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Composite local-linear state-space models for the behavioral modeling of digital devices

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Abstract *– This paper discusses the generation of accurate and efficient behavioral models of digital ICs for the assessment of signal integrity effects in high-performance electronics systems. The proposed approach is based on the estimation of composite local-linear state-space models either from simulated or measured device port responses. These representations help to overcome some limitations of traditional parametric relations used so far and the obtained models are readily implemented in any simulation tool as SPICE subcircuits or VHDL-AMS hardware descriptions. The application of the advocated approach to the modeling of a real device exhibiting a strong nonlinear behavior and high order dynamical effects concludes the contribution.*

Keywords *– Digital IC ports, Macromodels, Behavioral models, Identification.*

I. INTRODUCTION

Nowadays, the design of modern high-performance electronic systems requires, in the early stage of the design process, the accurate prediction of signals propagating on system interconnects. Such a prediction, that allows designers to perform both signal integrity analyses and EMC assessments, is mainly carried out via the numerical simulation of critical interconnection paths like high-speed serial links. Within this framework, the availability of accurate and efficient models of digital integrated circuits (ICs) plays a key role. IC port behavior cannot be considered ideal any longer, nor be represented by a lumped linear termination. Hence, suitable behavioral models (or macromodels) accounting for the non ideal analog operation of device ports are required.

Device models are usually based simplified equivalent circuits from some information on the internal structure of devices as suggested by the Input Output Information Specification (IBIS) [1]. Recently, other approaches to IC macromodeling, that supplement the IBIS resource and provide improved accuracy for recent device technologies, have been proposed [2], [3]. These approaches are based on the estimation of suitable parametric relations from port voltage and current responses to a suitable set of stimuli applied to the IC ports.

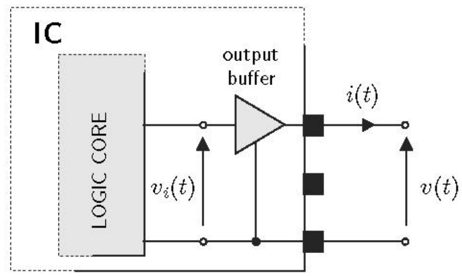

Fig. 1. Typical IC output buffer with the relevant electrical variables.

The parametric relations used so far for the generation of IC models have been sought for within the class of discrete-time Nonlinear Auto Regressive with eXtra input (NARX) parametric relations expressed in terms of gaussian or sigmoidal expansions. This choice arises from the large availability of methods for parameter estimation, as well as from the nice features of these models to approximate *almost any* nonlinear dynamical system [4]. NARX parametric relations have been proven to accurately reproduce the behavior of a wide class of commercial devices [2], [3]. Besides, they turn out to be very compact, *i.e.,* leading to models with a very small size. Owing to this, the estimated models, implemented in a simulation environment, are very efficient and allow simulation speed-ups on the order of $10 \div 1000$ w.r.t. physical descriptions of devices. In spite of these advantages, NARX relations have some inherent limitations. Mainly: *(i)* local stability of models cannot be easily imposed *a-priori* or even during the training process without impacting on model accuracy. It is worth remarking that locally unstable models must be avoided, even if they well reproduce the reference responses used in the model estimation. In fact, numerical simulation of these models for different signal and load conditions may lead to poor results. *(ii)* fully nonlinear optimization algorithms are required for the computation of model parameters and model accuracy depends on the initial guess of parameters and on local minima of the cost function. *(iii)* higher order dynamical effects may not be readily represented by these models. *(iv)* model estimation for real devices with multiple ports is troublesome and impacts on the quality of estimated models. As an example, the generation of device port models including the effects of the neighboring ports suffers from the increase of complexity of the approximation problem.

In order to address the previous limitations, along with the requirement of avoiding the use of complex model structures, impacting on the simulation efficiency, model representations based on composite Local Linear State-Space models (LLSS) [8] are assessed. LLSS models are nonlinear discrete-time state-space parametric equations defined by a weighed sum of linear state-space models that can be effectively used to approximate the port behavior of a nonlinear dynamic system and whose parameters can be automatically computed from system responses only. LLSS models provide a very good compromise between model accuracy and model efficiency for the modeling of real systems with a complex dynamic behavior, and are good candidates to be used for the modeling problem at hand.

II. MODEL STRUCTURE

For the sake of simplicity, the following discussion is based on single-ended output buffers like the one shown in Fig. 1. The results, however, are extensible to input ports and different device technologies. A macromodel for output buffers reproduces the electric behavior of the port current $i(t)$ and voltage $v(t)$ variables and is defined by the following two-piece relation [2].

$$i(t) = w_H(t) i_H(v(t), d/dt) + w_L(t) i_L(v(t), d/dt) \qquad (1)$$

where i_H and i_L are submodels describing the nonlinear dynamic behavior of the port in the fixed high and low logic states, respectively, and w_H and w_L are weighting signals describing state transitions (they play the same role of internal non measurable variables driving the buffer state).

The estimation of model (1) amounts to selecting a model representation for submodels i_H and i_L and to computing the model parameters. It is worth noting that the selection of the model representation along with a good algorithm for the estimation of model parameters are the most critical steps of the modeling process. In fact, once the submodels are completely defined, the computation of the weighting coefficients w_H and w_L in (1) is carried out by a simple linear inversion of the model equation. This is done from voltage and current waveforms recorded during state transitions events, as suggested in [2].

Finally, the last step of the modeling process amounts to coding the model equations in a simulation environment. This can be done by representing equations (1) in terms of an equivalent circuit and then implementing the equivalent as a SPICE-like subcircuit. The circuit interpretation of model equations is a standard procedure that is based on the use of resistors, capacitors, and controlled source elements. As an example, the SPICE-like implementation of a generic nonlinear dynamic parametric model is discussed in [2], [3]. As an alternative, model (1) can be directly plugged into a mixed-signal simulation environment by describing model equations via hardware description languages like Verilog-AMS or VHDL-AMS.

III. LOCAL LINEAR STATE-SPACE MODELS

The idea underlying the LLSS modeling methodology is the approximation of the complex dynamic behavior of a nonlinear dynamic system by means of the composition of local linear models [8]. The whole operating range of the system is partitioned into smaller operating regions where the system behavior is approximated by a linear state-space equation. Even if this idea has been already investigated in the literature, the implementation in [8] has several strengths, including the nice feature of providing the automatic computation of local linear models as well as the generation of the weights for the local models from input-output system responses only.

As an example, for the submodel $i_H(v(t), d/dt)$ of (1), a LLSS representation is defined by the following discrete-time state-space equation

$$\begin{cases} \mathbf{x}(k+1) = \sum_{i=1}^{s} p_i(\phi(k)) \left(\mathbf{A}_i \mathbf{x}(k) + \mathbf{b}_i v(k) + \mathbf{o}_i \right) \\ i_H(k) = \mathbf{c}^T \mathbf{x}(k) + d v(k) \end{cases} \qquad (2)$$

where k is the discrete-time variable, vector $\mathbf{x}$ collects the internal states and $p_i(\cdot)$ is the weighting coefficient of the $i-th$ local model. Each local model is defined by the state matrix $\mathbf{A}_i$ and by vectors $\mathbf{b}_i$ and $\mathbf{o}_i$. The argument of the weighs, *i.e.*, the scheduling vector $\phi_i(k)$, corresponds to the operating point of the system and is in general a function of both input and state variables. Among the possible choices for $p_i(\phi(k))$, a common solution in local linear modeling that is also used in [8] amounts to define the weights as normalized radial basis functions depending on the input sequence $v(k)$ only. The radial functions varies between zero and one and their sum is forced to be one at each operation point of the system. It is worth to remark that under some specific assumptions, the above parametrized state-space equation can be proven to arbitrarily approximate any nonlinear dynamic system [8].

In general, the approximation of a device behavior by means of a parametric relation like (2) amounts to compute model parameters from device port responses by matching model and reference responses. For the modeling problem at hand, the port voltage and current responses are obtained by stimulating the output port of the device by means of a voltage source. The source must be designed to span the range of the operating voltages and to excite the device dynamic behavior within the frequency band of interest. For the class of nonlinear systems, the typical choice is a multilevel signal superimposed by a small gaussian noise, as suggested in [7].

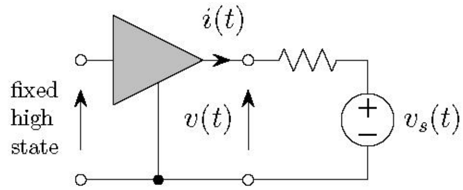

Fig. 2. Ideal setup for generation of the port responses required by the estimation of a LLSS model (2) for the submodel i_H of (1).

Since the computation of model parameters in (2), *i.e.,* the local model matrices and the parameters defining the weights, requires the solution of a nonlinear non-convex approximation problem, a modified version of the well established Levenberg-Marquardt (LM) iterative method is proposed in [8]. The basic version of the LM algorithm has been suitably modified to handle the non-uniqueness of a state-space representation that may cause ill-conditioning of matrices during model estimation. In addition, parameter initialization is carried out by means of a deterministic procedure, thus avoiding the dependence of the estimated model to the initial guess of parameters. Roughly speaking, the gradient direction search in the LM algorithm is modified to avoid the directions in the parameter space that do not change the cost function due to a similarity transformation of model matrices. Besides, the initial guess of the matrices defining the local models are set equal to the matrices of a single global stable linear model. The parameters of the global linear model are computed by means of the application of an efficient subspace identification method [9]. The latter subspace method also provide the automatic computation of the number of internal state variables, *i.e.,* the size of vector $\mathbf{x}$ in (2). Besides, the initial radial weighting functions p_i are distributed uniformly over the range of the input sequence.

It is worth to remark that in the proposed implementation of the algorithm, no additional constraints are included to enforce stable models during training and stability is only verified a posteriori. If needed, suitable modification of the algorithm may be devised. However, the device models obtained so far by using the proposed approach have been verified to be stable.

LLSS models designed as outlined in this Section have additional strengths. Mainly, the state-space nature of this class of representations benefits the approximation of devices with multiple inputs and they have been proven to be effective for the characterization of the strongly nonlinear behavior of real devices, possibly with higher order dynamical effects. Finally, LLSS models that have a relatively small size ($5 \div 10$ local models are sufficient for the modeling problem at hand), lead to efficient model implementations that can be effectively used in a simulation environment for replacing transistor-level models of devices and thus speeding-up the simulation of realistic structures.

IV. NUMERICAL RESULTS

In this Section, the methodology for the generation of IC macromodels based on both NARX parametric relations and on LLSS models is applied to a commercial device. The device is the output port of a Texas Instruments transceiver, whose HSPICE physical description is available from the official website of the vendor. The lumped circuit equivalent of the IC package is provided by the supplier as well. The example device is an 8-bit bus transceiver with four independent buffers (model name SN74ALVCH16973, power supply voltage VDD=1.8 V). The example device operates at 167 Mbps, i.e., the bit time is 6 ns. The HSPICE simulations of the physical model are assumed as the reference curves hereafter and are used for both generating the estimation signals and the validation responses.

For this example, different models are estimated. In one case, eight different NARX models are obtained by means of the application of either static [5] or dynamic [6] estimation algorithms. All these models have 2 functions and a dynamic order 3. The second case refers to a LLSS model composed of 3 local linear submodels, and is estimated as outlined in Sec. III. As an example, Fig. 3 shows the output port voltage and current responses used for the estimation of submodel i_H. The port responses are computed as shown in the ideal setup of Fig. 2, where the driver is forced in fixed high output state and a noisy multilevel signal is used for the voltage source $v_s(t)$. Similar curves can be obtained for the alternate submodel i_L.

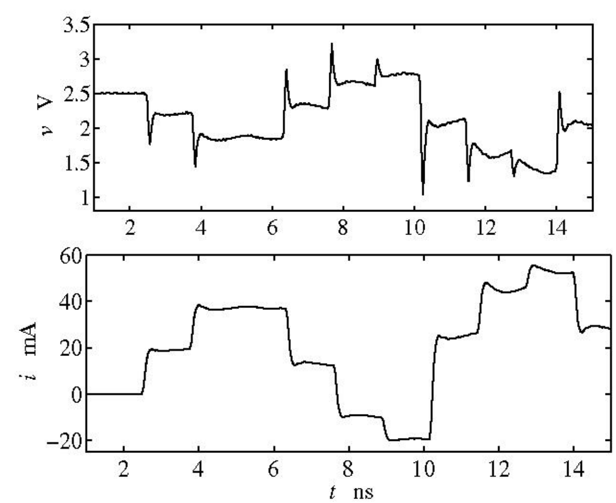

Fig. 3. Device output port voltage $v(t)$ and current $i(t)$ responses computed for the ideal setup in Fig. 2. The driver is replaced by the transistor-level description of the example and a multilevel noisy signal is used for the source $v_s(t)$.

In order to assess the quality of the different estimated models, a validation test circuit consisting of the same setup of Fig. 2 is considered. The voltage source produces a multilevel signal different from the one used for the estimation of model parameters. The test conditions (driver in high state) and the output variable (dynamic component of the port current) are adopted for the sake of simplicity, and not limiting the validity of our conclusions.

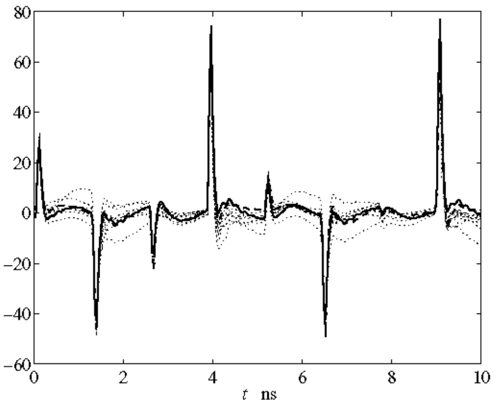

Fig. 4. Dynamic part of the port current response for the example validation test. The curves are computed by means HSPICE and the ideal setup in Fig. 2. Solid line: reference; dashed-line: LLSS model; dotted lines: different NARX models.

Figure 4 compares the dynamic component of the reference output port current response and the responses of models of two different classes. The responses of NARX models (whose estimation differs only for the random initialization of model parameters and the application of different estimation algorithms [5], [6]) produce a band of waveforms around the reference response. The variability of the curves is an indication of the strong dependence of the model quality to the initial guess of model parameters and to the difficulty for NARX models to reproduce much richer (*i.e.,* higher order) dynamics, like those introduced by the die package of this example. On the contrary, LLSS models are adequate to reproduce the reference device behavior.

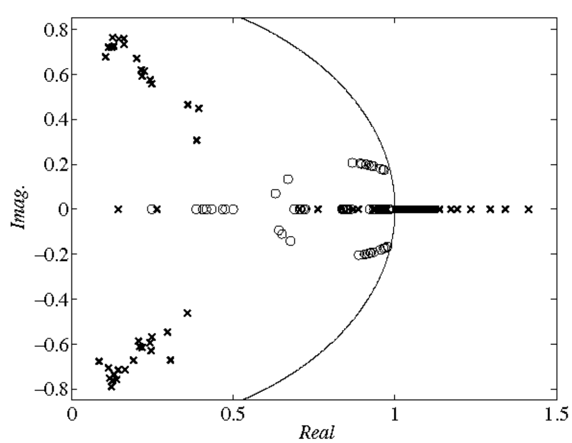

Fig. 5. Comparison of the eigenvalues of the linearized model. The eigenvalues for each point are explored during the transient simulation of the example. Circles: eigenvalues of the LLSS model; crosses: eigenvalues of the NARX models.

For an additional performance evaluation, we addressed ourselves to the assessment of model stability, by means of an analysis of the eigenvalues of the linearized model. The eigenvalues are computed for each point explored by the voltage and current responses during the transient simulations of a validation test [10]. Figures 5 compares the eigenloci of the linearized model equation for the dynamic component of i_H, for LLSS (circles) and NARX (crosses) models. This figure clearly shows that all the eigenvalues of the LLSS model turn out to be located within the unitary circle, as expected. On the other hand, NARX models have a potential dynamic instability (see the eigenvalues lying outside the unitary circle of Fig. 5).

The CPU time required by the estimation of LLSS models and of NARX models is comparable (both methods requires the solution of a nonlinear least squares problem). On the other hand, it is worth to remark that the size (and therefore the efficiency) of two classes of models is comparable. For the example considered in this study, the speed up introduced by the HSPICE implementation of NARX models is 25 and the one introduced by the alternate LLSS class is 22.

V. CONCLUSIONS

This paper addresses the generation of accurate and efficient behavioral models of digital devices. The nonlinear dynamic port behavior of a digital IC is approximated by means of composite local-linear state-space models, whose parameters are computed from device responses via a well established technique. The obtained models are implemented as SPICE subcircuits or hardware descriptions like VHDL-AMS. The feasibility of the approach for the modeling of a real device has been verified by applying it to the characterization of a commercial transistor-level model of a device exhibiting a strong nonlinear behavior and higher order dynamical effects.

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