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The Two-Echelon Capacitated Vehicle Routing Problem: models and math-based heuristics

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Multi-echelon distribution systems are quite common in supply-chain and logistics. They are used by public administrations in their transportation and traffic planning strategies as well as by companies to model own distribution systems. In the literature, most of the studies address issues relating to the movement of flows throughout the system from their origins to their final destinations. Another recent trend is to focus on the management of the vehicle fleets required to provide transportation among different echelons.

The aim of this paper is twofold. First, it introduces the family of Two-Echelon Vehicle Routing Problems, a term which broadly covers such settings, where the delivery from one or more depots to customers is managed by routing and consolidating freight through intermediate depots. Second, it considers in detail the basic version of Two-Echelon Vehicle Routing Problems, the Two-Echelon Capacitated Vehicle Routing Problem, which is an extension of the classical VRP where the delivery is compulsorily delivered through intermediate depots, named satellites.

A mathematical model for Two-Echelon Capacitated Vehicle Routing Problem, some valid inequalities, and two math-heuristics based on the model are presented. Computational results of up to 50 customers and 4 satellites show the effectiveness of the methods developed.

Key words: Vehicle Routing; Two-Echelon Systems; City Logistics
History:

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1. Introduction

The freight transportation industry is a major source of employment and supports the economic development of the country. However, freight transportation is also a disturbing activity, due to congestion and environmental nuisances which negatively affect the quality of life, in particular in urban areas.

In freight transportation there are two main distribution strategies: direct shipping and multi-echelon distribution. In direct shipping, vehicles starting from a depot transport their freight directly to the customers, while in multi-echelon systems the freight is delivered from the origin to the customers through intermediate depots. The growth in the volume of freight traffic as well as the need to take into account factors such as the environmental impact and traffic congestion has led research in recent years to focus on multi-echelon distribution systems, and, in particular, two-echelon systems (Crainic et al., 2004). In two-echelon distribution systems, freight is delivered to an intermediate depot and from this depot to customers.

Multi-echelon systems presented in the literature refer to the movement of flows throughout the system from their origins to their final destinations and, eventually, explicitly consider only the routing problem at the last level of the transportation system (Ricciardi et al., 2002; Daskin et al., 2002; Shen et al., 2003).

Moreover, in the past decade multi-echelon systems with LTL dispatching policies have been introduced by practitioners in different areas such as express delivery service companies (http://www.tntlogistics.com), grocery and hypermarkets product distribution, e-commerce and home delivery services (http://www.sears.com), Newspaper and press distribution (Jacobsen and Madsen, 1980), and city logistics (Crainic et al., 2004).

The main contribution of this paper is to introduce *Two-Echelon Vehicle Routing Problems*, a new family of routing problems where routing and freight management are explicitly considered at their different levels. The basic variant of Two-Echelon Vehicle Routing Problems, the *Two-Echelon Capacitated Vehicle Routing Problem* (2E-CVRP) is introduced and examined in detail. In 2E-CVRP, the freight delivery from the depot to the customers is managed by shipping the freight through intermediate depots. Thus, the transportation network is decomposed into two levels, the 1st level connecting the depot to the intermediate depots and the 2nd connecting the intermediate depots to the customers. The objective is to minimize the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate depots are considered, while the timing of the deliveries is ignored.

A flow-based model for the 2E-CVRP is introduced, as well as valid inequalities used to strengthen the continuous lower bound. Moreover, the same model is used to derive two fast math-heuristics.

The paper is organized as follows. In Section 2 we recall the literature related to Multi-Echelon Distribution Systems, showing similarities and differences between Two-Echelon Vehicle Routing Problems and other problems present in the literature. In Section 3 we give a general description of Two-Echelon Vehicle Routing Problems. In Section 4 2E-CVRP is introduced and a mathematical model is given, which is strengthened by means of valid inequalities in Section 5. Section 6 presents the two heuristics using different simplified variants of the base model to quickly find feasible solutions for the 2E-CVRP. Finally, test instances for 2E-CVRP are introduced and some computational results are discussed in Section 7.

2. Literature review

Freight distribution and vehicle routing have been playing in the past decade a central role not only in the supply chain and production planning, but also for their leading role in several environmental and politic aspects. Moreover, several transportation and production systems have been moved from a single-level to a multi-echelon distribution schema. As stated in the introduction, this paper focuses on the extension to multi-echelon systems of vehicle routing problems, which have been poorly studied so far from the routing point of view. For this reason, the literature review is presented below along two directions. First, some references for the Vehicle Routing Problems are shortly recalled. Second, a more detailed review of the literature on multi-echelon systems is discussed.

Vehicle Routing has become a central problem in the fields of logistics and freight transportation. In some market sectors, transportation costs constitute a high percentage of the value added of goods. Therefore, the use of computerized methods for transportation can result in savings ranging from 5% to as much as 20% of the total costs (Toth and Vigo, 2002). Unfortunately, to our knowledge, even if VRP problems are present in the literature in many variants, only the single-level version of the Vehicle Routing Problem has been studied (see Toth and Vigo (2002); Baldacci et al. (2007) for the main contributions in the area and Perboli et al. (2008) for a comparison of the main heuristic methods in the Capacitated VRP case).

In the literature, the multi-echelon systems, and the two-echelon systems in particular, refer mainly to supply chain and inventory problems (Daskin et al., 2002; Shen et al., 2003; Verrijdt and de Kok, 1995). These problems do not use an explicit routing approach for the different levels, but focus more on the production and supply chain management issues.

Several papers deal with the design of the Multi-Echelon system with different levels of detail (for a survey on continuous location models, network location models, mixed-integer programming models, and applications see Klose and Drexl (2005)). These papers include the location and relocation of uncapacitated (Galvão and Santibaez-Gonzalez, 1992; Khumawala and Whybark, 1976; Van Roy and Erlenkotter, 1982; Tadei et al., 2009) and capacitated (Hormozi and Khumawala, 1996; Barros, 1998; Barros and Labbé, 1994; Ricciardi et al., 2002; Tadei et al., 2010) intermediate depots, as well as with multiple objective functions (Melachrinoudis and Min, 2000; Min and Melachrinoudis, 1999) and budget constraints (Wang et al., 2003). For a classification of the different types of problems, as well as for a general mathematical framework for the dynamic multi-commodity capacitated facility location of intermediate depots, please refer to Melo et al. (2006). However, a common issue of these articles is the fact that the routing aspects are simplified by approximating the true routes by direct shipping.

The first application of a two-echelon distribution system with an explicit minimization of the total transportation costs can be found in Jacobsen and Madsen (1980). In this study, a comparison is presented between several fast heuristics for solving a two-echelon Location Routing Problem in which transfer points are not known in advance. The distribution system and input data are based on a real case, in which two newspaper publishers combined their printing and transportation facilities to decrease transportation costs. A more recent real application of two-tier distribution networks is due to Crainic et al. (2004) and is related to the freight distribution in a large urban area. The authors developed a two-tier freight distribution system for congested urban areas, using small intermediate platforms, called satellites, as intermediate points for the freight distribution. In Crainic et al. (2007, 2009b) the same authors introduced the first formal definition of 2E-CVRP as a two-level, time-dependent, synchronized, multi-tour, multi-depot, multi-product, heterogeneous fleet (on each level) VRP with hard (at satellites) and soft (at customers) time windows.

As stated before, multi-echelon systems presented in the literature usually explicitly consider the routing problem only at the last level of the transportation system, while at higher levels a simplified routing problem is considered. While this relaxation may be acceptable if the dispatching at higher levels is managed with a truckload policy (TL), the routing costs of the higher levels are often underestimated and decision-makers cannot directly use the solutions obtained from the models in the case of the less-than-truckload (LTL) policy (Daskin et al., 2002; Shen et al., 2003; Verrijdt and de Kok, 1995).

In the case where LTL policy with vehicle trips serving several customers is applied only at the second level, the problem is similar to a multi-depot VRP. However, since the most critical decisions are related to which satellites will be used and to the assignment of each customer to a satellite, more pertinent methods will be found in Location Routing Problems (LRP). In these problems, the location of the distribution centers and the routing problem are not solved as two separate problems, but are considered as a unique more complex problem (for a more detailed survey of LRP, see Nagy and Salhi, 2007; Albareda-Sambola et al., 2005; Prins et al., 2007). Moreover, even if the routing from the intermediate depots to the customers is considered, as for example in the Capacitated Location Routing Problem, the routing cost between the central depot and the satellites is ignored (Boulanger and Semet, 2009). Other LRP studies refer to direct shipping strategies in order to simplify the routing costs and some heuristics have been developed for specific multi-echelon problems, even if no extension to a general multi-echelon routing scheme has been developed (Jacobsen and Madsen, 1980; Gendron et al., 2009). Another application of a specific two-echelon routing problem is the Truck and Trailer Routing Problem (TTRP). In this problem some customers can be serviced by either a complete vehicle (i.e. a truck pulling a trailer) or a single truck, while others can only be serviced by a single truck for various reasons (e.g. government regulations, limited maneuvering space at customer site, road conditions, etc.) (Semet and Taillard, 1993; Lin et al., 2009; Villegas et al., 2010). Moreover, a limited number of parking points let the drivers to detach the trailer in order to service a subset of customers which require the truck only. This implies the presence in the solution of three types of routes: a pure truck route traveled by a single truck; a pure vehicle route without any sub-tour traveled by a complete vehicle; and a complete vehicle route consisting of a main tour traveled by a complete vehicle, and at least one sub-tour traveled by the truck alone. The last type of route is a two-level route, where the trailer at the parking point acts as a small satellite. The network structure is partially a two-echelon one, but on the contrary of 2E-CVRP just one single type vehicle and no consolidation at satellites are considered.

3. Two-Echelon Vehicle Routing Problems

Freight consolidation from different shippers and carriers associated with some kind of coordination of operations is among the most important ways to achieve a rationalization of the distribution activities. Intelligent Transportation Systems technologies and operations research-based methodologies enable the optimization of the design, planning, management, and operation of City Logistics systems (Crainic et al., 2009a; Taniguchi et al., 2001).

Consolidation activities take place at so-called *Distribution Centers* (DCs). When such DCs are smaller than a depot and the freight can be stored for only a short time, they are also called *satellite platforms*, or simply *satellites*. Long-haul transportation vehicles dock at a satellite to unload their cargo. Freight is then consolidated in smaller vehicles, which deliver them to their final destinations. Clearly, a similar system can be defined to address the reverse flows, i.e., from origins within an area to destinations outside it.

As stated in the introduction, in the Multi-Echelon Vehicle Routing Problems the delivery from the depot to the customers is managed by rerouting and consolidating the freight through different intermediate satellites. The general goal of the process, which is also known in the literature as cross-docking (Barthold and Gue, 2004), is to ensure an efficient and low-cost operation of the system, while the freight is delivered on time and the total cost of the traffic on the overall transportation network is minimized. Usually, capacity constraints on the vehicles and the satellites are considered.

More precisely, in the Multi-Echelon Vehicle Routing Problems the overall transportation network can be decomposed into $k \geq 2$ levels:

- the 1st level, which connects the depots to the 1st-level satellites;
- $k-2$ intermediate levels interconnecting the satellites;
- the last level, where the freight is delivered from the satellites to the customers.

The most common version of Multi-Echelon Vehicle Routing Problem arising in practice is the *Two-Echelon Vehicle Routing Problem*, where only one intermediate level of satellite depots is present.

Let us denote the depot by v_0, the set of intermediate depots called satellites by V_s and the set of customers by V_c. Let n_s be the number of satellites and n_c the number of customers. The depot is the starting point of the freight and the satellites are capacitated. The customers are the destinations of the freight and each customer i shows a demand d_i, i.e. the quantity of freight that has to be delivered to that customer. The demand of each customer cannot be split among different vehicles at the 2nd level. For the first level, we consider that each satellite can be served by more than one 1st-level vehicle, so the aggregated freight assigned to each satellite can be split into two or more vehicles. Each 1st level vehicle can deliver the freight of one or more customers, as well as serve more than one satellite in the same route.

The distribution of the freight cannot be managed by direct shipping from the depot to the customers. Instead the freight must be consolidated from the depot to a satellite and then delivered from the satellite to the desired customer. This implicitly defines a two-echelon transportation system: the 1st level interconnecting the depot to the satellites and the 2nd one the satellites to the customers (see Figure 1).

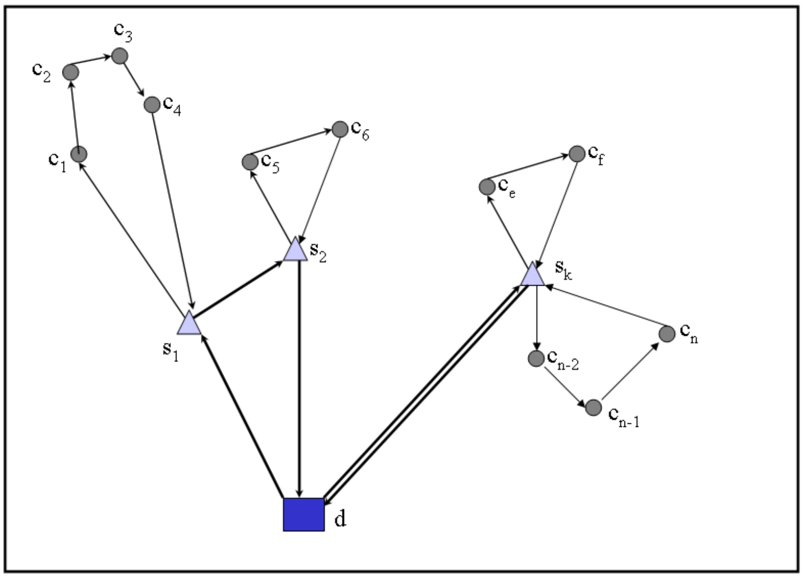

Figure 1 Example of 2E-CVRP transportation network.

We define as arc (i, j) the direct route connecting node i to node j. If both nodes are satellites or if one is the depot and the other is a satellite, we define the arc as belonging to the 1st-level network, while if both nodes are customers or if one is a satellite and the other is a customer, the arc belongs to the 2nd-level network.

We define as *1st-level route* a route run by a 1st-level vehicle which starts from the depot, serves one or more satellites and ends at the depot. A *2nd-level route* is a route run by a 2nd-level vehicle which starts from a satellite, serves one or more customers and ends at the same satellite.

The problem is easily seen to be NP-Hard via a reduction to VRP, which is a special case of 2E-CVRP arising when just one satellite is considered.

In the following, we will focus on Two-Echelon Vehicle Routing Problems, using them to illustrate the various types of constraints that are commonly defined on Multi-Echelon Vehicle Routing Problems. We can define three groups of variants:

Basic variants with no time dependence:

- Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP). This is the simplest version of Multi-Echelon Vehicle Routing Problems. At each level, all vehicles belonging to that level have the same fixed capacity. The size of the fleet of each level is fixed, while the number of vehicles assigned to each satellite is not known in advance. The objective is to serve customers by minimizing the total transportation cost, satisfying the capacity constraints of the vehicles. There is a single depot and a fixed number of capacitated satellites. All the customer demands are fixed, known in advance and must be compulsorily satisfied. Moreover, no time window is defined for the deliveries and the satellite operations. For the 2nd level, the demand of each customer is smaller than each vehicle's capacity and cannot be split in multiple routes of the same level.

Basic variants with time dependence:

- Two-Echelon VRP with Time Windows (2E-VRP-TW). This problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered. The time windows can be hard or soft. In the first case the time windows cannot be violated, while in the second, if they are violated a penalty cost is due.
- Two-Echelon VRP with Satellites Synchronization (2E-VRP-SS). In this problem, time constraints on the arrival and the departure of vehicles at the satellites are considered. In fact, the vehicles arriving at a satellite unload their cargo, which must be immediately loaded into a

2nd-level vehicle. Also this kind of constraints can be of two types: hard and soft. In the hard case, every time a 1st-level vehicle unloads its freight, 2nd-level vehicles must be ready to load it (this constraint is formulated through a very small hard time window). In the second case, if 2nd-level vehicles are not available, the demand is lost and a penalty is paid. If the satellites are capacitated, constraints on loading/unloading operations are incorporated, such that in each time period the satellite capacity in not violated.

Other 2E-CVRP variants are:

- Multi-depot problem. In this problem the satellites are served by more than one depot. A constraint forcing to serve each customer by only one 2nd-level vehicle can be considered. In this case, we have a Multi-Depot Single-Delivery Problem.
- 2E-CVRP with Pickup and Deliveries (2E-VRP-PD). In this case we can consider the satellites as intermediate depots where both the freight that has been picked-up from the customers and that which must be delivered to the customers are stored.
- 2E-CVRP with Taxi Services (2E-VRP-TS). In this variant, direct shipping from the depot to the customers is allowed if it helps to decrease the cost, or to satisfy time and/or synchronization constraints.

4. The Two-Echelon Capacitated Vehicle Routing Problem

As stated in Section 3, 2E-CVRP is the two-echelon extension of the well known CVRP problem. In this section we describe in detail the 2E-CVRP and introduce a mathematical formulation solving small and medium-sized instances. We do not consider any time windows or satellite synchronization constraints. In order to help the reader, we summarize the definitions of variables and constants in Table 1.

4.1. A Flow-based Model for 2E-CVRP

According to the definition of 2E-CVRP, if the assignments between customers and satellites are determined, the problem reduces to $1+n_s$ VRP (1 for the 1st-level and n_s for the 2nd-level).

The main question when modeling 2E-CVRP is how to connect the two levels and manage the dependence of the 2nd-level from the 1st one.

The freight must be delivered from the depot v_0 to the customers set $V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$. Let d_i be the demand of the customer c_i. The number of 1st-level vehicles available at the depot is m_1. These vehicles have the same given capacity K^1.

The total number of 2nd-level vehicles available for the second level is equal to m_2. The total number of active vehicles cannot exceed m_2 and each satellite k has a maximum capacity m_{s_k}. The 2nd-level vehicles have the same given capacity K^2. No additional limitation on the route size, neither in length, nor in number of visited customers is introduced.

In our model we will not consider the fixed costs of the vehicles, since we suppose that they are available in fixed number. Let us consider the travel costs c_{ij}, which are of two types:

- costs of the arcs traveled by 1st-level vehicles, i.e. arcs connecting the depot to the satellites and the satellites between them;
- costs of the arcs traveled by 2nd-level vehicles, i.e. arcs connecting the satellites to the customers and the customers between them.

Another cost that can be used is the cost of loading and unloading operations at the satellites. Supposing that the number of workers in each satellite v_{s_k} is fixed, we consider only the cost incurred by the management of the freight and we define F_k as the unit cost of freight handling at the satellite v_{s_k}.

The formulation we present derives from the multi-commodity network design and uses the flow of the freight on each arc as main decision variables.

We define five sets of variables, that can be divided in three groups:

Table 1 Definitions and notations.

$V_0 = \{v_0\}$	Depot
$V_s = \{v_{s_1}, v_{s_2}, ..., v_{s_{n_s}}\}$	Set of satellites
$V_c = \{v_{c_1}, v_{c_2}, ..., v_{c_{n_c}}\}$	Set of customers
n_s	Number of satellites
n_c	Number of customers
m_1	Number of the 1st-level vehicles
m_2	Number of the 2nd-level vehicles
m_{s_k}	Maximum number of 2nd-level routes starting from satellite k
K^1	Capacity of the vehicles for the 1st level
K^2	Capacity of the vehicles for the 2nd level
d_i	Demand required by customer i
c_{ij}	Cost of the arc (i,j)
F_k	Cost for loading/unloading operations of a unit of freight in satellite k
Q^1_{ij}	Flow passing through the 1st-level arc (i,j)
Q^2_{ijk}	Flow passing through the 2st-level arc (i,j) and coming from satellite k
x_{ij}	Number of 1st-level vehicles using the 1st-level arc (i,j)
y^k_{ij}	Boolean variable equal to 1 if the 2st-level arc (i,j) is used by the 2nd-level routing starting from satellite k
z_{kj}	Variable set to 1 if the customer c_i is served by the satellite k

- The first group represents the arc usage variables. We define two sets of such variables, one for each level. The variable x_{ij} is an integer variable of the 1st-level routing and is equal to the number of 1st-level vehicles using arc (i,j). The variable y^k_{ij} is a binary variable representing the 2nd-level routing. It is equal to 1 if a 2nd-level vehicle runs a route that starts from satellite k and goes directly from node i to node j, 0 otherwise.
- The second group of variables represents the assignment of each customer to one satellite and is used to link the two transportation levels. More precisely, we define z_{kj} as a binary variable that is equal to 1 if the freight to be delivered to customer j is consolidated in satellite k and 0 otherwise.
- The third group of variables, split into two subsets, one for each level, represents the freight flow passing through each arc. We define the freight flow as a variable Q^1_{ij} for the 1st-level and Q^2_{ijk} for the 2nd level, where k represents the satellite which the freight is passing through. Both variables are continuous.

In order to lighten the model formulation, we define the auxiliary quantity

$$D_k = \sum_{j \in V_c} d_j z_{kj}, \forall k \in V_s, \tag{1}$$

which is non-negative and represents the freight passing through each satellite k.

The model to minimize the total cost of the system may be formulated as follows:

$$\min \sum_{i,j \in V_0 \cup V_s, i \neq j} c_{ij} x_{ij} + \sum_{k \in V_s} \sum_{i,j \in V_s \cup V_c, i \neq j} c_{ij} y^k_{ij} + \sum_{k \in V_s} F_k D_k \tag{2}$$

Subject to

$$\sum_{i \in V_s} x_{0i} \leq m_1 \tag{3}$$

$$\sum_{j \in V_s \cup V_0, j \neq k} x_{jk} = \sum_{i \in V_s \cup V_0, i \neq k} x_{ki} \;\; \forall k \in V_s \cup V_0 \tag{4}$$

$$\sum_{k\in V_s}\sum_{j\in V_c} y^k_{kj} \le m_2 \tag{5}$$

$$\sum_{j\in V_c} y^k_{kj} \le m_{s_k} \;\; \forall k \in V_s \tag{6}$$

$$\sum_{j\in V_c} y^k_{kj} = \sum_{j\in V_c} y^k_{jk} \;\; \forall k \in V_s \tag{7}$$

$$\sum_{i\in V_s\cup v_0, i\neq j} Q^1_{ij} - \sum_{i\in V_s\cup v_0, i\neq j} Q^1_{ji} = \begin{cases} D_j & j \text{ is not the depot} \\ \sum_{i\in V_c} -d_i & \text{otherwise} \end{cases} \;\; \forall j \in V_s \cup V_0 \tag{8}$$

$$Q^1_{ij} \le K^1 x_{ij} \;\; \forall i, j \in V_s \cup V_0, i \neq j \tag{9}$$

$$\sum_{i\in V_c\cup k, i\neq j} Q^2_{ijk} - \sum_{i\in V_c\cup k, i\neq j} Q^2_{jik} = \begin{cases} z_{kj} d_j & j \text{ is not a satellite} \\ -D_j & \text{otherwise} \end{cases} \;\; \forall j \in V_c \cup V_s, \forall k \in V_s \tag{10}$$

$$Q^2_{ijk} \le K^2 y^k_{ij} \;\; \forall i, j \in V_s \cup V_c, i \neq j, \forall k \in V_s \tag{11}$$

$$\sum_{i\in V_s} Q^1_{iv_0} = 0 \tag{12}$$

$$\sum_{j\in V_c} Q^2_{jkk} = 0 \;\; \forall k \in V_s \tag{13}$$

$$y^k_{ij} \le z_{kj} \;\; \forall i \in V_s \cup V_c, \forall j \in V_c, \forall k \in V_s \tag{14}$$

$$y^k_{ji} \le z_{kj} \;\; \forall i \in V_s, \forall j \in V_c, \forall k \in V_s \tag{15}$$

$$\sum_{i\in V_s\cup V_c} y^k_{ij} = z_{kj} \;\; \forall k \in V_s, \forall j \in V_c \tag{16}$$

$$\sum_{i\in V_s} y^k_{ji} = z_{kj} \;\; \forall k \in V_s, \forall j \in V_c \tag{17}$$

$$\sum_{i\in V_s} z_{ij} = 1 \;\; \forall j \in V_c \tag{18}$$

$$y^k_{kj} \le \sum_{l\in V_s\cup V_0} x_{kl} \;\; \forall k \in V_s, \forall j \in V_c \tag{19}$$

$$y^k_{ij} \in \{0,1\}, \;\; \forall k \in V_s \cup V_0, \forall i, j \in V_c \tag{20}$$

$$z_{kj} \in \{0,1\}, \;\; \forall k \in V_s \cup V_0, \forall j \in V_c \tag{21}$$

$$x_{kj} \in \mathbb{Z}^+, \;\; \forall k, j \in V_s \cup V_0 \tag{22}$$

$$Q^1_{ij} \ge 0, \forall i, j, \in V_s \cup V_0, \; Q^2_{ijk} \ge 0, \; \forall i, j \in V_s \cup V_c, \forall k \in V_s. \tag{23}$$

The objective function minimizes the sum of the traveling and handling operations costs. Constraints (4) show that, if $k = v_0$, then each 1st-level route begins and ends at the depot. Otherwise, if k is a satellite, they impose the balance of vehicles entering and leaving that satellite. The limit on the satellite capacity is satisfied by constraints (6). These limit the maximum number of 2nd-level routes starting from every satellite (notice that constraints limit at the same time the freight capacity of the satellites as well). Constraints (7) force each 2nd-level route to begin and end at one satellite, while implying that the outgoing and the incoming routes associated to each satellite are equal. The number of routes at each level must not exceed the number of vehicles for that level, as imposed by constraints (3) and (5).

Constraints (8) and (10) indicate that the flows balance on each node is equal to the demand of this node, except for the depot, where the exit flow is equal to the total demand of the customers, and for the satellites at the 2nd-level, where the flow is equal to the demand (unknown) assigned to the satellites. Moreover, constraints (8) and (10) forbid the presence of subtours not containing the depot or a satellite, respectively. In fact, each node receives an amount of flow equal to its demand, preventing the presence of subtours. Consider, for example, that a subtour is present between the nodes i, j and k at the 1st level. It is easy to check that, in such a case, there exists no value for variables Q^1_{ij}, Q^1_{jk} and Q^1_{ki} satisfying constraints (8) and (10). The capacity constraints are formulated in (9) and (11), for the 1st-level and the 2nd-level, respectively. Constraints (12) and (13) do not allow residual flows in routes, making the returning flow of each route to the depot (1st-level) and to each satellite (2nd-level) equal to 0.

Constraints (14) and (15) indicate that a customer j is served by a satellite k ($z_{kj} = 1$) only if it receives freight from the same satellite ($y^k_{ij} = 1$). Constraint (18) assigns each customer to one only satellite, while constraints (16) and (17) indicate that there is only one 2nd-level route passing through each customer. At the same time, they impose the condition that a 2nd-level route departs from a satellite k to deliver freight to a customer if the customer is assigned to that satellite, and only in such case. Constraints (19) allow a 2nd-level route to start from a satellite k only if a 1st-level route has served it.

Finally, (20)-(23) specify the domains of the variables. In particular, notice that while the arc variables y^k_{ij} can be defined as Boolean, each customer being served by at most one route, the 1st-level arc variables x_{kj} must be integer. This is due to the fact that each satellite could be served by more than one vehicle and that the different vehicles could share the same arc.

5. Valid inequalities for 2E-CVRP

In order to strengthen the continuous relaxation of the flow model, we introduce cuts derived from VRP formulations. In particular, we use two families of cuts, one applying to the assignment variables derived from the subtour elimination constraints (edge cuts) and the other flow-based.

The *edge cuts* explicitly introduce the well-known subtours elimination constraints derived from the TSP. They can be expressed as follows:

$$\sum_{i,j \in V'} y^k_{ij} \leq |V'| - 1, \ \forall k \in V_s, \forall V' \subset V_c, \ 2 \leq |V'| \leq |V_c| - 2, \tag{24}$$

where V' is a subset of the customers.

One could also consider the similar constraints on variables x_{ij} for the 1st-level routes. They are not included here due to their marginal improvement, which in turn is probably due to the limited number of satellites considered.

Inequalities (24) explicitly forbid the presence in the solution of subtours not containing the depot, already forbidden by constraints (10).

These inequalities can be strengthened by considering that, given a subset of second-level edges y^k_{ij} belonging to the same satellite, the cardinality of the subset of customers V' appearing in (24) can be substituted by the sum of variables of any subset of V' such that the number of variables z_{kj} is equal to the size of V' minus one. More precisely, the inequality (24) can be rewritten as follows:

$$\sum_{i,j \in V'} y^k_{ij} \leq \sum_{j \in V' \setminus \{l\}} z_{kj}, \ \forall k \in V_s, \ \forall V' \subset V_c, \ 2 \leq |V'| \leq |V_c| - 2, \ \forall l \in V', \tag{25}$$

where V' is a subset of the customers. In the following, we will refer to inequalities (25) as *edge cuts*. The number of potential valid inequalities (24) and (25) is exponential, so we should need a separation algorithm to add them. As these cuts correspond to the Generalized Subtour Elimination Constraints for the TSP problem when adapted to the 2E-CVRP, we could use as separation procedure the exact procedure presented in Wolsey, 1998. According to our test, the inequalities involving sets V' with cardinality higher than 3 are rare. Moreover, no violation with sets over 5 nodes are present in the instances we tested (up to 50 customers, 5 customers) and the effect of the cuts with size more than 3 is negligible. Thus the separation algorithm has been substituted by a direct inspection of the constraints up to cardinality equal to 3.

The formulation can be strengthened by strengthening the BigM constraints (11). The idea is to reduce the constant K^2 by considering that each customer reduces the flow by an amount equal to its demand d_i. Thus the following inequalities are valid:

$$\begin{cases} Q^2_{ijk} \leq (K^2 - d_i)y^k_{ij}, \forall i,j \in V_c \ \forall k \in V_s \\ Q^2_{ijk} - \sum_{l \in V_s} Q^2_{jlk} \leq (K^2 - d_i)y^k_{ij} \ \ \forall i,j \in V_c, \forall k \in V_s. \end{cases} \tag{26}$$

Constraints (26) are of the same order of magnitude of (11) and dominate them. Thus, they simply replace constraints (11) in the model.

From the point of view of flow variables Q^2_{ijk}, the feasibility of a node j, restricted to a satellite k, is assured in a general way from constraints set in the basic formulation. The following constraints

$$\sum_{i \in V_c \cup V_s} Q^2_{ijk} - \sum_{l \in V_c \cup V_s} Q^2_{jlk} = d_j z_{kj} \quad \forall j \in V_c, k \in V_s \tag{27}$$

have the same meaning of (10), restricted to V_c nodes set. When the route is a 2nd-level route such that it does not serve only one customer, we can state that, for any integer solution, at most only one of variables y^k_{ij} and y^k_{ji} is non-zero.

Thereby, if a continuous solution of the continuous relaxation of the model contains both flow variables referred to a given edge (i,j), then the following inequalities apply:

$$Q^2_{ijk} - \sum_{\substack{m \in V_c \cup V_s \\ m \neq i}} Q^2_{jmk} \leq d_j y^k_{ij} \quad \forall i \in V_c \cup V_s, \forall j \in V_c, \forall k \in V_s \tag{28}$$

$$\sum_{\substack{i \in V_c \cup V_s \\ i \neq m}} Q^2_{ijk} - Q^2_{jmk} \geq d_j y^k_{jm} \quad \forall j \in V_c, \forall m \in V_c, \forall k \in V_s. \tag{29}$$

The inequalities (28) and (29) describe the possible node infeasibility problem generated by target incoming arc and target outgoing arc when both flow variables are active, respectively. A possible violation can be detected considering the couples (y^k_{ij}, y^k_{ji}) and the separation procedure that can be done is $O\left(|V_c|^3\right)$.

Additional cuts derived from the $CVRP$ literature could be added, but we verified that their improvement is quite marginal with respect to their high computational effort (Perboli et al., 2010) and we decided not to consider them.

6. Math-based Heuristics for 2E-CVRP

In this section we introduce heuristics for 2E-CVRP based on the information that can be obtained by solving the linear relaxation of the model presented in the previous section. Algorithms of this type are often called math-heuristics (or model based heuristics). If we consider the model of 2E-CVRP presented in Section 4, we can notice that, given feasible values to the variables dealing with the customer-satellite assignment, or z_{kj} variables, the problem is simply partitioned in at most $n_s + 1$ CVRP instances, one for the 1st-level and one for each satellite with at least one customer assigned. Thus, given the values of z_{kj}, the associated solution can be computed by means of any heuristic or exact method developed for the CVRP. Thus, our idea is to focus our search on z_{kj}, using model (2)-(23) to guide the search process. According to these guidelines, we develop two model-based heuristic methods to find feasible solutions based on the usage of simplified versions of the 2E-CVRP model.

The first heuristic considers a continuous model derived from (2)-(23) plus the valid inequalities (25) and the tighter constraints (26). Given the optimal solution of the continuous model, the heuristic apply a diving procedure on z_{kj} (Atamturk and Savelsberg, 2005). Differently from similar procedures, in our case we would rather set variables to 0. In this way we slightly perturb the model, letting it to adapt the values of the remaining variables while reducing the probability of obtaining infeasible solutions at the same time. The variable fixed to zero is the variable whose value does not exceed 0.1 and having the largest pseudocost. We remind that a pseudocost is an estimation of the change in the objective function in consequence of the fact that the corresponding variable is set to an integer value; for the basic variables in the optimal continuous solution, the pseudocost plays a role similar to that of the reduced cost for non-basic variables (Atamturk and Savelsberg, 2005). Moreover, in order to recover possible infeasibilities due to the setting, a restarting procedure is incorporated. More precisely, the procedure works as follows (see Algorithm 1 for the pseudocode and Table 5 for the list and the meaning of parameters):

- the set of compulsory forced variables $forcedVars$ is emptied;
- while an integer solution of z_{kj} variables is not found or a maximum number of trials is not reached proceed with the diving;
  - —set to 1 the z_{kj} in $forcedVars$;
  - —solve the continuous model (2)-(23);
  - —if the solution is integer in the z_{kj} variables and the corresponding assignment of 2nd-level vehicles satisfies the capacity constraints on the satellites, solve the corresponding CVRP instances;
  - —otherwise
    - ∗ get the $p < P$ z_{kj} variables with value near to zero and largest pseudocost and force them to zero;
    - ∗ if $p = 0$, get the Q z_{kj} variables with value greater or equal to 0.5 and force them to 1.
    - ∗ optimize the continuous model (2)-(23);
    - ∗ if the model is infeasible, take the last fixed variable and add to $forcedVars$, unfix the other fixed variables, increase the number of trials and restart the process.

In the second heuristic method we consider that the number of variables z_{kj} in model (2)-(23) is quite small and a MIP solver can find a near-optimal solution with a limited computational effort of 2E-CVRP model with variables y_{ij}^k and x_{ij} considered as continuous. Thus, we consider a simplified version of model (2)-(23) where (20) and (22) are ignored. Moreover, we add to the simplified model the integer variables v_k, representing the vehicles used by satellite k, and the following constraints:

$$\sum_{j \in V_c} z_{k,j} d_j <= K^2 v_k, \forall k \in V_s, \tag{30}$$

$$\sum_{k \in V_s} v_k <= m_2, \forall k \in V_s, \tag{31}$$

$$\sum_{k \in V_s} v_k <= m_{s_k}, \forall k \in V_s. \tag{32}$$

Constraints (30)-(32) are used to ensure that capacity constraints of satellites are satisfied even when y_{ij}^k are not integral. Constraints (30)-(32) could be also added to the original formulation in order to introduce some redundancy. Unfortunately, computational tests show that this redundancy

Algorithm 1 Diving-based Heuristic

$numTrials = 0$
$forcedVars = \{\oslash\}$
while $numTrials < MaxTrials$ **or** $solutionFound =$ **false do**
set the variables in $forcedVars$ to 1
Solve the continuous model
while one or more variables z_{kj} are not integer **do**
if All the z_{kj} have integral value and the capacity constraints on the satellites are satisfied **then**
Solve $m + 1$ CVRP instances
$solutionFound =$ **true**
else
Get the $p \leq P$ variables with value $z_{kj} \leq 0.1$ and having the largest pseudocost
if $p \neq 0$ **then**
Set the p variables to 0
else
Get the $q \leq Q$ variables with value $z_{kj} \geq 0.5$
Set the q variables to 1
Solve the continuous model
if Model is infeasible **then**
$numTrials = numTrials + 1$
Get the last variable $z_{\overline{kj}}$ rounded
$forcedVars = forcedVars \cup \{z_{\overline{kj}}\}$

is not able to strengthen the linear relaxation enough and the marginal improvement obtained is dominated by the results of the other cuts presented in this paper. In the following, we will refer to this simplified model as *semi-continuous 2E-CVRP* model.

Thus, the semi-continuous heuristic works as follows (see Table 5 for the list and the meaning of the parameters):

- solve the continuous relaxation of the semi-continuous 2E-CVRP model and set the integer variables to the value obtained by the model;
- solve the semi-continuous 2E-CVRP model on the reduced set of variables by means of a MIP solver with a time-limit of 60 seconds and put in a list the best integer solutions. Let M be the size of the list, i.e. the maximum number of solutions of the semi-continuous model, which is taken into consideration in the next steps;
- for every solution in the list:
  — consider the assignments satellite-customer given by the z_{kj} variables;
  — build the corresponding instances for the 1st-level and the single satellites CVRP;
  — solve each CVRP instance with a CVRP solver (a fixed time limit is given);
- return the best 2E-CVRP solution found.

The threshold on the explored feasible solutions of the semi-continuous model M is used to explore more integer solutions of the semi-continuous 2E-CVRP model, ensuring at the same time an upper limit to the computational effort to the subsequent CVRP instances.

In both heuristics, any exact or heuristic method to solve the CVRP problems can be used to solve the underlying CVRP instances. A comparison of the results obtained by means of both exact (Ralphs et al., 2003) and heuristic (Perboli et al., 2008) methods for CVRP is provided in Section 7.1.

7. Computational tests

In this section, we analyze the behavior of the model and the heuristics in term of solution quality and computational efficiency. Two-echelon systems are known in the literature, but the routing model as well as the 2E-CVRP are introduced for the first time in this paper. Thus, in Subsection 7.1 we define some benchmark instances, extending the instance sets from the VRP literature. All the tests have been performed on a 3 GhZ Pentium PC with 1 Gb of Ram. The models and the routines have been implemented in Mosel language and tested by means of XPress 2008a solver (Dash Associates, 2008).

Section 7.2 is devoted to present the computational results on a wide set of benchmark instances and the impact of the valid inequalities of Section 5 on the computational results, while Section 7.3 presents the computational results of the model, the heuristics and the valid inequalities on the overall sets of instances.

7.1. Instance sets

In this section we introduce different instance sets for 2E-CVRP. The instances cover up to 51 nodes (1 depot and 50 customers) and are grouped in four sets. The first three sets have been built from the existing instances for VRP by Christofides and Eilon and have been denoted as E-n13-k4, E-n22-k4, E-n33-k4 and E-n51-k5 (Christofides and Eilon, 1969), while the fourth set, taken from Crainic et al., 2010, comprises randomly generated instances replicating distributions of customers and satellites typical of city logistics problems. All instance sets can be downloaded from the web site of OR-Library (Beasley, 1990).

The first instance set comprises 66 small-sized instances with 1 depot, 12 customers and 2 satellites. All the instances have the cost matrix of the instance E-n13-k4 (the costs of the matrix of the original instance is read as an upper triangular matrix and the corresponding optimal cost of the VRP instance is 290). The two satellites are placed over two customers in all the $\binom{12}{2} = 66$ possible ways (the case where some customers are used as satellites is quite common for different kinds of distribution, e.g. grocery distribution). When a node is both a customer and a satellite, the arc cost c_{ki} is set to 0. The number of vehicles for the 1st-level is set to 2, while the 2nd-level vehicles are 4, as in the original VRP instance. The capacity of 1st-level vehicles is 2.5 times that of 2nd-level vehicles, to represent cases in which the 1st-level is made by trucks and the 2nd-level is made by smaller vehicles (e.g., vehicles with a maximum weight smaller than 3.5 t). The capacity of 2nd-level vehicles is equal to the capacity of the vehicles of the VRP instance. The cost due to loading/unloading operations is set to 0, while the arc costs are the same as for the VRP instances. This is done in order that results can be better compared with the original instances of the CVRP. In this way, we can analyze the effect on the routing costs and the satellite usage of the customers' geographical dispersion.

The second set of instances is obtained in a similar way from the instances E-n22-k4, E-n33-k4 and E-n51-k5. The instances are built by considering 6 pairs of randomly generated satellites. For the instance E-n51-k5, which has 50 customers, we considered an additional group of 3 instances obtained by randomly placing 4 satellites instead of 2. The cost due to loading/unloading operations is set to 0, while the arc costs are the same as for the VRP instances.

The main issue in the original instances by Christofides and Eilon is that the depot is in an almost central position with respect to the area covered by the customers. For this reason, the third set of instances also considers the instances E-n22-k4, E-n33-k4 and E-n51-k5, but the distribution of the satellites is more realistic. In fact, we consider six pairs of satellites randomly chosen between the customers on the external border of the area determined by the customers distribution. Moreover, the depot is external to the customers areas, being placed at the coordinate $(0,0)$ (the southwest corner of the customers area).

Table 2 Summary of the benchmark tests.

Set	Instances	ns	nc	m1	m2	C1	C2	Satellite distribution	Customer distribution
1	66	2	12	3	4	15000	6000	All pairs	Christofides and Eilon (1969) E-n13-k4 instance
2	6	2	21	3	4	15000	6000	Random	Christofides and Eilon (1969) E-n22-k4 instance
2	6	2	32	3	4	20000	8000	Random	Christofides and Eilon (1969) E-n33-k4 instance
2	6	2	50	3	5	400	160	Random	Christofides and Eilon (1969) E-n51-k5 instance
2	3	4	50	4	5	400	160	Random	Christofides and Eilon (1969) E-n51-k5 instance
3	6	2	21	3	4	15000	6000	Border Random	Christofides and Eilon (1969) E-n22-k4 instance
3	6	2	32	3	4	20000	8000	Border Random	Christofides and Eilon (1969) E-n33-k4 instance
3	6	2	50	3	5	400	160	Border Random	Christofides and Eilon (1969) E-n51-k5 instance
4	18	5	50	3	6	12500	5000	Crainic et al, 2009	Crainic et al, 2009

Finally, the fourth set comprises 18 instances with 50 customers and 5 satellites from Crainic et al., 2010. Those instances, generated in order to represent different scenarios in city logistics, present 3 realistic distributions for the customers and 3 different strategies for the location of the satellites. The depot is external to the customers areas and both capacities for the 2nd-level fleet and each satellite are present.

A summary of the main features of the different sets is reported in Table 2. The first column reports the instance set, while the number of instances in shown in Column 2. Columns 3 and 4 contain the number of satellites and customers, respectively. The number of vehicles for the 1st and 2nd level can be read in Columns 5 and 6, while Columns 7 and 8 provide the capacity of the vehicles at the two levels. In the remaining columns the rule used to locate the satellites and the customers are specified. More in detail, the value *All pairs* indicates for satellites that all the possible pairs have been computed, while *Random* and *Border Random* shows that satellites are randomly selected. About the instance names showed in Column 10, they are those used by Christofides and Eilon, 1969.

7.2. Valid inequalities computational results

In this section we present the computational results of the model (2)-(23) solved by means of XPress 2008a on instances belonging to Set 1 and Set 2 using the valid inequalities introduced in Section 5 within a computation time limit of 10000 seconds.

With respect to the edge cuts, a series of tests was carried out using a simple procedure which tested all the subtours up to cardinality 5. The procedure, coded in Mosel, iteratively solves the continuous problem and checks the violated cuts up to 10 iterations. According to our test, the inequalities involving subsets with cardinality more than 3 are rare. Moreover, no violation with sets over 4 nodes could be found in the instances that we tested (up to 50 customers and 4 satellites) and the effect of the cuts with size more than 3 is negligible (they increase the overall objective function of less than 0.1 units in the best case). Thus the separation algorithm for the Generalized Subtour Elimination Constraints has been substituted by a direct inspection of the constraints up to cardinality equal to 3.

In Table 3 the results of the 66 instances corresponding to the problem with 12 customers and 2 satellites are provided. The optimum is reported in the second column, while columns 3 and 4 contain the time in seconds needed to solve the instances without and with the valid inequalities introduced in Section 5, while column 5 reports the number of edge cuts added. Finally, the last column presents the percentage of decreasing/increasing of computational time due to the usage of valid inequalities. We do not present the lower bounds at the root node with and without cuts, the difference being less than 2%. This behavior, as we will show with the results of Set 2, is mainly due to the small size of the instances themselves.

According to the results most instances are solved in less than one minute, and only 10 of them need more than 2 minutes to be solved. There are however seven instances for which the computational time is greater than 10 minutes. This gap is mostly related to the satellite location. In fact, the greatest computational times are related to the situation where choosing which satellite to use has little or no effect on the final solution. In this situation, the model finds an optimal solution quickly, but spends much time closing the nodes of the decision tree. This is due to the poor quality of the lower bound obtained by the continuous relaxation of the model. A better behavior is obtained with the valid inequalities. As a counter effect, on some instances, the computational time still increases, due to the management of the additional inequalities. Moreover, the number of added cuts is quite limited, with a mean of 56 cuts added to the original formulation.

By considering all the pairs of customers as possible satellite location and comparing the results with the optimal solution of the original CVRP instance with optimum 247, in the following we discuss advantages and disadvantages of the proposed two-level distribution system.

From the quality point of view, it is clear the advantage of using the 2E-CVRP distribution model instead of the CVRP one. Indeed, the former is able to achieve a smaller cost in 42 instances, while the decreasing/increasing of the costs is, except for satellites $11, 12$ with $+24\%$, in the range $[-15\%, +12\%]$ of the corresponding CVRP instance. The mean decreases in the 42 instances with a reduced transportation cost of 8.5%, which could be used to balance the costs of loading/unloading operations at satellites. In the city logistics field, this means that the 2E-CVRP distribution model could be introduced without raising the total transportation cost, while obtaining indirect advantages, such as the reduction of the traffic flows and pollution level. For a more detailed discussion of the satellite location, see Crainic et al., 2010.

The results on Set 2 instances are presented in Table 4, where the behavior of the lower bound computed with a continuous relaxation of the model found without and with the valid inequalities is considered. More precisely, columns 1 and 2 contain, respectively, the number of customers in the original Christofides and Eilon's instances and the position of the satellites given as customer number. The values and the percentage gap with the best integer solution of the first lower bound (calculated at the root node) without and with the valid inequalities are reported in columns 3-6, while the same data on the final lower bound (calculated at the end of the optimization process), increased by letting the solver apply lift-and-project cuts, included as a standard feature of the MIP solver during the optimization, and its gap are presented in columns 7-10. All the gaps are computed as $(UB - LB)/LB$, where UB is the upper bound reported in column $Best\,sol$ and LB is the lower bound under study. The number of cuts added at the root node is shown in column 11, while the best solution after 5000 seconds and 10000 seconds are reported in columns 12 and 13, respectively (bold values mean optimal values).

From these results it can be seen that the use of cuts helps the model to reduce the gap by up to 9 percentage points. The behavior is confirmed by considering the values of the feasible solutions found by the model without and with the valid inequalities. According to these results, for up to 32 customers the model is able to find good quality solutions in 5000 seconds at most. When the number of customers increases to 50, more than 5000 seconds are required to find a good solution. Moreover, the use of the cuts increases the average model quality in terms of the initial solutions and the lower bounds. The gaps between the best solutions and the best bounds are quite small for instances involving up to 32 customers, but increase for 50-customer instances, with a gap up to 42% for the 4 satellite instances.

Table 3 **12 customers and 2 satellites instances: valid inequalities improvements.**

Satellites	OPT	Time		Cuts	% Time
		Without cuts	With cuts		
1,2	280	1312.34	835.56	55	-36.33%
1,3	286	861.94	247.99	66	-71.23%
1,4	284	1445.05	251.78	83	-82.58%
1,5	218	2.06	1.68	61	-18.40%
1,6	218	7.92	2.08	62	-73.71%
1,7	230	19.95	5.32	100	-73.34%
1,8	224	2.50	2.78	77	11.01%
1,9	236	13.34	6.28	117	-52.92%
1,10	244	14.27	5.18	114	-63.72%
1,11	268	28.70	7.81	67	-72.80%
1,12	276	45.05	28.71	5	-36.27%
2,3	290	849.17	392.11	100	-53.82%
2,4	288	895.19	639.22	93	-28.59%
2,5	228	4.48	2.56	55	-42.99%
2,6	228	4.20	12.24	66	191.16%
2,7	238	7.09	3.41	157	-51.96%
2,8	234	6.00	4.05	62	-32.43%
2,9	246	11.69	6.95	97	-40.49%
2,10	254	26.25	7.49	157	-71.48%
2,11	276	37.27	9.60	93	-74.25%
2,12	286	226.48	136.92	9	-39.55%
3,4	312	1704.41	810.09	40	-52.47%
3,5	242	4.61	2.28	86	-50.46%
3,6	242	13.13	1.89	36	-85.58%
3,7	252	17.05	1.65	31	-90.32%
3,8	248	7.08	1.96	58	-72.28%
3,9	260	6.17	3.46	50	-43.99%
3,10	268	33.27	8.37	83	-74.84%
3,11	290	17.50	5.79	88	-66.92%
3,12	300	13.39	7.82	76	-41.62%
4,5	246	6.39	2.80	31	-56.16%
4,6	246	10.17	2.93	32	-71.18%
4,7	258	12.16	5.38	56	-55.77%
4,8	252	5.25	0.64	48	-87.85%
4,9	264	6.56	3.71	18	-43.50%
4,10	272	15.28	3.56	50	-76.72%
4,11	296	11.11	5.02	86	-54.78%
4,12	304	13.91	5.79	52	-58.34%
5,6	248	3.28	3.64	36	10.87%
5,7	254	1.97	2.68	31	36.01%
5,8	256	9.34	4.06	42	-56.50%
5,9	262	6.59	4.39	48	-33.37%
5,10	262	2.08	2.84	27	36.90%
5,11	262	1.73	1.48	14	-14.51%
5,12	262	1.41	1.77	14	25.72%
6,7	280	17.70	19.35	62	9.31%
6,8	274	7.64	5.44	13	-28.83%
6,9	280	15.22	11.09	28	-27.15%
6,10	280	7.73	9.13	11	18.10%
6,11	280	7.11	4.84	10	-31.88%
6,12	280	14.88	4.36	14	-70.67%
7,8	292	4.42	1.84	18	-58.41%
7,9	300	8.97	7.93	38	-11.60%
7,10	304	12.63	14.11	41	11.79%
7,11	310	23.88	6.89	28	-71.15%
7,12	310	19.94	8.65	18	-56.60%
8,9	326	40.81	16.42	96	-59.77%
8,10	326	17.86	15.15	135	-15.17%
8,11	326	11.55	5.29	16	-54.18%
8,12	326	6.84	4.96	14	-27.59%
9,10	338	24.27	17.14	101	-29.38%
9,11	350	17.52	17.80	86	1.59%
9,12	350	16.25	10.87	12	-33.13%
10,11	358	40.98	55.11	32	34.46%
10,12	358	23.19	21.34	24	-7.97%
11,12	400	40.45	34.02	59	-15.91%

7.3. Overall computational results

In this section we present the results of the tests in Set 2, 3, and 4. All results have been obtained using the model having valid inequalities activated. According to the results discussed in Section 7.2, we limited the generation of the cuts to cycles of length 3, while inequalities (26) were directly added to the standard formulation. The results are related to Set 2, 3 and 4, being the sets with the largest size in terms of customers and satellites. The accuracy of both diving and semi-continuous heuristics are affected by the tuning of several parameters. The tuning has been done on a subset of 20% of the overall instances. In order to reduce the length of the paper, we do not discuss the tuning, but we simply report in Table 5 the optimal values for each parameter.

The results of the model on each set are summarized in Tables 6a and 6b. Each table contains the instance name and the number of satellites in columns 1 and 2. Columns 3 and 4 contain the best solution and the lower bound computed by continuous relaxation of the model. Finally, the percentage gap of the best solution compared with the lower bound is presented in Column 5.

These results indicate that the gap is quite small up to 32 customers, while it increases in the 50-customer tests, and particularly in those with 4 satellites.

The instances generated from the classical CVRP instances (sets 2 and 3) present a distribution of the customers which is quite different from the distribution in realistic applications in urban and regional delivery. The model is able to find solutions with an average gap of about 6% in Set 2 and 8.5% in Set 3, which is quite large, but understandable considering that the lower bounds come from the simple continuous relaxation of the model with cuts.

Table 4 Results on instances of Set 2.

		First Bound				Final Bound						
		Without cuts		With cuts		Without cuts		With cuts				
CVRP Instance	Satellites	Bound	Gap	Bound	Gap	Best Bound	Gap	Best Bound	Gap	Cuts	Sol 5000 ss	Best Sol.
E-n22-k4	7,18	399.20	4.48%	411.12	1.45%	417.07	0.00%	417.07	0.00%	234	417.07	**417.07**
	9,15	358.24	7.46%	369.92	4.06%	384.96	0.00%	384.95	0.00%	95	384.95	**384.96**
	10,20	423.48	11.13%	441.10	6.69%	457.07	2.96%	470.60	0.00%	197	470.60	**470.60**
	11,15	348.22	6.68%	360.56	3.03%	371.50	0.00%	371.48	0.01%	93	371.48	**371.50**
	12,13	374.11	14.20%	395.73	7.96%	417.61	2.30%	427.22	0.00%	210	427.22	**427.22**
	13,17	349.70	12.32%	366.31	7.23%	372.66	5.40%	392.78	0.00%	185	392.78	**392.78**
E-n33-k4	2,10	626.48	16.55%	696.70	4.80%	688.05	6.12%	730.16	0.00%	266	730.16	**730.16**
	3,14	610.21	17.11%	675.84	5.74%	656.75	8.81%	709.76	0.69%	190	714.64	714.64
	4,18	611.44	15.71%	657.33	7.63%	641.59	10.27%	698.81	1.24%	351	707.49	707.49
	5,6	636.93	23.61%	713.81	10.29%	711.73	10.62%	757.39	3.95%	187	856.93	787.29
	8,26	648.41	17.27%	718.35	5.85%	707.48	7.48%	745.71	1.96%	305	762.95	760.36
	15,23	662.62	17.81%	750.99	3.94%	741.57	5.26%	764.49	2.11%	155	781.56	780.60
E-n51-k5	3,18	536.23	11.47%	542.60	10.16%	548.80	8.92%	579.74	3.10%	840	755.11	597.74
	5,47	502.85	11.72%	509.36	10.30%	503.64	11.55%	515.24	9.04%	210	715.19	561.80
	7,13	505.31	10.87%	510.41	9.76%	507.98	10.28%	528.84	5.93%	781	732.17	560.22
	12,20	544.35	8.02%	551.06	6.70%	547.77	7.35%	559.59	5.08%	524	588.01	588.01
	28,48	499.29	7.79%	505.86	6.39%	501.82	7.25%	526.34	2.25%	1244	540.88	538.20
	33,38	513.01	7.70%	517.36	6.79%	519.56	6.34%	542.83	1.78%	802	692.54	552.49
E-n51-k5	3,5,18,47	465.35	30.99%	503.67	21.02%	479.91	27.02%	512.18	19.01%	2065	724.09	609.56
	7,13,33,38	462.99	23.50%	501.87	13.93%	480.45	19.01%	507.49	12.67%	1572	685.45	571.80
	12,20,28,48	476.98	51.81%	500.41	44.70%	482.01	50.22%	507.64	42.64%	936	915.43	724.09

Table 5 Summary of the optimal values of the parameters used by Diving and Semi-Continuous heuristics.

Heuristic	Parameter	Meaning	Value
Diving	P	Maximum number of variables fixed to 0	4
	Q	Maximum number of variables fixed to 1	4
	CVRP Time Limit	Time Limit for every CVRP	10 ss
	EVEOpt	Stopping criteria	Default from Perboli et al. (2008)
	Max restart	Maximum number of restart of the process when the fixing gives an infeasible solution	5
Semi-Continuous	Semi-continuous model Time Limit	Time Limit given to the MIP solver	60 ss
	M	Maximum number of integer solutions of the model which are considered for finding 2E-VRP solutions	10
	CVRP Time Limit	Time Limit for every CVRP	5 ss
	EVEOpt	Stopping criteria	Default from Perboli et al. (2008)

Given the complexity of the model and in particular the number of integer variables and constraints involved, it is not surprising that the solver requires more than 3 hours to obtain a reasonable solution. On the other hand, heuristic methods can help to close the gap with the lower bound with a limited computational effort. Tables 7a and 7b present the results of math-based heuristics derived from the complete 2E-CVRP model. Each table contains the instance name and the number of satellites in Columns 1 and 2. Column 3 reports the best solution obtained by the model. Columns 4, 5, 6, and 7 show the behavior of the diving and semi-continuous heuristic, providing for each heuristic the value of the objective function and the computational time, while the best solution obtained by combining the two heuristics and their total computational time is

Table 6 Results of the MIP model on Set 2 and Set 3.

Instance	Satellites	Final Solution	Best Bound	Gap
E-n22-k4-s6-17	2	**417.07**	**417.07**	0.00%
E-n22-k4-s8-14	2	**384.96**	**384.95**	0.00%
E-n22-k4-s9-19	2	**470.60**	**470.60**	0.00%
E-n22-k4-s10-14	2	**371.50**	**371.50**	0.00%
E-n22-k4-s11-12	2	**427.22**	**427.22**	0.00%
E-n22-k4-s12-16	2	**392.78**	**392.78**	0.00%
E-n33-k4-s1-9	2	**730.16**	**730.16**	0.00%
E-n33-k4-s2-13	2	714.64	709.76	0.69%
E-n33-k4-s3-17	2	707.49	698.81	1.24%
E-n33-k4-s4-5	2	787.29	757.39	3.95%
E-n33-k4-s7-25	2	760.36	745.71	1.96%
E-n33-k4-s14-22	2	780.60	764.49	2.11%
E-n51-k5-s2-17	2	597.74	579.74	3.10%
E-n51-k5-s4-46	2	561.80	515.24	9.04%
E-n51-k5-s6-12	2	560.22	528.84	5.93%
E-n51-k5-s11-19	2	588.01	559.59	5.08%
E-n51-k5-s27-47	2	538.20	526.34	2.25%
E-n51-k5-s32-37	2	552.49	542.83	1.78%
E-n51-k5-s2-4-17-46	4	609.56	512.18	19.01%
E-n51-k5-s6-12-32-37	4	571.80	507.49	12.67%
E-n51-k5-s11-19-27-47	4	724.09	507.64	42.64%
Mean				6.19%

(a) - Set 2

Instance	Satellites	Final Solution	Best Bound	Gap
E-n22-k4-s13-14	2	**526.10**	**526.10**	0.00%
E-n22-k4-s13-16	2	**521.04**	**521.04**	0.00%
E-n22-k4-s13-17	2	**496.34**	**496.34**	0.00%
E-n22-k4-s14-19	2	**498.81**	**498.81**	0.00%
E-n22-k4-s17-19	2	**512.80**	**512.80**	0.00%
E-n22-k4-s19-21	2	**520.41**	**520.41**	0.00%
E-n33-k4-s16-22	2	675.36	634.26	6.48%
E-n33-k4-s16-24	2	669.60	622.88	7.50%
E-n33-k4-s19-26	2	680.38	648.16	4.97%
E-n33-k4-s22-26	2	680.90	650.39	4.69%
E-n33-k4-s24-28	2	675.95	633.67	6.67%
E-n33-k4-s25-28	2	651.25	616.59	5.62%
E-n51-k5-s12-18	2	705.52	661.35	6.68%
E-n51-k5-s12-41	2	823.05	635.67	29.48%
E-n51-k5-s12-43	2	710.39	688.09	3.24%
E-n51-k5-s39-41	2	845.94	679.80	24.44%
E-n51-k5-s40-41	2	827.77	669.74	23.60%
E-n51-k5-s40-43	2	906.68	707.85	28.09%
Mean				8.41%

(b) - Set 3

shown in columns 8 and 9. Column 10 provides the value of the best lower bound known for each problem. Finally, columns 11 and 12 present the percentage gap of the best model solution and the best heuristic solution compared with the best lower bound, respectively. All the computational times include the time needed for solving the CVRP instances generated by the heuristics. Both diving and semi-continuous heuristics have been tested solving the CVRP subproblems by means of *EVEOpt*, the hybrid algorithm developed by Perboli et al., 2008. In semi-continuous heuristic the parameter M, relating to the maximum number of integer solutions of the semi-continuous model used by the heuristic, is set to 5.

According to the results, the semi-continuous heuristic dominates the diving one on Set 2, while there is not a heuristic dominating the other on Set 3. Moreover, the combination of diving and semi-continuous enabled us to reduce the mean gap from the lower bound. In particular, this is true for Set 3, where the mean gap is reduced from 8.41% to 4.86% . This is more evident in 50-customer instances, where the mean gap is reduced from 11% of the MIP model to 7% of the heuristics. The benefits of the heuristics are also clear from the computational point of view, presenting a mean value of 17 seconds and a worst case of 52 seconds in instance E-n51-k5-s6-12-32-37.

Obviously the results could be affected by the method used to solve the CVRP subproblems in solution quality and efficiency. Moreover, using a heuristic method to solve the CVRP instances had as side effect a worsening of the quality of the final solution. In order to test the heuristics we replaced *EVEOpt* with the Branch and Cut by Ralphs et al., 2003 and stopped after 5 seconds. Results are not presented, having the same solution quality, i.e. the objective function values of the solutions obtained with *EVEOpt* and the truncated Branch and Cut are the same for the two sets. From the efficiency point of view, the computational effort is much higher due to the usage of the Branch and Cut, while the size of the instances makes impossible to use the Branch and Cut for instances with more than 50 customers.

The results of the model with cuts are not satisfactory when the number of satellites increases, as shown by the results on Set 2. In order to obtain better results, we hybridized the math-heuristics and the exact model, providing as initial solution of the exact model the best solution found by the math-heuristics and we introduced the cuts in a simple Branch and Cut scheme. More in detail, the Branch and Cut works as follows:

- the diving and semi-continuous heuristics are applied at the root node only and their best integer solution is provided to the Branch and Cut;
- cut generation process is applied at every node up to node depth equal to 10;

Table 7 Results of the math-heuristics on Set 2 and Set 3.

Instance	Satellites	Model	Diving	Time ss	SC	Time ss	Best Heur	Time ss	Best LB	Gap Model	Gap Best Heur
E-n22-k4-s6-17	2	417.07	417.07	1.0	417.07	1.6	417.07	2.65	417.07	0.00%	0.00%
E-n22-k4-s8-14	2	384.96	441.41	0.9	408.14	0.9	408.14	1.74	384.95	0.00%	6.02%
E-n22-k4-s9-19	2	470.60	472.23	2.2	470.60	1.2	470.60	3.34	470.60	0.00%	0.00%
E-n22-k4-s10-14	2	371.50	435.92	1.5	440.85	0.0	435.92	1.51	371.48	0.01%	17.35%
E-n22-k4-s11-12	2	427.22	427.22	0.6	429.39	1.1	427.22	1.70	427.22	0.00%	0.00%
E-n22-k4-s12-16	2	392.78	425.65	1.1	425.65	1.0	425.65	2.03	392.78	0.00%	8.37%
E-n33-k4-s1-9	2	730.16	772.57	3.0	736.92	0.2	736.92	3.26	730.16	0.00%	0.93%
E-n33-k4-s2-13	2	714.64	749.94	5.0	736.37	0.7	736.37	5.72	709.76	0.69%	3.75%
E-n33-k4-s3-17	2	707.49	801.19	7.4	739.47	0.7	739.47	8.07	698.81	1.24%	5.82%
E-n33-k4-s4-5	2	787.29	838.31	2.0	816.59	1.5	816.59	3.41	757.39	3.95%	7.82%
E-n33-k4-s7-25	2	760.36	756.88	2.3	756.88	4.8	756.88	7.15	745.71	1.96%	1.50%
E-n33-k4-s14-22	2	780.60	779.06	1.6	779.06	0.5	779.06	2.07	764.49	2.11%	1.91%
E-n51-k5-s2-17	2	597.74	666.83	4.9	597.49	42.9	597.49	47.80	579.74	3.10%	3.06%
E-n51-k5-s4-46	2	561.80	543.24	8.4	543.20	20.7	543.20	29.15	515.24	9.04%	5.43%
E-n51-k5-s6-12	2	560.22	560.22	8.5	554.80	7.1	554.80	15.55	528.84	5.93%	4.91%
E-n51-k5-s11-19	2	588.01	584.09	4.9	592.06	10.5	584.09	15.43	559.59	5.08%	4.38%
E-n51-k5-s27-47	2	538.20	538.20	10.3	538.20	20.8	538.20	31.10	526.34	2.25%	2.25%
E-n51-k5-s32-37	2	552.49	584.59	8.9	578.23	29.5	578.23	38.34	542.83	1.78%	6.52%
E-n51-k5-s2-4-17-46	4	609.56	590.63	16.4	542.37	34.4	542.37	50.83	512.18	19.01%	5.89%
E-n51-k5-s6-12-32-37	4	571.80	571.80	23.5	584.88	28.8	571.80	52.30	507.49	12.67%	12.67%
E-n51-k5-s11-19-27-47	4	724.09	600.34	17.8	600.34	31.6	600.34	49.41	507.64	42.64%	18.26%
Me								17.74		6.19%	6.16%

(a) - Set 2

Instance	Satellites	Model	Diving	Time ss	SC	Time ss	Best Heur	Time ss	Best LB	Gap Model	Gap Best Heur
E-n22-k4-s13-14	2	526.10	561.10	1.7	526.54	0.6	526.54	2.25	526.10	0.00%	0.08%
E-n22-k4-s13-16	2	521.04	521.04	2.6	521.10	0.2	521.04	2.81	521.04	0.00%	0.00%
E-n22-k4-s13-17	2	496.34	496.34	3.0	496.39	0.5	496.34	3.45	496.34	0.00%	0.00%
E-n22-k4-s14-19	2	498.81	551.95	3.1	523.61	1.0	523.61	4.08	498.76	0.00%	4.98%
E-n22-k4-s17-19	2	512.80	512.81	13.8	521.84	5.4	512.81	19.21	512.75	0.00%	0.01%
E-n22-k4-s19-21	2	520.41	527.57	2.8	527.57	2.0	527.57	4.75	520.36	0.00%	1.39%
E-n33-k4-s16-22	2	675.36	674.71	3.0	672.19	15.6	672.19	18.57	634.26	6.48%	5.98%
E-n33-k4-s16-24	2	669.60	668.82	4.7	674.69	0.9	668.82	5.52	622.88	7.50%	7.38%
E-n33-k4-s19-26	2	680.38	744.42	1.8	680.38	5.2	680.38	7.01	648.16	4.97%	4.97%
E-n33-k4-s22-26	2	680.90	735.25	1.9	680.38	4.9	680.38	6.79	650.39	4.69%	4.61%
E-n33-k4-s24-28	2	675.95	702.86	8.1	692.66	6.6	692.66	14.75	633.67	6.67%	9.31%
E-n33-k4-s25-28	2	651.25	682.42	3.5	650.55	12.0	650.55	15.47	616.59	5.62%	5.51%
E-n51-k5-s12-18	2	705.52	719.64	16.9	692.54	21.2	692.54	38.04	661.35	6.68%	4.72%
E-n51-k5-s12-41	2	823.05	743.91	13.8	708.29	22.2	708.29	36.04	635.67	29.48%	11.42%
E-n51-k5-s12-43	2	710.39	711.73	4.8	712.48	9.2	711.73	13.94	688.09	3.24%	3.44%
E-n51-k5-s39-41	2	845.94	742.07	7.8	729.94	27.9	729.94	35.71	679.80	24.44%	7.38%
E-n51-k5-s40-41	2	827.77	733.60	7.7	732.42	10.5	732.42	18.16	669.74	23.60%	9.36%
E-n51-k5-s40-43	2	906.68	803.24	18.6	757.30	25.7	757.30	44.33	707.85	28.09%	6.99%
Me								16.16		8.41%	4.86%

(b) - Set 3

- pseudocost-based branching scheme with priority on z_{kj} variables;
- global time limit (heuristics and Branch and Cut) equal to 10000 seconds.

The results are summarized in Table 8, where the meaning of the columns is the same as in Table 6. As one can notice, initializing the Branch & Cut with the heuristics supports in both finding new integer solutions and improving the final lower bound, as the mean gap is reduced to 2% in Set 2 and 4% in Set 3. The effect is particularly relevant on 4-satellite instances in Set 2. Moreover, the behavior of the results is more stable as the number of satellites increases (compare the results of the instances with 4 satellites in Set 2).

In order to confirm the behavior, we tested the same procedure (math-heuristics and exact model) on the larger instances of Set 4 with 50 customers and 5 satellites. These instances present different realistic distribution both of customers and satellites. Their results are summarized in Table 9, where the meaning of the columns is the same as in Table 6. From the results, we can see that the mean gap is larger, but still limited to 11%. Moreover, preliminary tests by Perboli et al., 2010 show that this gap is mainly due to the model relaxation and that it can be still slightly reduced by a Branch & Cut procedure involving cuts based on CVRP, the network flow formulation, and the connectivity of the transportation system graph (Perboli et al., 2010). On the other hand, the solution quality obtained by applying the math-heuristics is quite satisfactory (most of the initial solutions found on Set 4 by the heuristics are slightly improved by the Branch & Cut).

Table 8 Results of the Branch & Cut with heuristic initial solution: results on MIP model on Set 2 and Set 3.

Instance	Satellites	Final Solution	Best Bound	Gap
E-n22-k4-s6-17	2	**417.07**	**417.07**	0.00%
E-n22-k4-s8-14	2	**384.96**	**384.95**	0.00%
E-n22-k4-s9-19	2	**470.60**	**470.60**	0.00%
E-n22-k4-s10-14	2	**371.50**	**371.50**	0.00%
E-n22-k4-s11-12	2	**427.22**	**427.22**	0.00%
E-n22-k4-s12-16	2	**392.78**	**392.78**	0.00%
E-n33-k4-s1-9	2	**730.16**	**730.16**	0.00%
E-n33-k4-s2-13	2	714.64	703.87	1.53%
E-n33-k4-s3-17	2	707.49	695.77	1.69%
E-n33-k4-s4-5	2	785.33	767.43	2.33%
E-n33-k4-s7-25	2	756.85	744.40	1.67%
E-n33-k4-s14-22	2	779.05	766.77	1.60%
E-n51-k5-s2-17	2	597.49	582.21	2.63%
E-n51-k5-s4-46	2	530.76	520.96	1.88%
E-n51-k5-s6-12	2	554.81	531.83	4.32%
E-n51-k5-s11-19	2	581.64	559.85	3.89%
E-n51-k5-s27-47	2	538.20	527.32	2.06%
E-n51-k5-s32-37	2	552.28	548.31	0.72%
E-n51-k5-s2-4-17-46	4	541.07	515.67	4.93%
E-n51-k5-s6-12-32-37	4	538.82	512.81	5.07%
E-n51-k5-s11-19-27-47	4	531.12	519.59	2.22%
Mean				2.03%

(a) - Set 2

Instance	Satellites	Final Solution	Best Bound	Gap
E-n22-k4-s13-14	2	**526.10**	**526.10**	0.00%
E-n22-k4-s13-16	2	**521.04**	**521.04**	0.00%
E-n22-k4-s13-17	2	**496.34**	**496.34**	0.00%
E-n22-k4-s14-19	2	**498.81**	**498.81**	0.00%
E-n22-k4-s17-19	2	**512.80**	**512.80**	0.00%
E-n22-k4-s19-21	2	**520.41**	**520.41**	0.00%
E-n33-k4-s16-22	2	672.17	634.09	6.01%
E-n33-k4-s16-24	2	668.81	625.73	6.88%
E-n33-k4-s19-26	2	680.89	648.20	5.04%
E-n33-k4-s22-26	2	680.89	652.12	4.41%
E-n33-k4-s24-28	2	672.60	633.03	6.25%
E-n33-k4-s25-28	2	653.67	615.87	6.14%
E-n51-k5-s12-18	2	692.56	662.16	4.59%
E-n51-k5-s12-41	2	716.58	646.50	10.84%
E-n51-k5-s12-43	2	712.48	690.28	3.22%
E-n51-k5-s39-41	2	729.94	682.98	6.88%
E-n51-k5-s40-41	2	732.42	678.19	8.00%
E-n51-k5-s40-43	2	757.30	709.46	6.74%
Mean				4.17%

(b) - Set 3

Table 9 Results of the Branch & Cut with heuristic initial solution: results on MIP model on Set 4.

Instance	Satellites	First Bound No Cuts	Cuts	Diving	SC	Final Solution	Best Bound	Gap
Instance50-s5-37.dat	5	1259.56	4615	1731.94	1587.95	1587.95	1405.64	12.97%
Instance50-s5-38.dat	5	972.86	3296	1434.98	1186.02	1185.58	1076.48	10.14%
Instance50-s5-39.dat	5	1239.73	4685	1591.48	1525.24	1525.24	1421.58	7.29%
Instance50-s5-40.dat	5	970.86	3418	1358.79	1226.79	1199.42	1068.57	12.25%
Instance50-s5-41.dat	5	1356.84	2863	1726.04	1726.04	1703.03	1541.88	10.45%
Instance50-s5-42.dat	5	1000.03	3034	1345.51	1324.38	1223.09	1097.89	11.40%
Instance50-s5-43.dat	5	1124.13	4532	1607.46	1453.11	1453.11	1283.21	13.24%
Instance50-s5-44.dat	5	843.92	3024	1394.63	1063.64	1039.39	935.42	11.12%
Instance50-s5-45.dat	5	1118.27	3906	1624.3	1497.91	1484.64	1299.06	14.29%
Instance50-s5-46.dat	5	853.99	1193	1244.13	1173.12	1095.69	930.53	17.75%
Instance50-s5-47.dat	5	1230.77	3880	1627.83	1620.7	1598.88	1444.15	10.71%
Instance50-s5-48.dat	5	893.37	3039	1209.89	1122.18	1096.96	998.69	9.84%
Instance50-s5-49.dat	5	1196.22	7207	1610.03	1508.87	1479.16	1339.67	10.41%
Instance50-s5-50.dat	5	879.14	2096	1392.03	1170.89	1090.60	968.47	12.61%
Instance50-s5-51.dat	5	1117.00	5143	1495.43	1456.12	1436.30	1310.90	9.57%
Instance50-s5-52.dat	5	895.08	3314	1185.05	1191.93	1128.33	1003.03	12.49%
Instance50-s5-53.dat	5	1240.22	4795	1611.89	1569.59	1552.75	1450.87	7.02%
Instance50-s5-54.dat	5	928.90	1898	1189.14	1189.14	1135.39	1034.88	9.71%
Mean								11.29%

8. Conclusions

In this paper, we introduced a new family of VRP models, the Two-Echelon VRP. In particular, we considered the Two-Echelon Capacitated VRP, giving a MIP formulation and valid inequalities for it. The model and the inequalities have been tested on new benchmarks derived from the CVRP instances according to the literature, showing a good behavior of the model for small and medium sized instances. Moreover, two different heuristics based on the MIP model have been presented. Both heuristics present good performance both from the computational and the solution quality point of view.

Presently, new clustering-based heuristics for the problem have been developed (Crainic et al., 2008), as well as larger instance sets of up to 200 customers and 7 satellites (Crainic et al., 2010). Moreover, in Crainic et al., 2010 the reader can find an in-depth analysis of the impact of customer and satellite realistic distributions, as well as a comparison of the standard VRP approach with the 2E-CVRP.

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