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Vorträge zum 7. Darmstädter Geotechnik-Kolloquium am 23. März 2000

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Basic features for the assessment of the behaviour of supports in rock tunnelling using back-analysis

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1 Introduction

The behaviour of underground cavities and therefore also of the supports that are necessary to guarantee stability of the voids, depends, to a great extent, on the geotechnical characteristics of the rock mass (above all strength and deformability). The description of the natural material, from the geotechnical point of view, at both small and large scale, in fact results to be a large part of the planning study of tunnels and underground voids in general. The definition of the geotechnical characteristics is today made in two different ways:

- through the geomechanical characterisation of the natural materials, with attribution of the physical, mechanical and hydraulic parameters at small and large scale (that is, at the laboratory sample scale and the problem scale, respectively);
- through the measurement of sizes described by the stress and strain behaviour of the rock during the construction of the work or during the construction of nearby auxiliary works that have to be built.

In the first case the evaluation of the geotechnical parameters occurs before and during the construction of the work and is based on laboratory tests and *in situ* sampling, with the help of experience gained from other analogous sites, experience that is organised in a rational manner through the different widespread technical classifications. A detailed preliminary geotechnical characterisation of the rock mass is also required today for less important tunnels and those of modest length, so as to be able to always define an adequate and reliable executive project of the work that is able to reduce or eliminate the instability risks and to establish in advance and with a certain precision the times and costs that will be necessary. Unfortunately, however, the preliminary knowledge of the rock mass is never certain; in some cases (very deep mountain pass tunnels or tunnels in complex and chaotic formations) it is even rather approximate and basically based on surface geological studies, therefore the use of geomechanical characterisation during the first stages of construction the work results to be fundamental.

The measurement of displacements and stresses in the rock mass and in the support structures represents a different methodology for the evaluation of the geotechnical characteristics of a rock mass and therefore also of the support work conditions. The measurements of stress and displacement (these later surely being more reliable than the first), are influenced not only by the characteristics of the rock, but also by those of the supports together with other technical parameters, such as the dimensions of the tunnel and the depth from the ground surface. It is therefore not possible to obtain a direct indication of the characteristics of the rock from the measured values. To correctly interpret the measurements it is necessary to make use of a more complex procedure, called back-analysis, that, starting from an estimation of the unknown parameters of the rock mass obtained through a preliminary characterisation, integrated and modified by sampling of the rock mass during the construction stage, and by the performed stress and displacement measurements, is able to define the unknown parameters of the rock mass. The unknown parameters of the rock mass are those that are able to produce, with the calculation and with a certain approximation, the same displacement and stress measurements actually encountered.

Once the geotechnical parameters of the rock mass are known, it is then possible to evaluate the strain state in the support structure and, if necessary, modify or calibrate the project previsions concerning the supports to make them adequate to the conditions actually encountered on the site. It is not possible to carry out a verification of the working conditions of the support structures or modify, and make these adequate, if a correct back-analysis of the *in situ* measurements was not previously performed in order to obtain a good reliability of the geomechanical parameters of the rock mass.

Back-analysis in engineering in the rock field occurs, however, in an uncertainty context, which complicates the problem. The preliminary estimation of the geotechnical characteristics of the rock mass have in fact a degree of reliability that is a function of the intensity of the preliminary investigations. The performed measurements present a certain precision in relation to the various typologies of error that can occur. The final result of the back-analysis therefore also consists in the definition of the geotechnical parameters of the rock mass that are considered to be of influence in the problem under

examination, with a certain reliability and precision that is obviously greater than that relative to the initial estimation of the same parameters.
The purpose of this work is to present a global approach to back-analysis in a probabilistic context that is aimed to obtain a reliable calibration of the parameters of the rock mass that are necessary to study the behaviour of the support structures.
After having mentioned the nature of the uncertainties that are inherent to the initial estimation of the geomechanical parameters of the rock mass deriving from the preliminary characterisation, the physical, mechanical and geometrical factors that influence the stresses and the displacements in the rock mass around a tunnel, in the presence of a support structure, are analysed. The most commonly used measurement techniques for tunnels and their precision and reliability characteristics are then examined. Finally, theoretic instruments to develop the back-analysis in a probabilistic context are then presented. These procedures are able to consider the uncertainties in the initial estimations and in the measurements, with the purpose of obtaining not only a new and better estimation of the geomechanical parameters of the rock mass, but also to know the precision of this estimation. Once this stage is finished, it is possible to proceed with the verification of the stress and strain state of the support structures and, if necessary, modify or integrate the planning regulations.
The logical process of the new proposed procedure, which is developed in the following paragraph, as far as the main components closely connected to the theme of this work are concerned, is shown in figure 1.

2 The uncertainties in the preliminary geomechanical characterisation of the rock mass

The geomechanical behaviour of the rock mass, following the opening of an underground excavation, is governed by the mechanical parameters of resistance and deformability, which are correlated to the "geomechanical quality of the rock mass". This has been defined over the years in different ways on the basis of observations of the behaviour of the rock mass during construction of tunnels and chambers.
Today, the geomechanical quality is evaluated on the basis of an estimation of the physical, mechanical and geometrical characteristics of the intact rock and, above all, of the natural discontinuity. The geomechanical quality can however only be evaluated with a certain degree of reliability due to the following important uncertainties:
- the intrinsic variation of the geological and geomechanical conditions along a tunnel;

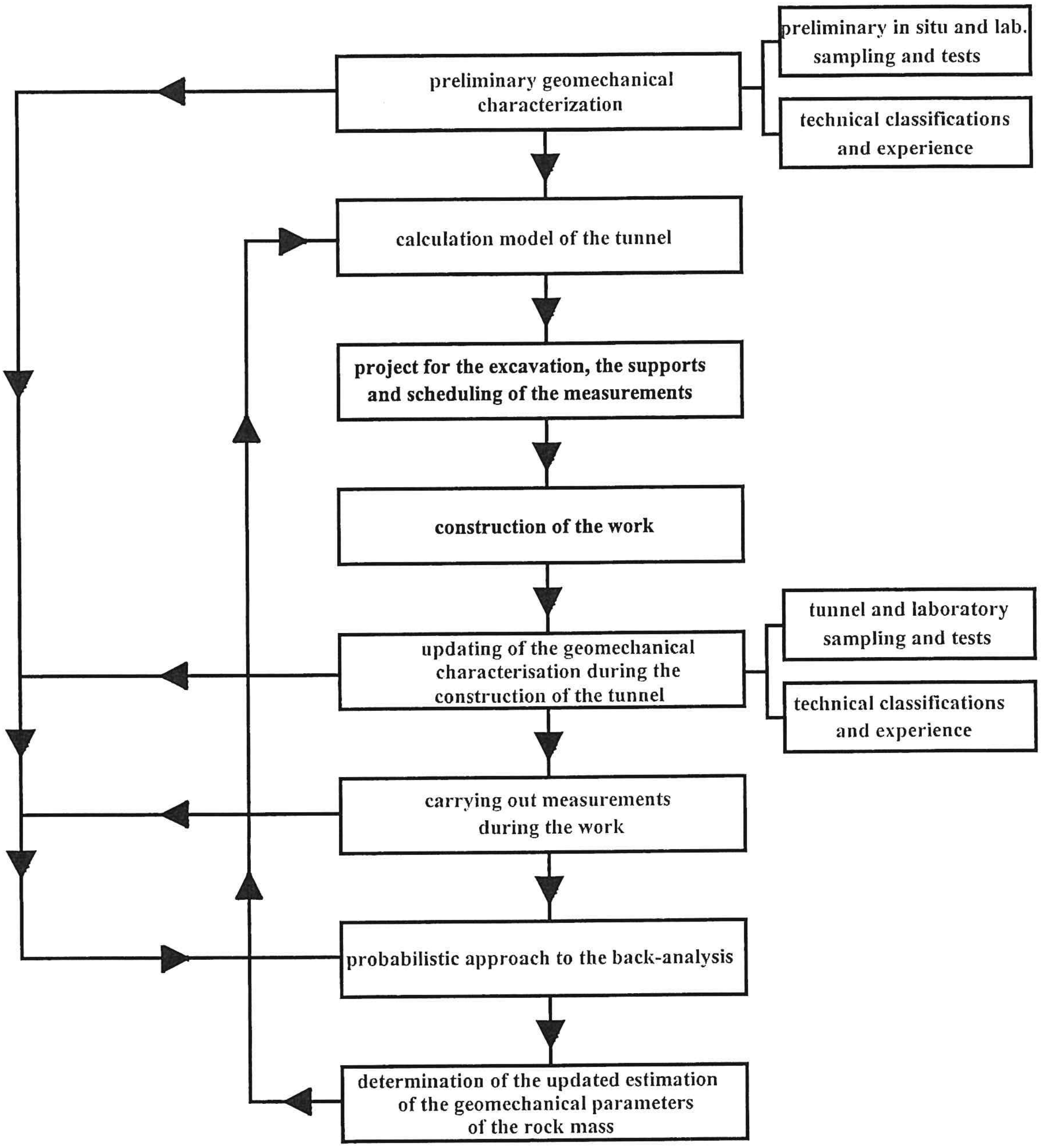

Fig. 1 Proposed procedure for the refining of the project of the support structure of a tunnel, thanks to the use of back-analysis, according to the probabilistic approach, which requires a first initial estimation of the rock parameters (and its degree of reliability) and the results of the measurements (with its precision) performed in a tunnel during construction.

- the modality used to obtain data, which can be of an extended but cortical type (geological-technical survey) or precise and localised (geognostic investigations, preliminary excavations) but on a reduced scale, which can be different from the analysed problem;
- the estimation procedure of the physical and mechanical parameters of the rock mass, which are conventional, because they are based on successive experience in time and with diversified final purposes (excavation, support classifications etc.,).

A correct procedure for the classification of the rock mass, that also permits the optimisation of the project in economic terms and not only in technological terms, requires a probabilistic type of approach and not simply deterministic. This is because the deterministic way of proceeding is not technically incorrect, but often leads to conservative evaluations due to the *a priori* overestimation of the safety factors in relation to the unknowns that are present.

The probabilistic approach can be of various types, depending on the detail that one wishes to obtain and, above all, to the available data. When it is possible, one can define a probabilistic distribution of the quality indexes of the rock mass in the homogeneous sections from the geological and geolithological point of view. As the quality is obtained, starting from numerous other physical, mechanical and geomechanical parameters of the rock and the discontinuities that are present, each of which has its own probabilistic distribution, the distribution that results from the quality index is usually assumed as a Gaussian type (symmetrical curve), whose probability density is described by equation 1:

$$f(x) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (1)$$

where: f(x) is the probability density;

x is the probabilistic variable, in the case under examination, the quality index of the rock mass;

μ is the mean of the distribution;

σ is the variance of the distribution.

The Gaussian distribution, also called normal, is univocally determined once the mean value and the variance of the distribution are known, which are calculated once the histogram of the relative frequencies for a section of a tunnel under examination have been determined. An example of a histogram of the relative frequencies of the RMR quality index is given in figure 2 for a section in gneiss and granite gneiss along 1.5 km of an Italian road tunnel.

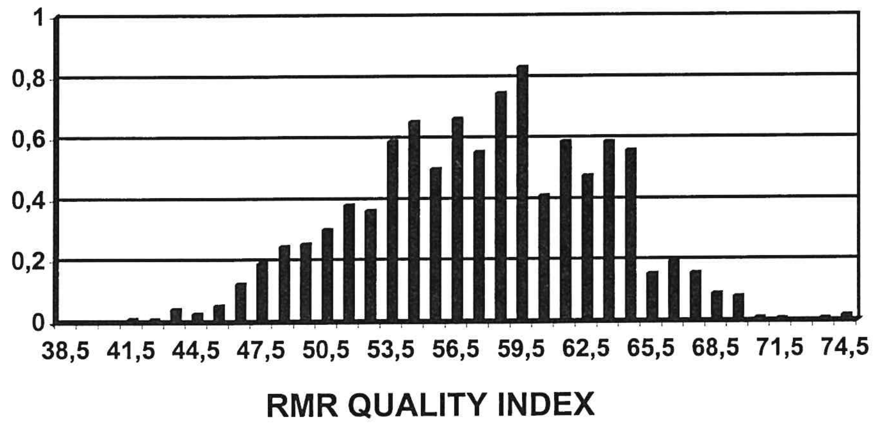

Fig. 2 Probabilistic density of the RMR quality index for a 1.500 m long tract of a 12.3 m wide and 8.9 m high Italian road tunnel in gneiss and granite gneiss

High values of the variation of the distribution can mean two different situations:

- the level of reliability of the estimation (of the mean) is low because of the lack of available data; further geomechanical investigations, even during the work, could lead to a reduction of the variance and a greater reliability of the estimation;
- the section of the tunnel under examination, which had been considered homogeneous in the preliminary zoning, is in reality unhomogeneous, from the mechanical behaviour point of view; as a consequence, the engineer tends, for caution reasons, to draw up a project that considers the worst conditions in the considered section. The alternative is to redesign the preliminary geomechanical zoning of the tract, foreseeing shorter tunnel sections (with low variance values) to make the support system more adherent to the quality of the rock along the tunnel axis.

3 The stress and strain state around a tunnel in the presence of support structures

The trend of the stresses and displacements that develop around a tunnel and in the supports during and after its construction are briefly dealt with in this section. These magnitudes are the subject of the measurements that are carried out during work. The technician in charge of planning, performing and interpreting the measurements should therefore preventively and qualitatively know the trend and the order of magnitude of these values.

The stress and strain state around a tunnel and on the inside of the supports is conditioned by different factors, which summarised, can be thus divided:

- quality of the rock mass, identified from a quality index;
- dimensions, geometry and depth of the tunnel;
- stress conditions of the rock and, in particular, the nature and intensity of the horizontal stresses in undisturbed conditions;
- the entity, number and type of supports installed;
- the distance from the excavation face along the tunnel axis.

A simple and reliable calculation method that permits one to consider almost all these previously mentioned elements is that of the convergence-confinement curves (fig. 3), which, however, is based on important hypothesis:

- pressure field of a hydrostatic type;
- homogeneous and isotropic rock mass;
- deep tunnel for which it is right to assume that the boundary conditions of a means are infinitely extended;
- circular tunnel profile;
- bi-dimensional type problem, which can be changed into a three-dimensional problem through the “fictitious internal pressure” concept.

The method is however valid, at least in qualitative and descriptive terms of the phenomena, also in general conditions when some of the previously mentioned hypothesis are not completely admissible.

For a rock mass that presents a Mohr-Coulomb type strength criterion, determined on the basis of the cohesion c and the friction angle φ, it is possible to identify a value of the internal pressure of the tunnel called p_{cr} (eq.2), below which a ring of rock with plastic type behaviour begins to form around the cavity whose extreme radius is called plastic radius R_{pl} (eq.3, fig.4).

$$p_{cr} = p_0 \cdot (1 - \operatorname{sen}\varphi) - c \cdot \cos\varphi \tag{2}$$

$$R_{pl} = R \cdot \left[\frac{\left(p_0 + \dfrac{c}{tg\varphi} \right) - \left(p_0 + \dfrac{c}{tg\varphi} \right) \cdot \operatorname{sen}\varphi}{p_{in} + \dfrac{c}{tg\varphi}} \right]^{\frac{1-\operatorname{sen}\varphi}{2\operatorname{sen}\varphi}} \tag{3}$$

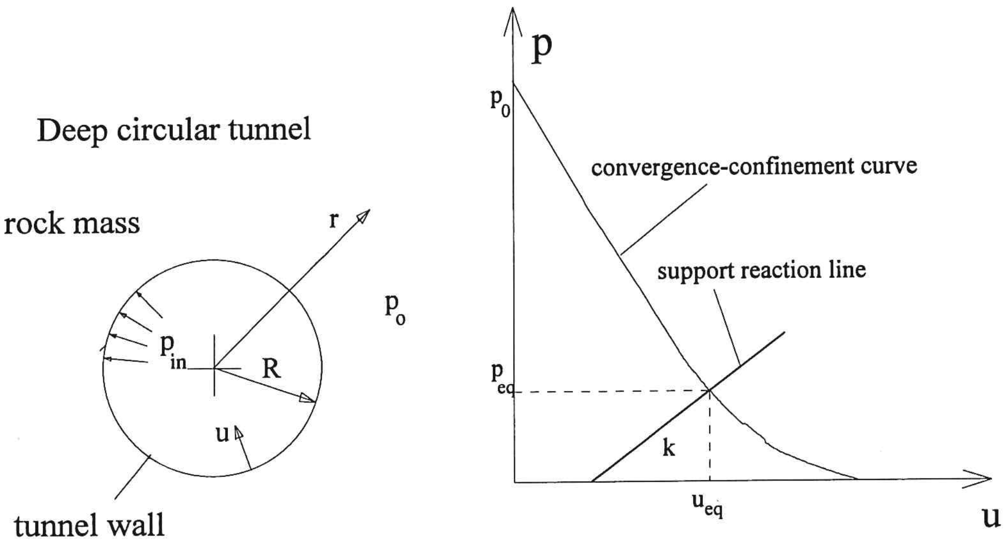

Fig. 3 Convergence-confinement curve of the tunnel and reaction line of the supports for a deep circular tunnel, in the presence of a hydrostatic stress state. Key: p_{eq}: final equilibrium pressure, equal to the load applied on the supports and to the radial pressure applied to the tunnel walls p_{in}; u_{eq} : radial displacement of the tunnel wall at the final equilibrium; p_0: undisturbed pressure in natural conditions; k: support stiffness.

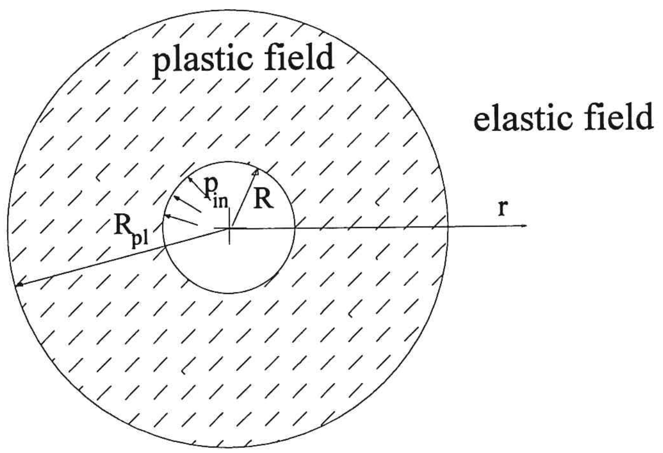

Fig. 4 Formation of the plastic zone around the tunnel, due to a certain internal pressure applied to the walls.

In the case where p_{cr} is negative, the rock mass appears elastic, even in the absence of an internal pressure. Even for internal pressures p_{in} greater than the p_{cr} , the rock mass appears elastic around the tunnel and the radial stresses σ_r , circumferential stresses σ_ϑ

and the radial displacements u_r at the generic distance r are determined by the simple relations 4-6. The radial displacement u of the tunnel wall is obtained from eq. 6, placing r = R (eq. 7).

$$\sigma_r = p_0 - (p_0 - p_{in}) \cdot \left(\frac{R}{r}\right)^2 \tag{4}$$

$$\sigma_\vartheta = p_0 + (p_0 - p_{in}) \cdot \left(\frac{R}{r}\right)^2 \tag{5}$$

$$u_r = -\frac{1+\nu}{E}(p_0 - p_{in})\frac{R^2}{r} \tag{6}$$

$$u = -\frac{1+\nu}{E}(p_0 - p_{in}) \cdot R \tag{7}$$

Eq.s 4-6 are still valid, when $p_{in} < p_{cr}$, to describe the behaviour of the rock outside the plastic zone. However R should be substituted with the value of the plastic radius R_{pl} and p_{in} with the radial stress in correspondence to the plastic radius (p_{cr}).
The stresses and displacements inside the plastic ring are described by eq.s 8-10.

$$\sigma_r = \left(p_{in} + \frac{c}{tg\varphi}\right) \cdot \left(\frac{r}{R}\right)^{\frac{2\,\mathrm{sen}\,\varphi}{1-\mathrm{sen}\,\varphi}} - \frac{c}{tg\varphi} \tag{8}$$

$$\sigma_\vartheta = \left(\frac{1+\mathrm{sen}\,\varphi}{1-\mathrm{sen}\,\varphi}\right) \cdot \left(p_{in} + \frac{c}{tg\varphi}\right) \cdot \left(\frac{r}{R}\right)^{\frac{2\,\mathrm{sen}\,\varphi}{1-\mathrm{sen}\,\varphi}} - \frac{c}{tg\varphi} \tag{9}$$

$$u_r = \frac{1+\nu}{E} \cdot \left\{ \frac{R_{pl}^{k+1}}{r^k}\left(p_0 + \frac{c}{tg\varphi}\right) \cdot \mathrm{sen}\,\varphi + \left(p_0 + \frac{c}{tg\varphi}\right) \cdot (1-2\nu) \cdot \left(\frac{R_{pl}^{k+1}}{r^k} - r\right) + \right. \tag{10}$$

$$\left. + \frac{\left[1 + N_\varphi \cdot k - \nu \cdot (k+1) \cdot (N_\varphi + 1)\right] \cdot \left(p_{in} + \frac{c}{tg\varphi}\right)}{(N_\varphi + k) \cdot R^{N_\varphi - 1}} \cdot \left[\frac{R_{pl}^{N_\varphi + k}}{r^k} - r^{N_\varphi}\right] \right\}$$

where: $N_\varphi = \dfrac{1+\mathrm{sen}\,\varphi}{1-\mathrm{sen}\,\varphi}$ $\quad k = \dfrac{1+\mathrm{sen}\,\psi}{1-\mathrm{sen}\,\psi}$;

ν: Poisson ratio of the rock mass;

ψ: dilatancy angle of the rock mass.

The radial displacement of the tunnel wall u, in the presence of a ring of rock mass in the plastic field, is obtained by substituting the generic radius r with the radius of the tunnel R in eq. 10 (the trend of the displacement u in function of the internal pressure p_{in} is the convergence-confinement curve of the tunnel shown in figure 3).
The load produced on the supports of a tunnel increases with the distance from the excavation face and reach, in regime conditions, the value p_{eq} that is obtained in figure 3. In these same conditions, the pressure applied to the wall is in fact p_{eq} and therefore $p_{in} = p_{eq}$.
The internal pressure p_{in} and also the load produced on the supports, therefore also depend on the stiffness k of the support system (fig. 3).
Finally, the stress and strain state around a tunnel and in the supports depends on a series of parameters that should be known and it would be opportune to back-analyse on the basis of the measurements obtained during the work. The influencing parameters are those that appear in the previously illustrated relation (eq.s 2-10). The design of the supports should be made operating an evaluation of the load that is acting on them (p_{eq}) on the basis of reliable geometric parameters of the tunnel and mechanical parameters of the rock.

The choice of measurement systems of the stress and strain must depend on an at least preliminary evaluation of the entity of the magnitudes involved, in relation both to the precision required and the range of measurement.

4 The calculation model

The planning of open pit and underground excavations follow different methodologies:

- an analytic approach, usually reserved to simple geometric situations or parts of a general system;
- an empirical-observational approach, based on case histories of already constructed works or on the phenomenological knowledge of the rock on the whole;
- a numerical approach, in which the rock and the support works are modelled through different calculation codes, to be adopted in function of the structure of the rock and the type of work.

The numerical approach is nowadays quite common. The use of simple analytical calculation procedures, such as that illustrated in section 3, still play an important role in the preliminary comprehension of the phenomenon and definition of the physical and mechanical parameters of influence.

There however exists some limitations to a perfect adherence of a numerical modelling to the physical reality of the problems, and this is due to the heterogeneity of the geological formations, to the non simple geometry, to the spatial variability of the parameters of the rock, to the presence of water, and to the problematic knowledge of the original stress state in the rock mass. Then there are problems that are difficult to transfer to a model, which, however, are of fundamental importance: for example mention can be made of phenomena that are dependent on time, the influence of mechanical and physical anisotrophies, the relevance of technological aspects connected to the modality and times of excavation and of the installation of the supports.
For these reasons approximate schemes are adopted, usually formulating simplifying theories on the behaviour of the natural materials.
In spite of all this, the use of numerical modelling should not be played down as, from the engineering point of view, even before that of the physical or mathematical point of view, what is necessary to establish is a connection between cause and effect and to know how to understand the evolution of the phenomenon as a tendency.
In order to work in a reliable way, it is necessary to have some connections with reality. This meeting point is represented by *in situ* measurements carried out during excavation of the tunnel. Experience has shown how it is necessary to have an agreement between numerical modelling and measurements; that means that the choice of measurement parameters, the installation of the instruments, and the frequency of the measurements should be as connectable as the model can offer. On the other hand it is opportune that the model be chosen and validated taking the geostructural characteristics of the rock mass and the scale of the work into consideration (equivalent continuous, discontinuous, etc.).

The geomechanical characterisation of the rock mass, the numerical modelling of the problem under examination, the planning, the performing and the interpreting of the measurements cannot be carried out separately, but should be continuously integrable (figure 1) during the construction of the work in order to improve the numerical simulation and therefore the comprehension of the real phenomenon so as to be able to take the necessary countermeasures and, if necessary, to redefine the support system.

5 Monitoring measurements during the work

The investigations and measurements that are carried out before, during and even after the excavation of the tunnel, investigate a large series of parameters.
The geomechanical characterisation of the rock mass is entrusted to geognostic investigations, laboratory and *in situ* tests: identification of the resistance, strain and fracturing parameters of the rock, hydraulic conductibility, geostructural pattern, atypical and problematic behaviour (squeezing, swelling, slabbing, etc.).
Some of the monitoring measurements instead investigate the over all character of the work closer at hand, that is, of the rock mass and the support, in response to excavation operations. Today measurements carried out during the work are an integral part of the planning project of each work and modern personal computers are able to record (even at a distance) and facilitate the opportune interpretation of relative measurements and magnitudes of any kind.
In the field of underground construction the most frequently carried out measurements refer to one of the following groups of magnitudes, according to the particular problem: relative displacements, absolute displacements, stresses in the linings and in the rock mass, pressures on the lining and forces in the anchorages in the rock, or water table pressures. Preference is usually given to displacement measurements (such as convergence measurements of the tunnel walls) as they represent, from the mathematical point of view, integral magnitudes that are not subject to typical local effects. Stresses and strains are instead differential magnitudes that can present values that are very different from point to point and should therefore be calculated on sufficiently extended areas so as to be able to furnish appreciable indications.

With measurements carried out during the work, one usually searches for the following responses (Kovari and Amstad, 1979):

- a check on the safety;
- the determination of the properties of the materials and, if possible, of their undisturbed stress state (obtained through back-analysis techniques); the determination of the physical and mechanical characteristics of the rock mass results to be particularly interesting at this stage as they refer to the problem in real magnitude, this being different from the preliminary tests which instead involve only a limited portion of the rock mass;
- verification of the validity of the instrumental choice, with reference to the excavation method;
- comparison of the prediction theories with the real structural behaviour.

One single measurement campaign is usually enough to be able to supply several of these responses at the same time.

The most important aspects that concern monitoring with the purpose of connection with the numerical back-analysis procedures, are as follows:

1) choice of the magnitudes to measure;
2) localisation of the measurement positions;
3) definition of the area of influence on the measurements;
4) frequency of the measurements;
5) particular characteristics of the instruments.

It is useful to give some details on these aspects.

1) The displacement measurements that are usually carried out can be divided into the following types:
   - measurement of the variations of the dimensions of the tunnel (measurement of the relative convergence);
   - measurements of the displacements of the surrounding rock with reference to an area that is considered stable (measurement of distension, which can be performed working on the walls of the cavity or, when possible, from the external surfaces);
   - displacement measurements between the two borders of a relevant structural discontinuity;
   - strain measurements in the body of the lining;
   - strain measurements of the excavation face, especially when working in large sections, in soft or loose rock and when there are important reinforcing interventions of the nucleus (also called extrusion measurements).

   The loading measurements on the supports concern, in particular, the bolts, the tendons, the steel sets and the tie-bolt heads.

   The strain measurements concern, in particular, the aspects of interaction between the natural formation and the support works, whether they are of a preliminary type, such as cast concrete shells, or of a permanent type.

   The pressures due to the presence of water are likewise measured because of the great influence on the stability conditions, above all in tunnels excavated in greatly fractured masses, in low cohesive soils or when there are impermeable barriers in clay soil close to the excavations.

2) The measurement stations should be installed so as to be able to identify the expected variations without these being masked by other secondary phenomena. Furthermore, it is a good idea that the positions of the measurement stations can be

clearly identified in the numerical model used for the simulation of the tunnel. In this way one reduces the possibility of loosing information simply because of geometric type reasons (for example, because of a different orientation of a bi-dimensional model from the section to be measured) or for geological reasons (for example, because of a lithological variation between the installed measurement section and the section studied in a bi-dimensional model).

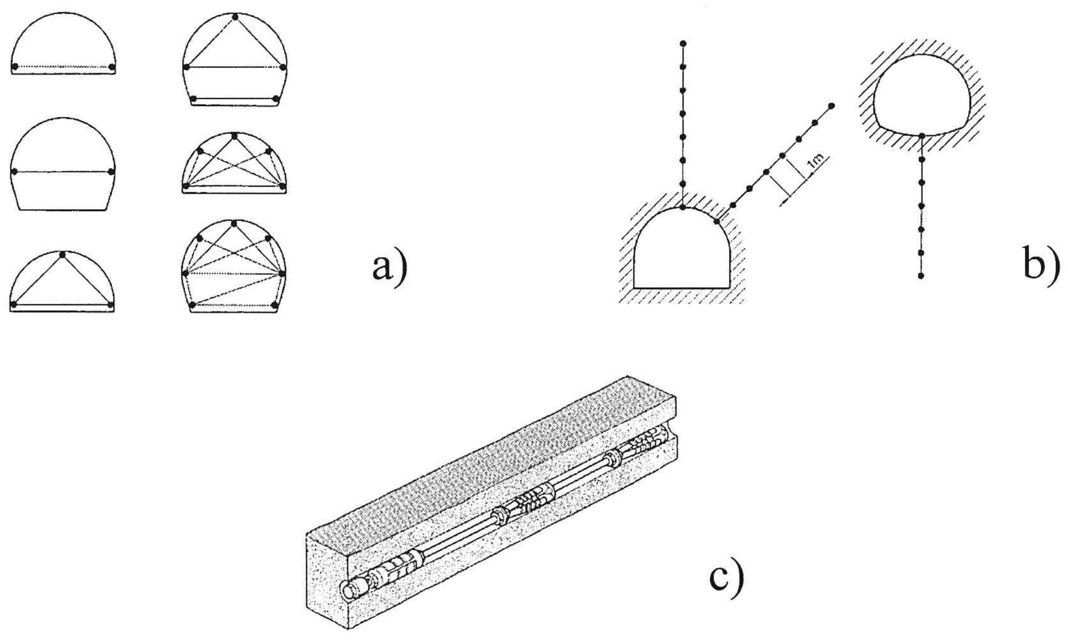

Fig. 5 Some examples of common tunnel monitoring devices. Key: a): convergences monitoring schemes; b): reference point for removable extensiometric probe; c): mechanical extensiometer.

It is also advisable to not put ones trust in a single measurement position, in order to prevent local malfunctioning or anomalies from conditioning first the good results of the measurements and then the comparison with the calculation.

3) The volume that is involved by the instrumentation is of interest in that it is from this that one derives the either local or extended value of the measurements that have been carried out. Some parameters of the rock (for example, the strain) or the stress conditions induced around the excavation are greatly influenced by the reference volume. The rock structure makes the measured value vary with the direction and with the entering of the rock, above all in fractured or anistropic rock masses. For example, the stress measurements along the border of an excavation, carried out with

flat jacks, are greatly conditioned by the redistribution of the stresses (see section 3), with steep gradients close to the angular areas.

As the volume of interest grows, the measurement data tend to become homogeneous while local measurements show a greater dispersion.

4) The frequency of the measurements should be compared to the foreseen evolution of the phenomena, to the capacity of the data acquisition system and to the advancement of the excavations. It is possible to simulate intermediate stages of the excavations in the numerical modelling through the removal of the excavation volumes, or the gradual intervention of the support means, with an application of increasing stresses at the excavation walls or rather with the decrement of the confinement pressures at the border of the excavation. It is also possible for this calculation to perform a back-analysis, not fixed to a single final value but rather to the evolution of the phenomenon.

   The enlargement of the tunnel sections, the modification of the support systems, the interventions on the invert, and reinforcing during advancement are all elements that condition the frequency of the measurements both for the purpose of checking the statics of the excavations and for a later comparison with the results of the modelling.

5) The instrumentation should be simple, well installed and robust and should allow the whole range of expected values to be measured (range). It should also maintain stability and repeatability of the measurements (reliability), guarantee closeness of the measured value to the real value (accuracy) and should be rapid (promptness).

   The measurement procedures should be carried out with care to avoid the occurrence of rough errors (for example, due to mistakes in reading the instrument) and to be able to recognise instrumental errors (for example, due to the unsuitable calibration of the instruments).

   The measurements should also be abounding, that is, it is necessary to have both a greater number of instruments available than the minimum necessary (to be able to reconstruct the trend of the measured magnitudes in time in the case where a measurement station is destroyed), and to carry out the readings with a frequency that is opportunely higher than that of the physical phenomenon. This allows one to have a cross-check between the instruments and the different magnitudes.

The precision required of the measurement instrumentation varies according to the order of magnitude of the expected phenomena. However, for practical reasons, it is common that the potentiality of the instruments to be used in tunnels is adaptable to the various requirements. It should be remembered, however, how the cost of a measurement device

is connected to the precision according to a power function: the choice should therefore respond to a cost-benefit analysis.
For example, the measurement of displacements are repeatable within 0.01 mm, the load measurements within 100 N, the pressure measurements within 10 kPa, the extensimetric deformation measurements within 5 $\mu\varepsilon$, and the inclinometric measurements within 1 mm / 30 m. Some main data concerning the most widespread instrumentation in tunnelling are given in table 1.

The responses of the various instruments are however different in terms of overall capacity to understand the physical trend of the phenomenon without feeling local effects: the convergence and the speed of convergence, the measurements with a flat jack, the load cells on the bolts or on the steel arches are, for example, measurements that are more easily interpreted than those based on extensometers mounted at the extradox of the supports. This is probably due to the damage that the instruments can undergo during the different procedures in the construction phase.

Tab. 1 Typical range, precision and possibility of errors of different measurement types used in tunnels.

Measurement type	***Range***	***Accuracy***	***Possibility of errors***
load cells	10^4-10^6 N	2-10 %	medium
strain gages deformations	500-6000 $\mu\varepsilon$	5-100 $\mu\varepsilon$	low
inclinometers	12°-30°	1-15 mm / 30 m	high
crack gages	10 mm- 50 cm	0.1-3 mm	low
convergences	10 cm- 1 m	0.1 mm- 1 cm	low
rock mass strains due to destressing	1 -10 cm	0.01- 1 mm	medium

6 Back-analysis in uncertain environments

In order to improve and make the first estimation of the geomechanical parameters of the rock obtained by the geomechanical characterisation more reliable, one should proceed with the treatment of the results of the measurements through adequate back-analysis techniques. By back-analysis in the excavation and rock engineering field one means that particular procedure, developed above all with numerical analysis methods of the stress-strain state of the rock mass and the supports which, starting from the displacement and strain measurements obtained *in situ* during the construction of the work, permits the calibration of the calculation model of the initial estimations:

- of the geomechanical parameters of the natural material;
- of the initial strain state (undisturbed) in the rock mass.

Modern back-analysis represents one of the most delicate stages of the whole planning programme as one has only limited times available to be able to supply, during the construction, the necessary guidelines for any possible improvements of the original project and for the design of the unforeseen interventions which are however necessary to guarantee the stability of the work and an economic efficiency (figure 1). This results to be even more important in the construction of underground tunnels and voids, when a certain variability of the geomechanical characteristics of the rock mass is encountered along the section which was not possible to ascertain in detail during the preliminary analysis.

Back-analysis therefore usually consists in the search for unknown parameters, of which one only has a preliminary estimation, that minimise the difference between the results of the calculation with the numerical model and the results of the performed measurements.

To perform a correct back-analysis it is usually necessary to choose:

a) a suitable calculation model that is able to determine the stress and strain state in the rock mass, with the evolution of the excavation stages;
b) the function error, which measures the distance between the forecasts and the available measurements, with a variation of the unknown parameters.

It is also necessary to notice (Sakurai, 1983) how different combinations of different values of the geotechnical parameters that characterise the rock mass can lead one to obtain, through the back-analysis process, the measured values of the stress-strain state in the rock mass and in the supports: it is therefore necessary to carry out an adequate characterisation of the rock mass during excavation of the tunnel, contextual with the monitoring, for a correct interpretation of the back-analysis, through *in situ* and laboratory tests.

From what has emerged from the previous sections of this work, both the preliminary estimation of the geomechanical parameters of the rock mass and the results of the measurements, present a certain level of uncertainty in relation to the intrinsic variability of the rock mass quality, to the reduced availability of data, and to the precision of the measurement instruments. The back-analysis should therefore be able to develop in an uncertain environment, furnishing a new estimation of the geomechanical parameters of the rock mass that are able to guarantee a greater reliability than the initial estimation (a smaller variance).

The uncertainty of the monitoring system can be described through the covariance matrix $C_{\Delta\bar{\eta}}$ of the errors $\Delta\bar{\eta}$ of the single m measurement. It is a square matrix of m x m dimensions expressed as:

$$C_{\Delta\bar{\eta}} = E\left|\Delta\bar{\eta}\Delta\bar{\eta}^T\right| \tag{11}$$

where the operator $E|\ldots|$ is the expected mean.

For statistically independent measurements, for which the operative conditions and instruments of one does not have repercussions on the others, $C_{\Delta\bar{\eta}}$ is a diagonal matrix with all positive values, which represent the variances of each single measurement. As a first approximation, the variance of a measurement can be intended as the square of the precision of a measurement instrument (see section 5).

The initial estimation of the unknown vector $x^0 = \left(x_1^0, x_2^0 \ldots, x_n^0\right)$ (see section 2) can also be considered as a probabilistic variable, for which one assumes:

$$x^{(0)} = E|x| \tag{12}$$

$$C_{x^{(0)}} = E\left|\left[x - x^{(0)}\right]\left[x - x^{(0)}\right]^T\right| \tag{13}$$

where x is the unknown vector column, of n terms;

$C_{x^{(0)}}$ is the square matrix of the covariance of the initial estimation, of n x n dimensions.

If the initial estimation of the unknown parameters are not correlated in probabilistic terms (unfortunately they often are as the initial estimation refers to the same quality index for the majority of the unknown parameters), the matrix $C_{x^{(0)}}$ is diagonal. With an increase of the uncertainty of the initial estimations, there is an increase in the values of the components of the matrix $C_{x^{(0)}}$.

If one wishes to search for a new estimation of the unknown parameters so that the distance between the unknown vector and the preliminary estimation (expressed as the difference between the unknown vector of the geomechanical parameters and the initial estimation, related to the variance of the initial estimation), added to the distance between the *in situ* measurements and the calculation results (expressed as the difference between the results of the calculation and the carried out measurements, related to the variance of the measurements) is minimal, the function error ε takes on the following form:

$$\varepsilon = \left[\bar{\eta} - \eta\right]^T \left[C_{\Delta\bar{\eta}}\right]^{-1} \left[\bar{\eta} - \eta\right] + \left[x^{(0)} - x\right]^T \left[C_{x^{(0)}}\right]^{-1} \left[x^{(0)} - x\right] \tag{14}$$

where $\bar{\eta}$ is the column vector of the mean of the *in situ* measurement, of m terms;

η is the vector column of the results of the numerical modelling, corresponding to the measurements carried out *in situ*.

In the simplest case in which the results of the numerical model η are linear functions of the unknown parameters x:

$$\eta = f(x) = \eta' + L\{x - x'\} \tag{15}$$

where η' is the column vector of m known terms, independent of x;

L is the matrix of dimensions m × n (m lines and n columns);

x' is the column vector of n known terms, equal to the value of the unknown vector x for which $\eta = \eta'$;

η', L and x' are obtained from the numerical model by varying one of the unknown parameters at a time and obtaining the results of the calculation;

deriving the function error ε (eq. 14) with respect to x, one obtains the following simple linear equation system:

$$\left[L^T C_{\Delta\bar{\eta}}^{-1} L + \left(C_{x^{(0)}}\right)^{-1}\right] x = L^T C_{\Delta\bar{\eta}}^{-1} \left[\bar{\eta} - \eta' + Lx'\right] + \left[C_{x^{(0)}}\right]^{-1} x^{(0)} \tag{16}$$

which, once solved, furnishes the solution of the back-analysis problem:

$$x^* = \left[I - M_0 L\right] x^{(0)} + M_0 \bar{\eta} - M_0 \left[\eta' - Lx'\right] \tag{17}$$

where I is the matrix identity;

$$M_0 = \left[L^T C_{\Delta\bar{\eta}}^{-1} L + \left(C_{x^{(0)}} \right)^{-1} \right]^{-1} L^T C_{\Delta\bar{\eta}}^{-1} ;$$

and the matrix of the x covariance, which permits one to describe the newly obtained estimation in probabilistic terms:

$$C_{x^*} = \left[I - M_0 L \right] C_{x^{(0)}} \left[I - M_0 L \right]^T + M_0 C_{\Delta\bar{\eta}} M_0^T \tag{18}$$

In the more general case, the function f, given in equation 15, is not linear but can be linearised through a Taylor series truncated at the first order terms. In this case, eq. 17 no longer directly produces the solution x^*, but only its approximation; one therefore proceeds iteratively, through the following calculation steps, starting from the initial estimation $x^{(0)}$:

a) the vector x' is placed equal to $x^{(i)}$ in eq.15;
b) the term η' and the matrix L in eq.15 are obtained through a parametric analysis with the numerical model (the matrix L is determined by approximation through the secant method in point $x^{(i)}$);
c) $x^{(i+1)}$ is obtained from eq. 17;
d) the iteration proceeds until the difference in norm between the two following approximations is lower than a pre-established tolerance; once the convergence is reached, the covariance matrix of the solution is calculated on the basis of eq. 18.

The availability of matrix C_{x^*} allows one to obtain a quantitative evaluation of the reliability of the results of the back-analysis and to choose the number and quality of measurements to carry out *in situ* to obtain the unknown parameters with the desired precision.

From the analysis of the probabilistic approach to the theory of the back-analysis, it is possible to conclude how it is necessary, to obtain a satisfactory calibration of the geomechanical parameters of the rock mass, to be able to refer to a preliminary characterisation based on geognostic investigations that are adequate for the problem under examination. A large uncertainty of the initial estimation would in fact reflect on the final results even in the presence of precise measurements and suitable calculation models.

The evaluation of the stability conditions of the supports can, once the back-analysis is terminated, be obtained by analysing the stress state induced inside the foreseen structure, carrying out parametric analysis which allows one to vary the geomechanical parameters of the rock in a variability interval, expected with a certain confidence.

7 Conclusions

The behaviour of supports in tunnels depends, to a great extent, on the geomechanical characteristics of the rock mass which, unfortunately, are only known with a certain approximation before starting the excavation works. The further investigations in the tunnel and the results of monitoring measurements of the behaviour of the cavity following excavation procedures, leads to the improvement of the estimation of the mechanical characteristics of the rock mass, and reduces the level of uncertainty. It is necessary to perform a back-analysis to correctly interpret monitoring measurements, that is, a type of procedure that permits the back definition of rock parameters that, considered in the calculation, produce results that are close to the performed measurements.

As both the preliminary estimation of the rock characteristics and the measurements made during excavation of the tunnel are not known with certainty, but present an uncertainty that can be described with a probability distribution, the back-analysis can be developed according to the probabilistic type approach.

The results of the procedure is the definition of a new and better estimation of the geomechanical characteristics of the rock mass and its variance. These, in the hypothesis of a probabilistic distribution of a Gaussian type, allow one to easily identify a variability interval with a certain level of confidence, for each of the uncertain geomechanical parameters. A parametric analysis has then to be performed allowing one to evaluate the stress conditions induced inside the support structure and therefore to verify their suitability and, if necessary, to carry out a new designing that takes the actually encountered conditions into account.

In order to make this procedure efficient, it is necessary to be able to perform preliminary investigations that are able to limit the uncertainty on the estimation of the geomechanical parameters as much as possible. Further investigations inside the tunnel during work are necessary to integrate the surveys and the previously performed investigations. It would be necessary to carry out investigations until the reached level of uncertainty results to be satisfactory.

The monitoring measurements should also be planned, carried out and interpreted in close connection to the tunnel calculation model, which nowadays is usually of a numerical type. At least those measurements needed in the back-analysis procedures should in fact be foreseen. The type, precision and number of measurements that should be carried out should be defined in relation to the available and set up numerical model and to the required precision of the final estimation of the geomechanical parameters of the rock mass that one wishes to obtain.

The application of this proposed procedure results to be very interesting and so the authors suggest to utilize it together with the more common typical back-analysis approach, in order to verify its suitability in different rock and support conditions.

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