\Phi(t)$  is a stationary process whose mean value is  $\langle \Phi \rangle = S\dot{I}$ . If we consider the mean value of Eq. (2) and we take its time derivative, we obtain, assuming that  $\langle H_c \rangle$  is constant in time,<sup>20</sup>

$$\langle \dot{H}_m \rangle = \dot{H}_a = \dot{I}/\mu = \langle \dot{\Phi} \rangle / S\mu. \quad (5)$$

The simplest assumption consistent with Eq. (5) is of course

$$\dot{H}_m = \dot{\Phi} / S\mu. \quad (6)$$

Despite its simplicity, this description of magnetostatic effects is quite satisfactory, and leads to predictions in good agreement with experiments.<sup>16</sup> The time derivative of Eq. (2) can then be written as

$$\frac{d\dot{\Phi}}{dt} + \frac{\dot{\Phi} - S\dot{I}}{\tau} = -\frac{1}{\sigma G} \frac{dH_c}{dt}, \quad (7)$$

where the time constant  $\tau$ , defined as

$$\tau = \sigma G S \mu, \quad (8)$$

controls the decay of magnetostatic fields.

Even Eq. (7) has a Langevin structure, but the random term  $dH_c/dt$  does not correspond to a Gaussian white noise, and must be evaluated from Eq. (3). Note, however, that  $\Phi$ , and not  $t$ , is the natural coordinate appearing in Eq. (3). This interplay between magnetic flux and time dependencies will have far-reaching consequences, particularly in determining the intermittent character of DW motion at low magnetization rates. On the other hand, we know that the DW will move, under the action of  $H_a$ , in one direction only, corresponding, with the present conventions, to  $\dot{\Phi} > 0$ . Therefore, Eq. (7) is physically meaningful only for  $\dot{\Phi} > 0$ , and special care should be paid to its behavior for  $\dot{\Phi} \rightarrow 0$ .

Let us introduce the dimensionless parameter

$$c = S\dot{I}(\sigma G)^2 / A\tau, \quad (9)$$

the dimensionless variables

$$z = c\dot{\Phi}/S\dot{I}, \quad h_c = -H_c/AS\mu, \quad (10)$$

and the normalized  $WL$  process  $w(\Phi) = W(\Phi)/AS\mu$ . After rewriting Eq. (3) as a function of time, Eqs. (3), (7), and (4) take the form

$$\frac{dz}{dt} + \frac{z-c}{\tau} = \frac{dw}{dt}, \quad (11)$$

$$\frac{dh_c}{dt} + \frac{z}{c} \frac{h_c - \langle h_c \rangle}{\tau_c} = \frac{dw}{dt}, \quad (12)$$

$$\langle |dw|^2 \rangle = 2z \frac{dt}{\tau}, \quad (13)$$

where the time constant

$$\tau_c = \xi/SI \quad (14)$$

governs the decay of local coercive field correlations. The most intriguing aspects of Eqs. (11)–(13) are the presence of the nonlinear coupling between  $z$  and  $h_c$  in Eq. (12) and the dependence of  $\langle |dw|^2 \rangle$  on  $z$  in Eq. (13). These complications play an important physical role, which can be better appreciated by first discussing the approximate description in which  $\tau_c \rightarrow \infty$ . This limit illustrates the basic features of the model, and can be fully worked out by analytic tools.

### III. THE $\tau_c \rightarrow \infty$ LIMIT

According to Eqs. (11)–(14), the condition  $\tau_c \rightarrow \infty$  (or more precisely  $\tau_c \gg \tau$ ) is attained at low DW velocities and/or when the correlation length  $\xi$  of local coercive field fluctuations is sufficiently large. By assuming  $\tau_c \rightarrow \infty$  in Eqs. (11)–(13), we obtain

$$\frac{dz}{dt} + \frac{z-c}{\tau} = \frac{dw}{dt}, \quad (15)$$

$$\langle |dw|^2 \rangle = 2z \frac{dt}{\tau}. \quad (16)$$

$z(t)$  is a Markov process, whose statistical properties are governed by the conditional probability density  $P(z, t | z_0)$ , where  $z = z_0$  is the initial condition at  $t = 0$ .  $P$  obeys the Fokker-Planck equation associated with Eqs. (15) and (16),

$$\tau \frac{\partial P}{\partial t} - \frac{\partial(z-c)P}{\partial z} - \frac{\partial^2 z P}{\partial z^2} = 0. \quad (17)$$

We must look for a solution of this equation in the region  $z > 0$ , decaying to zero exponentially when  $z \rightarrow +\infty$ , and reducing to  $\delta(z - z_0)$  when  $t \rightarrow 0^+$ . Such a solution has been obtained in Ref. 4, in terms of a series of Laguerre polynomials  $L_k^{c-1}(z)$ ,  $k = 0, 1, \dots$ . Here, we shall consider in more detail the relation between Eq. (17) and the two quantities that characterize the statistical behavior of the process  $z(t)$ , i.e., the stationary amplitude probability distribution  $P_0(z)$  and the autocorrelation function  $R_z(\Delta t)$ .

$P_0(z)$  is obtained from Eq. (17) after dropping the time derivative term. The solution is

$$P_0(z) = z^{c-1} \exp(-z)/\Gamma(c), \quad (18)$$

where  $\Gamma(c)$  is the Euler gamma function. Note that this expression implies  $\langle z \rangle = \int z P_0(z) dz = c$ , a result equivalent to  $\langle \Phi \rangle = SI$  and thus consistent with our initial physical assumptions.

The most interesting aspect of Eq. (18) is its power-law behavior for  $z \rightarrow 0^+$ . This is the direct consequence of the linear dependence of  $\langle |dw|^2 \rangle$  on  $z$  in Eq. (16), which in turn is physically related to the fact that the local coercive field  $H_c$  is a random function of  $\Phi$  and not of time. The behavior of  $P_0(z)$  for  $z \rightarrow 0^+$  changes drastically, from  $P_0(z) \rightarrow 0$  to  $P_0(z) \rightarrow \infty$ , when  $c$  crosses the value  $c = 1$ . We can say that  $c = 1$  defines the boundary between the regime ( $c > 1$ ) of continuous DW motion, where the condition  $z \sim 0$  (i.e.,  $v \sim 0$ ) has a negligible probability to occur, and that ( $c < 1$ ) of intermittent DW motion, where the power-law divergence of  $P_0(z)$  for  $z \rightarrow 0$  implies the presence of burstlike events separated by finite time intervals in which  $z(t) \sim 0$ . This intermittent behavior is characterized by interesting scaling properties. According to Eq. (18), when  $c < 1$  the system can be found arbitrarily close to  $z = 0$ , but not exactly at  $z = 0$ , where  $P_0(z)$  is undefined. Consequently, deciding whether the system is active ( $z > 0$ ) or not ( $z = 0$ ) depends on our ability to resolve the details of the process  $z(t)$ . Given the resolution parameter  $z_R$ , the system will appear not to be active whenever  $z(t) < z_R$ . Let  $\mathcal{L}$  be the set of time intervals where the condition  $z(t) < z_R$  occurs. If we decrease  $z_R$ , new  $z(t)$  details are resolved and  $\mathcal{L}$  splits into more and more disjoint subintervals. The length  $L$  of  $\mathcal{L}$ , being proportional to the probability that  $z < z_R$ , can easily be evaluated from Eq. (18). When  $z_R \ll 1$ , we find  $L \propto z_R^c$ , a relationship which points at  $\mathcal{L}$  as a fractal set of fractal dimension  $D = 1 - c < 1$ , to be compared with the Cantor and Levy dusts discussed by Mandelbrot in Ref. 14. This result provides a new conceptual frame for the description and interpretation of clustering processes in BE. A nice pictorial representation of these considerations can be obtained by computer simulations, in which Eqs. (15) and (16) are used to generate  $z(t)$  step by step in time. The numerical problems associated with the simulation algorithm are discussed in the Appendix. In Fig. 1 we give a typical example of the behavior of  $z(t)$  obtained at the computer, with an illustration of its scaling properties when  $z \rightarrow 0$ .

The autocorrelation (autocovariance) function  $R_z(\Delta t)$  is defined as

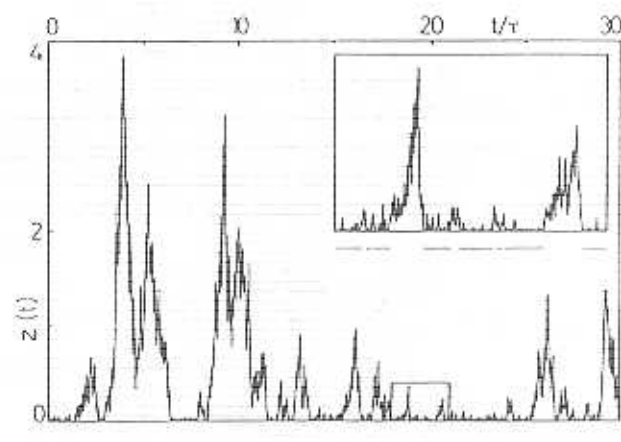


FIG. 1. Computer simulation of the time behavior of  $z(t)$  obtained from Eqs. (15) and (16) for  $c = 0.26$ . Inset: magnification of the signal inside the small frame. The set of time intervals where  $z(t) < z_R$  is represented below the figure for  $z_R = 0.1$ , and below the inset for  $z_R = 0.01$ .

$$R_z(\Delta t) = \int \int dz dz_0 (z - c)(z_0 - c) \times P(z, \Delta t | z_0) P_0(z_0). \quad (19)$$

The equation obeyed by  $R_z(\Delta t)$  can be derived from Eq. (17), written in the form  $\tau \partial P / \partial t + \partial J / \partial z = 0$ , where  $J(z, t)$  represents the probability current density. The fact that DW motion is confined within the region  $z > 0$  implies that  $J(0, t) = 0$ . This result can be directly calculated from the general solution for  $P(z, t | z_0)$  given in Ref. 4. Let us multiply Eq. (17) by  $(z - c)(z_0 - c)P_0(z_0)$ , integrate over  $z$  and  $z_0$ , and make use of the mentioned property of  $J$ . We obtain

$$\tau \frac{\partial R_z}{\partial \Delta t} + R_z = 0, \quad \Delta t > 0, \quad (20)$$

which shows that  $R_z$  is an exponential function,

$$R_z(\Delta t) = c \exp(-|\Delta t|/\tau), \quad (21)$$

where the preexponential factor, representing  $\langle (z - c)^2 \rangle$ , can be calculated from Eq. (18). The power spectrum  $F_z(\omega)$  of  $z$  is thus a Lorentzian,

$$F_z(\omega) = (2c/\tau)/(\omega^2 + \tau^{-2}). \quad (22)$$

According to the definition of  $z$  [Eq. (10)], the spectrum  $F(\omega)$  of  $\Phi$ , the quantity actually measured in BE experiments, is given by<sup>21</sup>

$$F(\omega) = 2(S\dot{I}/c)^2 F_z(\omega) = 4S\dot{I} [A/(\sigma G)^2]/(\omega^2 + \tau^{-2}). \quad (23)$$

Equations (22) and (23) imply that  $F(\omega) \simeq 4S\dot{I}A(S\mu)^2$  when  $\omega\tau \ll 1$ , and  $F(\omega) \simeq 4S\dot{I} [A/(\sigma G)^2]/\omega^2$  when  $\omega\tau \gg 1$ . Thus, the influence of magnetostatic fields, described by  $\mu$ , is limited to the region  $\omega\tau \lesssim 1$ . When  $\omega\tau \gg 1$ ,  $F(\omega)$  is uniquely controlled by short-range coercive field interactions, described by parameter  $A$ . This conclusion, together with the fact that the time constant of BE fluctuations is  $\tau \propto \mu$ , remains valid even when the limit  $\tau_c \rightarrow \infty$  does not apply, and provides a definite prediction which can be experimentally tested.

#### IV. THE GENERAL CASE

Following the results of the previous section, we shall now discuss the general properties of Eqs. (11)–(13), both by approximate analytic considerations and by computer simulations, in which the process  $z(t)$  is generated step by step in time and standard data analysis is then performed to obtain  $P_0(z)$  and  $F_z(\omega)$ . Details of the algorithm employed in the simulations are given in the Appendix.

It is convenient to introduce the new variable

$$y = (z - c) - (h_c - \langle h_c \rangle). \quad (24)$$

Equations (11) and (12) become then

$$\frac{dz}{dt} + \frac{z - c}{\tau} \left( 1 + \frac{\tau}{\tau_c} \frac{z}{c} \right) - \frac{y}{\tau_c} \frac{z}{c} = \frac{dw}{dt}, \quad (25)$$

$$\frac{dy}{dt} + \frac{z - c}{\tau} = 0, \quad (26)$$

and the corresponding Fokker-Planck equation for  $P(z, y, t | z_0, y_0)$  is

$$\tau \frac{\partial P}{\partial t} - \frac{\partial}{\partial z} \left[ (z - c) \left( 1 + \frac{\tau}{\tau_c} \frac{z}{c} \right) P \right] + \frac{\tau}{\tau_c} y \frac{\partial}{\partial z} \frac{z}{c} P - (z - c) \frac{\partial P}{\partial y} - c \frac{\partial^2}{\partial z^2} \frac{z}{c} P = 0. \quad (27)$$

#### A. Probability amplitude distribution

An analytic solution for  $P_0(z)$  can be obtained in the limit of high DW velocities, where, according to Eq. (9),  $c \gg 1$ . In this limit,  $z(t)$  will show only small fluctuations around its mean value,  $\langle z \rangle = c$ . We can thus assume that  $z \simeq c$  in Eqs. (12) and (13), or, equivalently, that all the terms  $z/c$  of Eq. (27) are equal to 1. The equation for the stationary amplitude distribution  $P_0$  becomes then

$$\frac{\partial}{\partial z} \left[ (z - c) \left( 1 + \frac{\tau}{\tau_c} \right) P_0 \right] + c \frac{\partial^2 P_0}{\partial z^2} + (z - c) \frac{\partial P_0}{\partial y} - \frac{\tau}{\tau_c} y \frac{\partial P_0}{\partial z} = 0. \quad (28)$$

The last two terms of Eq. (28) vanish if we look for a function  $P_0[(z - c)^2 + \tau y^2/\tau_c]$ , and the first two terms show that this function must be Gaussian:

$$P_0(z, y) \propto \exp\left[-(1 + \tau/\tau_c) [(z - c)^2 + \tau y^2/\tau_c]/2c\right]. \quad (29)$$

In terms of the parameters

$$\tilde{c} = c(1 + \tau/\tau_c), \quad \tilde{z} = (\tilde{c}/c)z = \tilde{c}\Phi/S\dot{I}, \quad (30)$$

we can simply write

$$P_0(\tilde{z}) \propto \exp\left[-(\tilde{z} - \tilde{c})^2/2\tilde{c}\right]. \quad (31)$$

Starting from this result, an intermediate-DW-velocity approximation can be worked out by assuming  $z \simeq c$  only in Eq. (12) and not in Eq. (13). This corresponds to estimating the decay of local coercive field correlations on the basis of the average rather than the instantaneous DW velocity. Instead of Eq. (28), we obtain

$$\frac{\partial}{\partial z} \left[ (z - c) \left( 1 + \frac{\tau}{\tau_c} \right) P_0 \right] + \frac{\partial^2 z P_0}{\partial z^2} + (z - c) \frac{\partial P_0}{\partial y} - \frac{\tau}{\tau_c} y \frac{\partial P_0}{\partial z} = 0. \quad (32)$$

Since we are interested in the  $z$  dependence only, we can integrate Eq. (32) over  $y$  and, as a first approximation, we can use the high-DW-velocity expression for  $P_0(z, y)$ , Eq. (29), in the calculation of  $\int dy y P_0$ . By doing so, the two last terms of Eq. (32) vanish and the resulting equation simply reads

$$\frac{\partial}{\partial \tilde{z}} [(\tilde{z} - \tilde{c}) P_0] + \frac{\partial^2 \tilde{z} P_0}{\partial \tilde{z}^2} = 0. \quad (33)$$

The solution is

$$P_0(\tilde{z}) \propto \tilde{z}^{\tilde{c}-1} \exp(-\tilde{z}), \quad (34)$$

which, as a function of the variables  $\tilde{z}$  and  $\tilde{c}$ , is coincident with Eq. (18), obtained in the limit  $\tau_c \rightarrow \infty$ . This remarkable result indicates that an amplitude distribution of the form of Eq. (34) is one of the basic, distinctive features of BE, which should be quite generally observed in BE experiments.

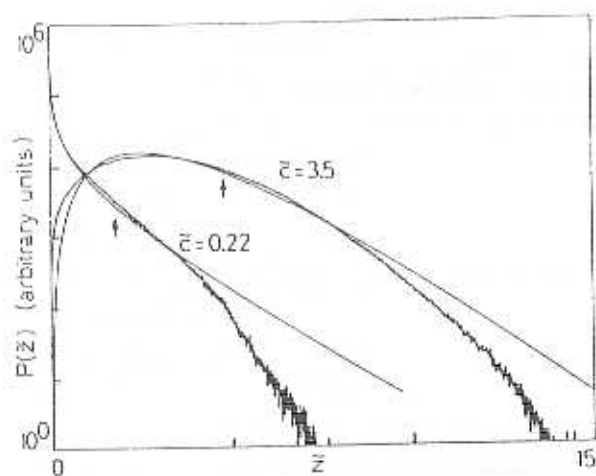


FIG. 2. Comparison between the predictions of Eq. (34) and of computer simulations for  $\bar{z} = 3.5$  and  $\tau/\tau_c = 0.79$ , and for  $\bar{z} = 0.22$  and  $\tau/\tau_c = 0.12$ . The arrows indicate the points where  $\bar{z} = \bar{z}\tau_c/\tau$ .

It is important to discuss the actual role of the approximations underlying Eq. (34), in particular the linearization obtained by assuming  $z \approx c$  in Eq. (12). In fact, the nonlinear term of Eq. (12) acts as an increasingly efficient feedback, when  $z(t)$  reaches high values, so that we expect  $P_n(\bar{z})$  to decay to zero more rapidly than Eq. (34). The second term of Eq. (27) indicates that this deviation should occur when  $(\tau/\tau_c)(\bar{z}/c) \gg 1$ , i.e.,  $\bar{z} \gg \bar{z}(\tau_c/\tau)$ . This conclusion is confirmed by computer simulations, as shown in Fig. 2. On the other hand, the nonlinear term of Eq. (12) becomes negligible when  $z/c \ll 1$ . Thus, the behavior of  $P_n$  for  $z \rightarrow 0$  should approach the one relative to the  $\tau_c \rightarrow \infty$  limit, and we expect a crossover from  $P_n \sim z^{-1}$ , predicted by Eq. (34), to  $P_n \sim z^{-1}$ , predicted by Eq. (18) when  $z$  is sufficiently low.

## B. Power spectrum

In the high-DW-velocity limit discussed in the previous section, the power spectrum  $F(\omega)$  is easily calculated assuming that  $z = c$  in Eqs. (12) and (13), and taking the Fourier transform of Eq. (12). This straightforwardly leads to the following expression for the power spectrum  $F_c(\omega)$  of  $h_c$ :

$$F_c(\omega) = (2c/\tau)/(\omega^2 + \tau_c^{-2}). \quad (35)$$

From the Fourier transform of Eq. (11), we then obtain<sup>21</sup>

$$F(\omega) = 2(SI/c)^2 F_c(\omega) \omega^2 / (\omega^2 + \tau^{-2}) \\ = 4SI \frac{A}{(\sigma G)^2} \frac{\omega^2}{(\omega^2 + \tau^{-2})(\omega^2 + \tau_c^{-2})}. \quad (36)$$

This equation and Eq. (23), obtained in the  $\tau_c \rightarrow \infty$  limit, represent our basic results on the behavior of the BE power spectrum. According to Eq. (36),  $F(\omega) \approx 4SI [A/(\sigma G)^2]/\omega^2$  when  $\omega\tau \gg 1$  and  $\omega\tau_c \gg 1$ . In this frequency range, Eqs. (36) and (23) become coincident and are controlled uniquely by the local coercive field parameter  $A$ . At lower frequencies, the behavior of Eq. (36) becomes more complex than the one of Eq. (23).  $F(\omega)$  reaches a maximum

$$F_M = 4SI [A/(\sigma G)^2] (1/\tau + 1/\tau_c)^{-2} \quad (37)$$

at the angular frequency

$$\omega_M = (\tau\tau_c)^{-1/2} \quad (38)$$

and decays to zero as  $F(\omega) \sim \omega^{-2}$  when  $\omega \rightarrow 0$ .

A power spectrum having approximately the shape of Eq. (36) has been known for many years to be a characteristic feature of BE, commonly observed in experiments.<sup>22</sup> In Ref. 22, this behavior is attributed to the existence of a time-amplitude correlation of magnetostatic origin between subsequent  $B$  pulses, and an equation identical to Eq. (36) is derived. However, our approach shows that Eq. (36) has a more general origin, not necessarily related to the presence of well-separated  $B$  pulses. In fact, the presence of a maximum in  $F(\omega)$  appears as the inevitable consequence of the interplay between magnetostatic effects, described by the time constant  $\tau$ , and the existence of a finite correlation length  $\xi$  in coercive field fluctuations. Note that, through Eqs. (8) and (14), Eq. (36) provides definite, stringent predictions on the behavior of  $F(\omega)$  as a function of  $I$  and  $\mu$ , which can be experimentally tested.

Filling up the gap between Eq. (36) (high-DW-velocity limit) and Eq. (23) ( $\tau_c \rightarrow \infty$  limit) by a convenient analytic treatment is far from being a simple task. In fact, the nonlinearity of Eq. (12) has crucial effects which are lost in any linearization. This problem will no longer be addressed in this paper. In the discussion of experimental data, one can resort to computer simulations to analyze the behavior of  $F(\omega)$  whenever neither Eq. (23) nor Eq. (36) are applicable.

## V. EXTENSION TO MANY ACTIVE DOMAIN WALLS

A basic point still to be discussed is the fact that the whole theory developed so far refers to a single DW, while we know that in reality many DWs participate in the magnetization process. We want to show now that, under certain conditions, the theory holds even when  $\Phi$  represents the total flux rate produced by many DWs.

Let us assume that the ferromagnetic slab contains, in a given cross section,  $n$  active DWs, each of which provides, under the action of the applied field  $H_a$ , a longitudinal flux rate  $\Phi_k$ ,  $k = 1, 2, \dots, n$ . We are interested in the time behavior of the total flux rate  $\dot{\Phi} = \sum_k \dot{\Phi}_k$ .  $\dot{\Phi}_k$  obeys the equation

$$\sigma G \frac{d\dot{\Phi}_k}{dt} = \dot{H}_a - \frac{dH_{mk}}{dt} - \frac{dH_{ck}}{dt}, \quad (39)$$

corresponding to the time derivative of Eq. (2), where the internal field  $H_k$  has been split into its basic contributions, the applied field  $H_a$  and the local magnetostatic field  $H_{mk}$ .<sup>23</sup> Even in this case,  $H_{mk}$  can be approximately evaluated by considering its average value. From Eq. (39), recalling that  $\dot{H}_a = I/\mu$ , and again assuming that  $d\langle H_{ck} \rangle/dt = 0$ , we obtain  $d\langle H_{mk} \rangle/dt = \sum_k \langle \dot{\Phi}_k \rangle / S\mu$ . The simplest assumption consistent with this result is

$$\frac{dH_{mk}}{dt} = \frac{\dot{\Phi}_k}{S\mu} + \sum_{p \neq k} \frac{\langle \dot{\Phi}_p \rangle}{S\mu}. \quad (40)$$

The term  $\dot{\Phi}_k/S\mu$  represents the instantaneous reaction of the surrounding medium to the motion of the  $k$ th DW, to

which a mean field evaluation of the demagnetizing effects associated with the motion of the remaining DWs is added. From Eqs. (39) and (40), summing up over  $k$  and taking into account that  $\langle \Phi_k \rangle = SI/n$ , we obtain an equation for  $\Phi$  having the same structure of Eq. (7):

$$\sigma G \frac{d\Phi}{dt} + \frac{\Phi}{S\mu} - \frac{I}{\mu} = - \frac{dH_c}{dt}, \quad (41)$$

where  $dH_c = \sum_k dH_{ck}$ . In analogy with Eq. (3),  $H_{ck}$  will obey the Langevin equation

$$\frac{dH_{ck}}{d\Phi_k} + \frac{H_{ck} - \langle H_{ck} \rangle}{\xi_{DW}} = \frac{dW_k}{d\Phi_k}, \quad (42)$$

where  $\langle |dW_k|^2 \rangle = 2A d\Phi_k$  and  $\xi_{DW}$  is the correlation length associated with the motion of a single DW. In order to evaluate the properties of  $dH_c$ , let us first consider the limit  $\xi_{DW} \rightarrow \infty$ . In this case,  $dH_{ck} \equiv dW_k$  and  $dH_c \equiv dW = \sum_k dW_k$ . Since  $W_k$  fluctuations originate from localized interactions, it is reasonable to assume that they are statistically independent for different DWs, so that

$$\langle |dH_c|^2 \rangle = \langle |dW|^2 \rangle = \sum_k \langle |dW_k|^2 \rangle = 2A d\Phi, \quad (43)$$

which is coincident with Eq. (4). Let us now consider the case where  $\xi_{DW}$  is not infinite. Summing up Eq. (42) over  $k$ ,

$$dH_c + \sum_k (H_{ck} - \langle H_{ck} \rangle) \frac{d\Phi_k}{\xi_{DW}} = dW. \quad (44)$$

Equation (44) becomes identical to Eq. (3) in the case where  $(H_{ck} - \langle H_{ck} \rangle)$  and  $d\Phi_k$  are uncorrelated, i.e.,

$$\sum_k (H_{ck} - \langle H_{ck} \rangle) d\Phi_k = (H_c - \langle H_c \rangle) \frac{d\Phi}{n}. \quad (45)$$

The total correlation length is then

$$\xi = n\xi_{DW}. \quad (46)$$

Equation (46) has a clear physical meaning. If  $n$  DWs are simultaneously moving, it is necessary that each of them reverses a flux  $\sim \xi_{DW}$  before the system loses the memory of its past interactions, and this implies a total flux reversal  $\xi \sim n\xi_{DW}$ . On the other hand, Eq. (46) suggests that further complications in the behavior of the BE power spectrum may arise from the fact that  $n$  may be a complex function of  $I$  and  $\mu$ . However, these complications are limited to the low  $\omega$  range. When  $\omega\tau \gg 1$  and  $\omega\tau_c \gg 1$ ,  $A$  is the only important parameter and  $n$  does not play any role.

## VI. CONCLUSION

The theoretical description of DW dynamics discussed in the preceding sections [Eqs. (11)–(13)] leads to definite predictions about BE statistical properties, in particular the amplitude probability distribution  $P_0(\Phi)$  [Eqs. (18), (31), (34)] and the power spectrum  $P(\omega)$  [Eqs. (23) and (36)]. These equations provide approximate solutions to the general model, which, as will be discussed in paper II, cover most of the situations of physical interest in BE experiments. On the other hand, we stress the fact, discussed in Sec. V, that the theory, originally developed as a single-DW theory, applies also to the description of BE when many DWs are active. This result makes the theory much more realistic and

justifies its direct application to the interpretation of BE phenomenology in ordinary materials.

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## APPENDIX

In this Appendix, we discuss the algorithm employed in the computer simulations of DW motion. Let us consider Eqs. (7), (3), and (4).  $\Phi(t)$  is generated by calculating its variation  $\Delta\Phi$  associated with given small steps  $\Delta\Phi$  and  $\Delta t$ , according to the equation

$$\Delta\Phi = SI\Delta t/\tau - \Delta\Phi/\tau - \Delta H_c/\sigma G. \quad (A1)$$

$H_c$  is calculated as

$$\Delta H_c = -(H_c - \langle H_c \rangle)\Delta\Phi/\xi + \Delta W, \quad (A2)$$

where  $\Delta W$  is the variation of the WL process  $W(\Phi)$  in the step  $\Delta\Phi$ . The variations  $\Delta\Phi$  and  $\Delta t$  are not independent, because they must be consistent with  $\Delta\Phi/\Delta t = \dot{\Phi}$ . Furthermore, they must be such that  $\Delta t/\tau \ll 1$  and  $\Delta\Phi/\xi \ll 1$ .

Each simulation step ordinarily proceeds by setting  $\Delta t = \Delta t_0$  and  $\Delta\Phi = \dot{\Phi}\Delta t_0$ , where  $\Delta t_0$  is a given, fixed value ( $\sim 10^{-6}$ – $10^{-5}$  s in most cases of physical interest), and by calculating  $\Delta\Phi$  and  $\Delta H_c$  through Eqs. (A1) and (A2), where  $\Delta W$  is chosen at random from the Gaussian distribution

$$P(\Delta W) \propto \exp[-(\Delta W)^2/4A\Delta\Phi], \quad (A3)$$

consistent with  $\langle |dW|^2 \rangle = 2A d\Phi$ . This scheme works well at high magnetization rates and/or low permeabilities ( $\hat{c} > 1$ ), but needs some refinements when  $\hat{c} < 1$ , due to the fact that  $\dot{\Phi}$  may come arbitrarily close to zero. Under these conditions, the value of  $\dot{\Phi} + \Delta\dot{\Phi}$  is tested after each calculation of  $\Delta\Phi$  and  $\Delta H_c$ . If  $\dot{\Phi} + \Delta\dot{\Phi} < 0$ , we are in an unphysical situation, where the simulation steps  $\Delta\Phi$  and  $\Delta t$  are not small enough to prevent the system from entering the region  $\dot{\Phi} < 0$ , which is a forbidden region for the process.  $\Delta\Phi$  and  $\Delta H_c$  are then discarded,  $\Delta\Phi$  and  $\Delta t$  are halved and the calculation is repeated. However, the value of the WL process, say  $W_1$ , and the corresponding value of the magnetic flux,  $\Phi_1$ , are not discarded, but stored for later use. This is a fundamental point, because otherwise the statistics of the process  $W(\Phi)$  would be affected by our decision to accept or not specific values of  $\Delta\Phi$  and  $\Delta H_c$ . This also implies that  $\Delta W$  must not be generated from the distribution (A3), but from the conditional distribution taking account of all the values  $W(\Phi_1) = W_1$ ,  $W(\Phi_2) = W_2, \dots$  possibly generated in previous attempts to calculate  $\Delta\Phi$  and  $\Delta H_c$  with larger steps  $\Delta\Phi$  and  $\Delta t$ . Attempts to reach the condition  $\dot{\Phi} + \Delta\dot{\Phi} > 0$  are made down to a minimum value  $\Delta\Phi_{\min}$  of  $\Delta\Phi$ . If even in this case  $\dot{\Phi} + \Delta\dot{\Phi} < 0$ , an exit approximation is adopted, in which it is assumed that the flux variation  $\Delta\Phi_{\min}$  takes place in the time interval

$$\Delta t = (\Delta\Phi_{\min} + \tau\Delta H_c/\sigma G - \tau\dot{\Phi})/SI. \quad (A4)$$

This is just the interval taken by the applied field to overcome the coercive and magnetostatic field barrier hindering the system at the time under consideration.

The algorithm generates the process  $\Phi(t)$ . Standard signal analysis is then performed to obtain  $P_0(\Phi)$  and  $F(\omega)$ . The procedure has been tested by direct comparison with those cases where analytical predictions of  $P_0(\Phi)$  and  $F(\omega)$  were available [Eqs. (18), (23), and (36)].

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<sup>11</sup> N. G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981); C. W. Gardiner, *Stochastic Methods* (Springer, Berlin, 1983).

<sup>12</sup> Some of the ideas at the basis of the theoretical approach discussed in this paper have been preliminary presented in the following conference papers: G. Bertotti, in *Physics of Magnetic Materials*, edited by W. Gorkowski, H. K. Lachowicz, and H. Szymczak (World Scientific, Singapore, 1986); G. Bertotti, in *Magnetic Excitations and Fluctuations II*, edited by U. Balucani, S. W. Lovesey, M. G. Rasetti, and V. Tognetti (Springer, Berlin, 1987), p. 135. See also Ref. 4.

<sup>13</sup> An alternative theoretical description of DW dynamics, which will no longer be considered in this paper, is discussed in G. Bertotti, Phys. Rev. B 39, 6737 (1989).

<sup>14</sup> B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, San Francisco, 1982).

<sup>15</sup> L. Néel, J. Phys. Rad. 15, 225 (1954).

<sup>16</sup> B. Alessandro, C. Beatrice, G. Bertotti, and A. Montorsi, J. Appl. Phys. 68, 2908 (1990) (part II).

<sup>17</sup> We employ SI units, with the convention  $B = \mu_0 H + I$ , and we assume that the term  $\mu_0 dI/dt$  can always be neglected with respect to  $dI/dt$  in any flux rate evaluation. This implies that we can approximate the differential permeability  $\mu = dB/dH$ , as  $\mu \approx dI/dH$ , and that we can identify the flux rate  $\Phi$  ( $\sim dI/dt$ ) of Eq. (2) with the flux rate ( $-dB/dt$ ) measured in a BE experiment. This approximation applies to soft materials and to magnetization fluctuations of frequency low enough to avoid eddy current shielding. Such limiting frequency is of the order of several kHz for the system considered in paper II.

<sup>18</sup> G. Bertotti, J. Appl. Phys. 57, 2110 (1985); 57, 2118 (1985).

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<sup>20</sup> This assumption is not really restrictive. In fact, if  $\langle H_c \rangle$  in Eq. (3) depends on  $\Phi$ , and  $d\langle H_c \rangle/d\Phi = \alpha$ , then  $\langle H_c \rangle = \alpha\Phi$ , which is similar to Eq. (6) and can be included in the magnetostatic term  $H_m$ , thus reducing the analysis to the case considered here.

<sup>21</sup> The normalization of  $F(\omega)$  in Eqs. (23) and (36) is such that  $\int_0^\infty F(\omega) d\omega/2\pi$  gives the total noise power. This is the normalization commonly adopted in noise experiments.

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<sup>23</sup> A further contribution to  $H_k$  may arise from the superposition of the eddy current patterns generated by different DWs. This contribution may be particularly important when several neighboring DWs, concentrated in a region smaller than the slab thickness, move at the same time. In this case, however, the group of DWs as a whole exhibits a dynamic behavior similar to that of an equivalent, single DW, and the total flux rate produced by the group of DWs still obeys Eq. (39). This point is fully discussed in Ref. 18.