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Diffraction on two opposite parallel PEC half-planes at skew incident

V. Daniele[†] G. Lombardi[&]

Abstract – **The diffraction of an arbitrary incident plane wave on two opposite PEC half-planes is formulated in terms of Wiener-Hopf equations. The factorization of the 4x4 matrix kernel is reduced to the factorization of a 2x2 matrix kernel. The factorization of the kernel is obtained by the solution of a Fredholm equation of second kind that provides very accurate results.**

1 INTRODUCTION

We consider an arbitrary plane wave incident on two opposite PEC half-planes, see Figure 1:

$$\begin{cases} \boldsymbol{E}_z^i = \boldsymbol{E}_o e^{j\tau_o \rho \cos(\varphi-\varphi_o)} e^{-j\alpha_o z} \\ \boldsymbol{H}_z^i = \boldsymbol{H}_o e^{j\tau_o \rho \cos(\varphi-\varphi_o)} e^{-j\alpha_o z} \end{cases} \quad (1)$$

where k is the free-space wave number, $\tau_o = k\sin\beta$, $\alpha_o = k\cos\beta$, and β and φ_o are respectively the zenithal and azimuthal angles of the incident wave.

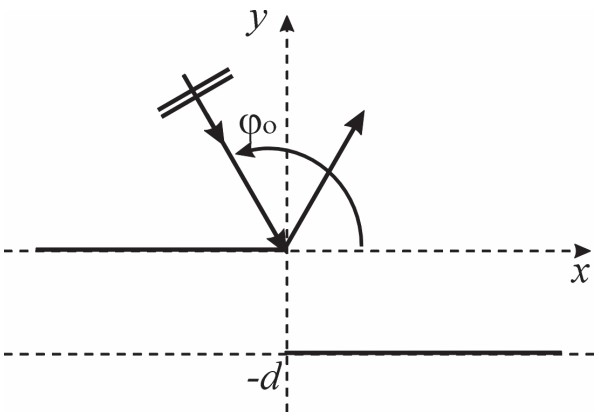

Figure 1: Two opposite parallel PEC half-planes, normal view.

We formulate the problem in the spectral domain by using the Fourier transforms:

$$\begin{aligned} \mathbf{V}(\eta, y) &= e^{j\alpha_o z} \int_{-\infty}^{\infty} \hat{y} \times \mathbf{E}_t(x, y, z) e^{j\eta x} dx \\ \mathbf{I}(\eta, y) &= e^{j\alpha_o z} \int_{-\infty}^{\infty} \mathbf{H}_t(x, y, z) e^{j\eta x} dx \end{aligned} \quad (2)$$

The spectral domain allows to describe the problem with an equivalent circuit representation, see Figure 2, where

$$\mathbf{Y}_c = \frac{Y_o}{k\sqrt{\tau_O^2 - \eta^2}} \begin{vmatrix} \tau_O^2 & -\eta\,\alpha_O \\ -\eta\,\alpha_O & k^2 - \eta^2 \end{vmatrix} \quad (3)$$

is the matrix characteristic admittance of the free space [1].

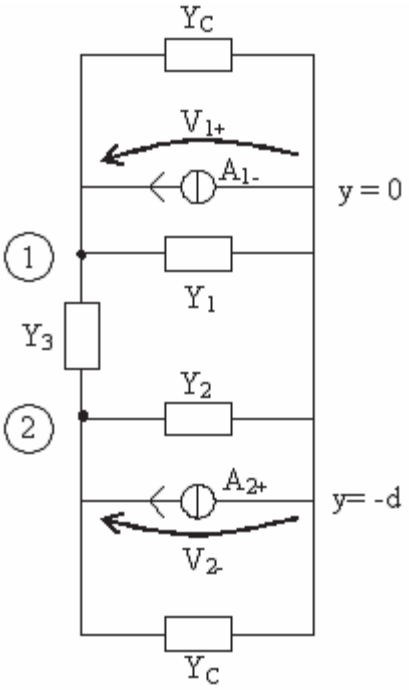

Figure 2: Equivalent circuit representation

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Figure 2 uses the following definitions:

$$\mathbf{Y}_1 = \mathbf{Y}_2 = j\mathbf{Y}_c \tan\frac{\tau d}{2},\ \mathbf{Y}_3 = -j\mathbf{Y}_c \frac{1}{\sin\tau d},$$
$$\tau = \sqrt{\tau_o^2 - \eta^2}$$

With reference to the circuit representation, the node equations yield directly the W-H equations of the problem:

$$\begin{cases} (Y_1 + Y_3 + Y_c)V_{1+} - Y_3 V_{2-} = A_{1-} \\ -Y_3 V_{1+} + (Y_2 + Y_3 + Y_c)V_{2-} = A_{2+} \end{cases} \quad (4)$$

where $\mathbf{V}_{1+} = \mathbf{V}(\eta, 0)$, $\mathbf{V}_{2-} = \mathbf{V}(\eta, -d)$, and $-A_{1-}$ and $-A_{2+}$ are the Fourier transforms of the total currents induced on the two half planes.

Taking into account that the source is constituted by an incident plane wave, the W-H equations present the following standard form:

$$G(\eta)\cdot F_+(\eta) = F_-(\eta) = F_-^S(\eta) + \frac{R_o}{\eta - \eta_o} \quad (5)$$

where the unknowns are

$$\begin{cases} F_+(\eta) = \left| V_{1+} \quad A_{2+} \right|^t \\ F_-(\eta) = \left| A_{1-} \quad V_{2-} \right|^t \end{cases} \quad (6)$$

The W-H kernel is

$$\mathbf{G}(\eta) = \begin{pmatrix} 2\mathbf{Y}_c & -e^{-j\tau d}\mathbf{1} \\ e^{-j\tau d}\mathbf{1} & \dfrac{1 - e^{-2j\tau d}}{2}\mathbf{Z}_c \end{pmatrix} \quad (7)$$

where

$$\mathbf{Z_c} = \mathbf{Y}_c^{-1} \frac{Z_o}{k\sqrt{\tau_O^2 - \eta^2}} \begin{vmatrix} k^2 - \eta^2 & \eta\,\alpha_O \\ \eta\,\alpha_O & \tau_O^2 \end{vmatrix}.$$

The source term of the W-H equation (5) is $R_o = \left| R_{o1} \quad R_{o2} \quad 0 \quad 0 \right|^t$, and $\eta_o = -\tau_o \cos\varphi_o$ (source pole).

R_{o1} and R_{o2} are known scalars depending on the parameters of the incident plane wave. We avoid to report the expression of R_{o1} and R_{o2}.

2 REDUCTION OF THE PROBLEM TO FACTORIZATION OF 2X2 MATRICES

We multiply and post multiply the kernel $G(\eta)$ by suitable minus and plus rational matrices. The result is $G_t(\eta)$ given by:

$$\mathbf{G}_t(\eta) = \begin{vmatrix} \mathbf{G}_{th}(\eta) & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{te}(\eta) \end{vmatrix} \quad (8)$$

Therefore the factorization of the matrix kernel $G(\eta)$ of order four can be reduced to the factorization of the matrix $G_t(\eta)$.

The two matrices of order two $\mathbf{G}_{th}(\eta)$ and $\mathbf{G}_{te}(\eta)$ are given by:

$$G_{te}(\eta) = \begin{pmatrix} 2 & -\dfrac{\sqrt{\tau_o - \eta}}{\sqrt{\tau_o + \eta}} e^{-j\tau d} \\ \dfrac{\sqrt{\tau_o + \eta}}{\sqrt{\tau_o - \eta}} e^{-j\tau d} & \dfrac{1 - e^{-2j\tau d}}{2} \end{pmatrix} \quad (9)$$

$$G_{th}(\eta) = \begin{pmatrix} 2 & -\dfrac{\sqrt{\tau_o + \eta}}{\sqrt{\tau_o - \eta}} e^{-j\tau d} \\ \dfrac{\sqrt{\tau_o - \eta}}{\sqrt{\tau_o + \eta}} e^{-j\tau d} & \dfrac{1 - e^{-2j\tau d}}{2} \end{pmatrix} \quad (10)$$

We observe that it is possible to obtain the factorization of the matrix $\mathbf{G}_{th}(\eta)$, *i.e.* $\mathbf{G}_{th}(\eta) = \mathbf{G}_{th-}(\eta)\cdot\mathbf{G}_{th+}(\eta)$ in terms of the factorization of matrix $\mathbf{G}_{te}(\eta)$, *i.e.* $\mathbf{G}_{te}(\eta) = \mathbf{G}_{te-}(\eta)\cdot\mathbf{G}_{te+}(\eta)$.

Consequently the factorization of the complete kernel $G(\eta)$ requires only the factorization of one matrix of order two: $\mathbf{G}_{te}(\eta)$.

3 SOLUTION

The factorization of matrix $\mathbf{G}_{te}(\eta)$ constitutes a problem that is well known in the literature. However no exact solution of this problem has been obtained up to now. Usually an approximate factorization is obtained with the technique described in [2].

Alternatively we can apply a general method that reduces the general problem of factorization to the solution of a Fredholm integral equation of second kind [3]:

$$F_+(\eta)+\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\left[G^{-1}(\eta)G(u)-\mathbf{1}\right]F_+(u)}{u-\eta}du=G^{-1}(\eta_o)\frac{R_o}{\alpha-\eta_o}$$

where $\mathrm{Im}[\alpha_o]>0$.

We have experienced that the Fredholm technique produces very accurate results for example as reported in [4] for wedge diffraction problems.
From the evaluation of the unkowns (6) we can obtain the total field of the problem.
Numerical results will be shown at the Conference.

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References

[1] V.Daniele, Electromagnetic propagation in plane stratified regions, Internal Report ELT-2006 , Politecnico di Torino, available at http://www.eln.polito.it/staff/daniele.

[2] A.Büyükaksoy, and A.H Serbest in, "Analytical and Numerical Methods in Electromagnetic Wave Theory," M. Hashimoto, M .Idemen and O.A. Tretyakov Editors, Science House Co.,Ltd., Tokio, pp.257-315, 1993.

[3] V.Daniele, An introduction to the Wiener-Hopf technique for the solution of electromagnetic, Internal Report ELT-2004-1, Politecnico di Torino, http://www.eln.polito.it/staff/daniele

[4] Daniele V., and G. Lombardi, "Wiener-Hopf solution for impenetrable wedges at skew incidence," IEEE Trans. Antennas Propagat., vol. AP-54, no. 9, pp. 2472-2485, Sept. 2006, doi:10.1109/TAP.2006.880723