Iron Loss Prediction With PWM Supply Using Low- and High-Frequency Measurements: Analysis and Results Comparison

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Abstract—In this paper, two different methods for iron loss prediction are analyzed. The first method is based on the classical separation of loss contributions (hysteresis, eddy-current, and excess losses). The model requires loss contribution separation using iron loss measurements with sinusoidal supply. In this paper, this method will be called the “low-frequency method.” The second method is based on the “high-frequency method.” It is based on the assumption that, under PWM modulation supply, the higher order flux density harmonics do not influence the magnetic work conditions. These magnetic conditions depend only on the amplitude of the fundamental harmonic of the flux density. In this paper, both the proposed methodologies and the related measurements are described in detail, and the obtained results are compared with the experimental ones. The experimental results show that both methods allow getting excellent results. The high-frequency method is better than the lower one but requires a more complex test bench. Depending on the accuracy required by the user, the more handy method can be chosen, with the guarantee that the estimation errors will be lower than 5%.

Index Terms—Iron losses, magnetic material, pulsedwidth modulation (PWM) supply.

I. INTRODUCTION

NOWADAYS, the use of pulsedwidth modulation (PWM) supply in variable-speed electrical machine drives is a standard solution. Consequently, the behavior of the magnetic lamination, used in the machine with a PWM supply voltage, is one of the most common problems to be solved in the electrical machine designer community. It is important to underline that an accurate iron loss prediction with PWM supply is not only an academic matter. In fact, the interest on this subject is also growing in the drives and control research fields to get better system efficiency [1]–[3].

In the last half decade, several researchers have proposed models and methods for predicting the iron losses in magnetic materials supplied by PWM sources. More relevant research activities on this topic can be found in [4]–[16]. It is well evident that several approaches have been analyzed, and the obtained results cannot be considered conclusive on this important subject.

In this paper, two different methods for iron loss prediction with PWM supply are analyzed and compared. The two methods are based on very similar theoretical considerations, but they require very different measurement tests for model parameter determination.

Both the proposed methodologies and the related measurements are described in detail, and the obtained results are compared with the experimental ones.

II. PROPOSED METHODOLOGY DESCRIPTION

In this section, the two considered methods for iron loss prediction with PWM supply are described and analyzed. The first method is based on the classical separation of loss contributions in hysteresis, eddy-current, and excess losses. The model requires loss contribution separation using iron loss measurements with sinusoidal supply at different frequencies and flux densities (at least two different frequencies are required). In this paper, this method will be called the “low-frequency method.” The second method is based on the assumption that, under PWM supply, the higher order flux density harmonics do not influence the magnetic work conditions. These magnetic conditions depend only on the amplitude of the fundamental harmonic of the flux density. In this method, the period of the fundamental harmonic is divided into very small intervals. In each interval, a different magnetic work condition is accepted, but the magnetic condition is assumed to be constant inside the time interval. Thus, in each time interval, it is possible to apply the superposition of the harmonics. Then, in this interval, it is possible to calculate the power losses that come from the higher order harmonics. The average power losses for each harmonic are obtained from the sum of the power losses, determined for all intervals divided by the number of intervals. This way, the power losses that come from a single higher order harmonic are determined. It is important to underline that the predicted power losses are defined under variable magnetic conditions with the time. For constant amplitude and frequency of the higher order harmonic, the power losses depend, in an essential way, on the flux density amplitude of the fundamental harmonic. The total power losses under the PWM condition are estimated by the addition of the power losses that come from the fundamental harmonic and all the higher order harmonics.
In this method, power losses that come from the fundamental harmonic can be estimated by well-known analytic methods. The proposed method requires the iron loss measurement with low-frequency sinusoidal supply (or dc supply), plus one high-frequency sinusoidal component. In this paper, this method will be called the “high-frequency method.” Hereafter, the two methods are described.

A. Low-Frequency Method

This method is based on the well-known separation of iron losses into hysteresis, eddy-current, and excess loss components. The complete description of this method is presented in [9]; hereafter, a short theoretical summary is reported.

The hysteresis component can be evaluated as follows:

\[ P_h = a \cdot f \cdot B_p^x \]  \hspace{1cm} (1)

where \( B_p \) is the peak value of the flux density, \( f \) is the frequency, and \( x \) is a Steinmetz coefficient.

It is important to underline that (1) is valid only if the supply voltage \( v(t) \) is alternating and the instantaneous value has the same sign as its first harmonic instantaneous value. In other words, no minor loops have to be present in the hysteresis cycle.

The eddy-current contribution can be evaluated as follows:

\[ P_{ec} = b \cdot f^2 \cdot B_p^2. \]  \hspace{1cm} (2)

The excess losses are due to the dynamic losses of the Weiss domains when a variable magnetic field is applied to the magnetic material. From a simple point of view, these losses are due to Block walls’ discontinuous movements with the production of Barkausen jumps. Since Barkausen jumps are very fast, they produce eddy currents and related joule losses [12]. Their contribution can be evaluated as follows:

\[ P_e = e \cdot f^{1.5} \cdot B_p^{1.5}. \]  \hspace{1cm} (3)

The coefficients “\( a \), \( b \), \( e \), and \( x \)” in the previous relations depend on the chemical and physical characteristics of the considered magnetic material. The total iron losses with sinusoidal supply can be written as the addition of the previous contributions, i.e.,

\[ P_{ir,sin} = a \cdot f \cdot B_p^x + b \cdot f^2 \cdot B_p^2 + e \cdot f^{1.5} \cdot B_p^{1.5}. \]  \hspace{1cm} (4)

To simplify the model, the skin effect related to the eddy currents in the lamination has been neglected. The iron loss separation can be obtained by iron loss measurements at several frequency and flux density values. Through these experimental results, it is possible to obtain the unknown material coefficients in (4), minimizing the following relation:

\[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{P_{ir,sin,i} - P_{ir,sin,p}}{P_{ir,sin,i}} \right)^2 \]  \hspace{1cm} (5)

where \( n \) is the number of experimental data, \( P_{ir,sin,i} \) is the measured iron losses, and \( P_{ir,sin,p} \) is the predicted iron losses in (4).

Experience from many experimental tests on several types of magnetic steel [9] shows that the excess losses can be neglected. Consequently, it is possible to simplify (4) because the coefficient “\( e \)” was always zero. This choice does not mean that the excess losses are zero but that the adopted test does not allow putting in evidence the difference between the eddy currents due to the classical losses and the eddy currents due to the excess losses. By an engineering and a very practical approach, the effects of the excess losses can be combined with the classical losses to define one global eddy-current loss.

In [13], the authors have shown that with PWM supply, the hysteresis losses are dependent on the rectified average value of the supply voltage, i.e.,

\[ P_h = \zeta V_{av} f^{1-x}. \]  \hspace{1cm} (6)

In (6), \( \zeta \) is a constant coefficient that depends on the considered material. The eddy-current losses are dependent on the rms value of the supply voltage, i.e.,

\[ P_{ec} = 2\sigma V_{rms}^2 \]  \hspace{1cm} (7)

where \( V_{rms} \) is the voltage rms value, and \( \sigma \) is a constant coefficient.

Starting from the iron loss contributions with sinusoidal supply, it is possible to predict the iron losses with a PWM supply voltage if the voltage characteristics are known. In particular, if the supply voltage is alternating and if it can be represented by two half-waves with a constant sign, then the following equation can be adopted for iron loss prediction:

\[ P_{ir} = \eta^2 P_{h,sin} + \chi^2 P_{ec,sin} \]  \hspace{1cm} (8)

where

\[ \eta = \frac{V_{av}}{V_{av,fund}} \]  \hspace{1cm} (9)

\[ \chi = \frac{V_{rms}}{V_{rms,fund}}. \]  \hspace{1cm} (10)

The mean of the quantities in (8)–(10) is listed as follows:

\( P_{h,sin} \) hysteresis losses with sinusoidal supply (at the same flux density peak of the first harmonic);

\( P_{ec,sin} \) eddy-current losses with sinusoidal supply (at the same flux density peak of the first harmonic);

\( V_{av} \) voltage mean rectified value;

\( V_{rms} \) voltage rms value;

\( V_{av,fund} \) mean rectified value of the fundamental voltage harmonic;

\( V_{rms,fund} \) rms value of the fundamental voltage harmonic.

The modern power analyzers can measure all the previously listed voltages, so the \( \eta \) and \( \chi \) coefficients can easily be computed. Consequently, by the separation of the iron loss contributions \( P_{h,sin} \) and \( P_{ec,sin} \), the prediction of the iron losses with PWM waveform can quickly be evaluated.

B. High-Frequency Method

The proposed method assumes that the higher order harmonics of the flux density do not change the magnetic work conditions. The magnetic work condition depends mainly on the amplitude of the fundamental flux density harmonic.
In this method, the subdivision of the fundamental harmonic period is requested. In each time interval, the magnetic conditions are considered as a constant, and they are defined by the instantaneous value of the fundamental flux density. This instantaneous value obviously makes reference to the considered time interval. It means that in the specified time interval, a constant magnetic permeability is accepted, and consequently, the linear condition is assumed. In the next time interval, the magnetic permeability value will be different. In the proposed method, the time interval has to be short to include both the harmonic effects and the variable magnetic work conditions. Moreover, in each time interval, the superposition of harmonics can be applied. The different magnetic conditions (with different values of the relative magnetic permeability) lead to different power loss values, coming from a single higher order harmonic. Measurements confirm this tendency. The measurements of the power losses that come from the higher order harmonic were executed in the measurement system reported in Fig. 3. The maximum value of the measured apparent power corresponds to the instant in which the maximum value of the flux density of the fundamental harmonic is present (see Fig. 1). The increase in the apparent power for these instants is not symmetrical. This is due not only to the increase in the reactive power, but also to the increase in the active
power. These waveforms were obtained by the registration of the current in HF windings and the induced voltage in the Sgn windings, which are connected in a proper way (as described in Section III). The LF windings were connected to the low-frequency supplier (50 Hz ac). This way, the low-frequency harmonic in the magnetic flux was enforced. By connecting the Sgn windings in a proper way, it is possible to observe that the induced voltage is dependent only on the high- or low-frequency signal. Then, the instant values of the current and the induced voltage were multiplied and stored. Based on these considerations, it is possible to state that the magnetic condition, determined by the fundamental harmonic of the flux density, has a direct influence on the value of the iron loss due to the higher order harmonic. A further analysis can be carried out with the exploitation of the results obtained from the tests with the dc-bias field. In these conditions, a single harmonic with specified amplitude and frequency coexists with the dc-bias field. Let us accept that the instant-by-instant shape of the flux density envelope can be considered as a straight line. Then, it is possible to accept that the temporary value of the alternating flux density corresponds to the flux density caused by the dc-bias field. Thus, in a defined time instant, the iron losses that come from the higher order harmonic can be analyzed, considering that the dc-bias field overlapped with a single higher order harmonic. For higher order harmonics, the dependence of the iron loss increase, coming from the dc-bias field, is described in [14] and [15]. The measurement results show that is possible to approximate the trend of the iron loss increase versus the dc-bias flux density value by the following polynomial equation:

\[
k(B) = 1 + A_1 B + A_2 B^2 + A_3 B^3 + A_4 B^4
\]  

(11)

where \(k(B)\) is the relative iron loss increase factor calculated for the specified dc-bias flux density value (this factor is the ratio between the iron loss for the higher order harmonic with the dc-bias condition and the power loss without the dc-bias field), \(A_1, A_2, A_3,\) and \(A_4\) are the coefficients of the polynomial, which describe the trend of \(k(B)\), and \(B\) is the value of the dc-bias flux density.

The \(A_k\) coefficients that occur in (11) must be determined by measurements under the dc-bias conditions. A direct use of this approximation for iron loss analysis is inappropriate when the fundamental harmonic and a single higher order harmonic coexist. In such conditions, we should calculate the average value of the iron loss increase coefficient. The average value computation has to be executed for the period of the fundamental harmonic. Thus, the coefficients have to be taken into account the change in the instantaneous magnetic conditions, which vary during the period of the fundamental harmonic. The same consideration can be made when the magnetic conditions are imposed not only by the fundamental harmonic (e.g., by the first and third harmonics). In general, the indispensable transformations, as described in [14] and [15], lead to

\[
k_{AV} = 1 + \alpha_1 A_1 B_{max} + \alpha_2 A_2 B_{max}^2 + \alpha_3 A_3 B_{max}^3 + \alpha_4 A_4 B_{max}^4
\]  

(12)

where \(B_{max}\) is the maximum flux density value of the waveform that establishes the magnetic work condition, \(\alpha_1, \alpha_2, \alpha_3,\) and \(\alpha_4\) are the shape waveform coefficients whose values depend on the shape of the aforementioned waveform, which imposes the magnetic condition, \(k_{AVi}\) is the average increase coefficient, calculated for the specified \(B_{max}\) flux density.

The computation of the average coefficient can be limited to a quarter of the fundamental harmonic period when the magnetic work conditions are established only by the fundamental harmonic. Then, it is possible to write

\[
k_{AV} = \frac{4}{T} \int_0^T k(B) dt
\]  

(13)

where \(k(B)\) is the relative iron loss increase factor (11), which is valid for the specified time instant, and \(T\) is the period of the fundamental harmonic.

When the fundamental harmonic is sinusoidal, we have

\[
B(t) = B_{max} \sin(\omega t).
\]  

(14)

Let us put (14) into (11) and, then, into (13). Then, it is possible to write

\[
k_{AV} = \frac{4}{T} \int_0^{T/4} (1 + A_1 B_{max} \sin(\omega t) + A_2 B_{max}^2 \sin^2(\omega t)
\]

\[
+ A_3 B_{max}^3 \sin^3(\omega t) + A_4 B_{max}^4 \sin^4(\omega t)) dt.
\]  

(15)

After transformations, we obtain (12) with the following coefficients:

\[
\alpha_1 = \frac{2}{\pi} \quad \alpha_2 = \frac{1}{2} \quad \alpha_3 = \frac{4}{3\pi} \quad \alpha_4 = \frac{3}{8}.
\]  

(16)

This way, the influence of the magnetic work condition on the iron loss produced by the higher order harmonic is taken into account.

The coefficient \(k_{AVi}\) estimated for each harmonic allows the calculation of the iron loss value with respect to the presence of the fundamental harmonic, i.e.,

\[
P_{iAC} = P_i k_{AVi}
\]  

(17)

where \(P_{iAC}\) represents the power losses of the higher order harmonic with the presence of the fundamental harmonic, \(P_i\) represents the power losses of the higher order harmonic without the fundamental harmonic, and \(k_{AVi}\) is the average coefficient of the iron loss increase, determined for the \(i\)th specified harmonic and estimated by (12) or measured in the presence of the fundamental harmonic and a single higher order harmonic.

Because the higher order harmonics do not change the magnetic work conditions, it is possible to add the iron losses due to each harmonic. The method presented above allows accurate iron loss calculation with respect to the fundamental harmonic. Finally, when the fundamental flux density harmonic defines only the magnetic work conditions, it is possible to write the
formula that estimates the total iron losses with PWM supply as follows:

\[ P_{\text{TOT}} = P_1 + \sum_{i=m}^{n} P_i k_{AV_i} \]  

(18)

where

- \( k_{AV_i} \) average coefficient of the iron loss increase determined for the \( i \)th harmonic;
- \( P_{\text{TOT}} \) total iron loss value under the PWM supply condition;
- \( m \) number of the first harmonic in the flux spectrum, which does not influence the magnetic work conditions;
- \( n \) number of the last harmonic in the flux density spectrum;
- \( P_1 \) iron loss for the fundamental harmonic, which is calculated by any analytical formula, e.g., (4), and establishes the magnetic work conditions;
- \( P_i \) iron losses of the \( i \)th harmonic when this harmonic is solely present in the magnetic core.

III. TEST BENCH DESCRIPTION

The two methods have been applied on medium- and low-quality magnetic steel normally used for industrial motor realization.

As requested by the high-frequency method, two toroidal cores have been assembled, overlapping 22 rings. In Fig. 2, the actual test bench is shown, whereas in Fig. 3, the test bench layout used for the high-frequency method is depicted.

As shown in detail in Fig. 4, the high-frequency method requires the following three windings:

1) LF1 and HF1 exciting windings for sample 1;
2) LF2 and HF2 exciting windings for sample 2;
3) Sgn1 and Sgn2 additional measuring windings for samples 1 and 2, respectively.

All the windings in the two samples have 110 turns and are uniformly distributed on the entire sample circumference to avoid leakage flux.

The HF windings are supplied by the HF supply source (in the test bench, this is an audio power amplifier with the frequency of the output voltage changing in a wide range, i.e., 500–5000 Hz). The LF windings are supplied by the low-frequency supply (50 Hz in the tests).

The Sgn windings are used for taking out the high-frequency signal of the two samples. With reference to the black points in Fig. 4, the LF1 and LF2 windings are connected in opposite series so that the high-frequency induced voltage in the LF1 + LF2 winding is equal to zero. The HF1 and HF2 windings and the Sgn1 and Sgn2 windings are connected in series so that the low-frequency induced voltage in the HF1 + HF2 and Sgn1 + Sgn2 windings is also equal to zero.

This type of connection allows decoupling of the low- and the high-frequency supply from the electrical point of view. The Sgn1 and Sgn2 windings are connected in series, allowing the measure of the Sgn1 + Sgn2 induced high-frequency voltage to overimpose on the low-frequency one. The current in the HF1 winding and the voltage in the Sgn1 + Sgn2 winding are measured and stored. This way, only the power losses produced by the higher order harmonic, in the presence of the fundamental harmonic, are measured.

The low-frequency method requires only one sample, and the iron loss measurement method is the same as the Epstein frame one.

IV. EXPERIMENTAL RESULTS

The low-frequency method has been implemented by performing tests at 10, 20, 30, 40, 50, 60, 70, 80, 100, 150, and 200 Hz, with flux densities in the range of 0.2–1.7 T. The loss
separation in (4), with “e” being equal to zero, has led to the following value of the material coefficients:

\[ a = 0.0513 \quad b = 0.00022 \quad x = 1.904. \]

One sample has been supplied by a PWM line-to-line voltage (three-level waveform) of an industrial three-phase inverter with the following setup.

1) The switching frequency is equal to 2 kHz.
2) The fundamental frequency is equal to 50 Hz.
3) The dc bus voltage is constant.
4) The variable modulation index “m” is between 0.2 and 1.

In the measurements, for each considered modulation index “m,” the PWM output voltage contained different harmonic spectra. Two of these spectra are presented in Figs. 5 and 6.

When the modulation index “m” is equal to one, the fundamental harmonic voltage has a significantly greater percentage amplitude than the higher order harmonics. Meanwhile, when the modulation index “m” is equal to 0.2, the fundamental harmonic voltage has a lower percentage amplitude with respect to high order harmonics. As previously discussed (14), the harmonic spectrum is requested for the implementation of the high-frequency method, which has to be measured with good accuracy.

By means of (8)–(10), the predicted specific iron losses with PWM supply have been computed and compared with the measured ones, as shown in Fig. 7. The good agreement between the measured and the predicted results is well evident, with an average percentage error lower than 5%.

As shown in Fig. 7, the prediction values are higher than the measured ones, so the results are precautionary for an applicative use.

The comparison between the measured and predicted iron losses (in watts per kilogram), using the high-frequency method described in Section II, is shown in Fig. 8. In addition, in this case, the excellent agreement between the predicted and the measure specific iron losses is well evident, with an average percentage error lower than 0.5%. Both the proposed methods allow getting interesting results compared with the measured ones.

As a final remark, the choice of the methods depends on the requested accuracy and, most importantly, on the capability to
perform the tests requested by the two methods (as discussed in Section II). In fact, the performed tests at a 50-Hz fundamental frequency show that the high-frequency method is more accurate than the lower one but requires a more complex test bench and a more complex computation procedure.

To understand if the high-frequency method is, in general, better than the low-frequency one, tests at different fundamental and switching frequencies will be performed, and the results will be reported in a future paper.

V. Conclusion

In this paper, the comparison between two methods for predicting iron losses in magnetic lamination supplied by PWM supply has been presented. One method is based on the classical iron loss contribution separation at a low frequency, whereas the second one uses tests performed at high frequencies over-imposed at a low-frequency supply. Both methods allow getting good results. The high-frequency method is more accurate than the lower one but required a more complex test bench and a more complex computation procedure. Depending on the accuracy required by the user, the more handy method can be chosen, with the guarantee that the errors will be lower than 5%. The application of the two methods for predicting the iron losses in inverter-fed induction motors is currently under analysis.

References


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