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On relative error minimization in passivity enforcement schemes

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Abstract

This paper presents a new technique for the elimination of passivity violations in linear lumped macromodels. The main algorithm is based on the perturbation of imaginary eigenvalues of suitably-defined Hamiltonian matrices, as documented in the existing literature. We introduce a modification aimed at the minimization of the relative error in the model responses during the passivity enforcement. This strategy allows the accurate modeling of structures characterized by a large dynamic range, as typically found in microwave filters or advanced packaging applications.

Introduction and motivations

Linear macromodeling provides a flexible and effective solution for fast and accurate simulation of complex interconnects. However, macromodel identification from time-domain or frequency-domain tabulated data [1]-[8] often leads to non-passive results. Though possibly accurate, non-passive models may lead to unstable behavior when used in a CAD tool for system design and verification. Therefore, passivity should be enforced in some way during the model identification process.

Despite the intense research efforts that have been devoted to the subject, the passivity enforcement of linear macromodels still poses several challenges. Currently available solutions can be grouped into tree main classes. On one hand, methods based on convex optimization [9]-[12] are guaranteed to find the optimal solution. Unfortunately, these techniques are limited to small-scale models due to their large computational complexity. A second class is provided by a posteriori passivity correction techniques based on linear or quadratic programming [13, 14]. These methods are based on inequality constraints imposed at discrete frequency samples. They are applicable to larger-size models but are not guaranteed to fully enforce passivity. Finally, other methods exploit the theory of Hamiltonian matrices [15]-[18]. They are applicable to larger-size models and do provide a global passivity characterization and enforcement. However, the convergence is not always guaranteed and the solution they offer is only sub-optimal.

All passivity enforcement techniques apply some perturbation to the model until its passivity is achieved. This perturbation is performed using special constraints insuring that the model accuracy is preserved. These constraints have always been formulated so that the absolute error in the responses is minimized, except for the very recent results in [19]. In this work, we present a method allowing for the systematic preservation of the relative error during the passivity enforcement. We show that the proposed technique leads to superior performance with respect to standard schemes in all cases characterized by responses with large dynamic range. This scenario is typical, e.g., in packaging applications and RF component modeling.

Formulation

The main formulation is here presented for a scalar (one-port) macromodel. The extension to multi-port structures will be fully documented in a forthcoming report, although some hints are provided in the next sections.

We consider a one-port linear macromodel described by one of the following equivalent representations

$$\begin{aligned} H(s) &= d + \sum_{n=1}^{N} \frac{r_n}{s - p_n} \\ &= d\, \frac{\prod_{n=1}^{N}(s - z_n)}{\prod_{n=1}^{N}(s - p_n)} \\ &= d + \mathbf{c}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{b}\,, \end{aligned} \tag{1}$$

where p_n, z_n, r_n, d are, respectively, poles, zeros, residues, and direct coupling constant. State-space matrices $\mathbf{A} \in \mathbb{R}^{N\times N}$, $\mathbf{b} \in \mathbb{R}^{N\times 1}$, and $\mathbf{c} \in \mathbb{R}^{1\times N}$ are easily derived from poles and residues using standard realization techniques. In the following, we will assume that $H(s)$ is regular $\forall s : \mathbb{R}\mathrm{e}\,\{s\} \geq 0$, including $s = 0$ and $s = \infty$. This implies that $H(0)$ and $H(\infty)$ are finite and non-vanishing, and that all poles are strictly stable, i.e., $\mathbb{R}\mathrm{e}\,\{p_n\} < 0,\ n = 1, \ldots, N$. Form (1) is the typical outcome of standard macromodeling algorithms providing rational approximations of tabulated data, Vector Fitting (VF) being the most popular in its various implementations [1, 2, 3, 4, 5, 6, 7].

Given the above regularity assumptions, a macromodel (1) is passive when $H(s^*) = H^*(s)$ and when

- $H(j\omega) + H^*(j\omega) \geq 0$, $\forall\omega$ in case $H(s)$ represents a driving-point impedance or admittance (henceforth denoted ”case I”)
- $|H(j\omega)| \leq 1$, $\forall\omega$ in case $H(s)$ represents a reflection coefficient (henceforth denoted ”case II”)

Once a model (1) is available, its passivity should be checked and enforced before safe use in a CAD environment. Several techniques are available for checking passivity. Here, we rely on algebraic tests based on the eigenstructure or Hamiltonian matrices [17], which do not require any frequency sampling process, and which provide global results over the complete frequency spectrum. More precisely, the model is passive when there are no imaginary eigenvalues of the associated Hamiltonian matrix $\mathcal{M}$, provided that $d + d^* \geq 0$ (case I) or $|d| \leq 1$ (case II). More details can be found in [17].

When a passivity violation is detected, it can be eliminated using an iterative perturbation technique aimed at the displacement of the imaginary Hamiltonian eigenvalues. The perturbation scheme is obtained by relating, via first-order expansions, the modification of the coefficients in (1) that is required for a desired eigenvalue perturbation. This procedure results [17] in an iterative linear system solution

$$\mathbf{Z}\,\boldsymbol{\delta} = \boldsymbol{\mu}\,, \tag{2}$$

where $\mathbf{Z}$ is a constant matrix, $\boldsymbol{\delta}$ collects the corrections to be applied to the coefficients in (1), and $\boldsymbol{\mu}$ is the desired amount of Hamiltonian eigenvalue perturbation. Accuracy is usually preserved during this process by adding a minimum-norm constraint. Choice of this norm is crucial for the performance of the algorithm, as discussed below.

Absolute and relative norms

We focus our analysis on perturbation schemes applied to the residues r_n. We will assume that the state-space realization is obtained as in [18], so that state-space matrix $\mathbf{c}$ collects all residues. Therefore, the model to be determined is associated to the state-space realization

$$H_p(s) = d + (\mathbf{c} + \boldsymbol{\delta})\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{b}\,. \tag{3}$$

Other perturbation choices are available in [19].

The standard approach for the preservation of model accuracy is to minimize the absolute energy $\mathcal{L}^2$ norm in the perturbation of the model response. This norm can be defined either in time-domain or directly in frequency-domain, the two formulations being equivalent due to the Parseval's identity. We have

$$\begin{aligned} ||H_p(s) - H(s)||_2^2 &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |H_p(j\omega) - H(j\omega)|^2 \mathrm{d}\omega \\ &= \int_0^{\infty} |h_p(t) - h(t)|^2 \mathrm{d}t \\ &= \boldsymbol{\delta}\,\mathbf{P}\,\boldsymbol{\delta}^T\,, \end{aligned} \tag{4}$$

where $h(t)$ denotes the inverse Laplace transform of $H(s)$ and matrix $\mathbf{P}$ is the controllability Gramian of the state-space realization (1), defined as the unique symmetric and strictly positive-definite solution of the following Lyapunov equation [20, 21]

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T + \mathbf{b}\mathbf{b}^T = \mathbf{0}\,. \tag{5}$$

The last row in (4) defines the norm that should be minimized during the solution of (2) in order to preserve the absolute accuracy during passivity enforcement.

Let us now introduce the relative perturbation as

$$\Theta(s) = (H_p(s) - H(s))\,H^{-1}(s)\,. \tag{6}$$

We want to derive an algebraic characterization of the $\mathcal{L}^2$ norm of this perturbation, in terms of the model perturbation coefficients $\boldsymbol{\delta}$. To this aim, we first derive the state-space realization of $H^{-1}(s)$ as

$$H^{-1}(s) = d^{-1} - \left(d^{-1}\mathbf{c}\right)\left(s\mathbf{I} - (\mathbf{A} - \mathbf{b}\,d^{-1}\mathbf{c})\right)\left(\mathbf{b}\,d^{-1}\right)\,. \tag{7}$$

Direct substitution into (6) leads, using (1) and (3), to

$$\Theta(s) = [\boldsymbol{\delta} \quad \mathbf{0}]\,(s\mathbf{I} - \mathbf{A}_\theta)^{-1}\,\mathbf{b}_\theta\,, \tag{8}$$

where

$$\mathbf{A}_\theta = \begin{bmatrix} \mathbf{A} & \mathbf{b}d^{-1}\mathbf{c} \\ \mathbf{0} & \mathbf{A} - \mathbf{b}d^{-1}\mathbf{c} \end{bmatrix}, \quad \mathbf{b}_\theta = \begin{bmatrix} \mathbf{b}d^{-1} \\ -\mathbf{b}d^{-1} \end{bmatrix}. \tag{9}$$

We compute now the controllability Gramian $\mathbf{P}_\theta$ of (8) as the solution of

$$\mathbf{A}_\theta\mathbf{P}_\theta + \mathbf{P}_\theta\mathbf{A}_\theta^T + \mathbf{b}_\theta\mathbf{b}_\theta^T = \mathbf{0}\,. \tag{10}$$

This Gramian matrix has a 2×2 block structure

$$\mathbf{P}_\theta = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix} \tag{11}$$

where each block is $N \times N$. A straightforward calculation leads to the following characterization of the relative norm

$$\begin{aligned} ||H_p(s) - H(s)||_{\mathrm{rel}}^2 &= ||\Theta(s)||_2^2 \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |\Theta(j\omega)|^2 \mathrm{d}\omega \\ &= \boldsymbol{\delta}\,\mathbf{P}_{11}\,\boldsymbol{\delta}^T\,. \end{aligned} \tag{12}$$

The last row is formally identical to (4), differing only in the definition of the Gramian matrix. The above derivation implies that if (4) is replaced by (12) during the passivity enforcement iterations, the relative error is indeed minimized.

Minimum phase constraints

The above derivation implies some important restriction on the models that can be handled. Precisely, the model (1) must be minimum-phase. Equivalently, all its zeros must satisfy $\mathbb{R}\mathrm{e}\,\{z_n\} < 0,\ n = 1, \ldots, N$. The necessity of this condition is clear from (7). Since these zeros are the eigenvalues of

$$\{z_n\} = \mathrm{eig}\{\mathbf{A} - \mathbf{b}\,d^{-1}\mathbf{c}\}\,, \tag{13}$$

a violation of the minimum-phase condition leads to unstable realizations of $H^{-1}(s)$ and $\Theta(s)$. Consequently, the Gramians $\mathbf{P}_\theta$ and $\mathbf{P}_{11}$ become non-positive definite, and (12) cannot be a well-defined norm.

The above limitation is easily overcome as follows. First, the set of zeros is split as

$$\{z_n\}_{n=1}^{N} = \{z_n^+\}_{n=1}^{N^+} \cup \{z_n^-\}_{n=1}^{N^-}\,, \tag{14}$$

where

$$\mathbb{R}\mathrm{e}\left\{z_n^+\right\} > 0 \quad \text{and} \quad \mathbb{R}\mathrm{e}\left\{z_n^-\right\} < 0\,. \tag{15}$$

Then, the modified definition for the relative perturbation

$$\Theta(s) = (H_p(s) - H(s))\,H^{-1}(s)\prod_{n=1}^{N^+}\frac{s - z_n^+}{s + z_n^+}\,. \tag{16}$$

is used instead of (6). The last term is an all-pass function that flips the positive zeros of $H(s)$ into the left hand plane. Being an all-pass function, the magnitude $|\Theta(j\omega)|$ on the imaginary axis is left unchanged, with no effect in the numerical value of the norm (12). However, the associated state-space realization of $\Theta(s)$ becomes strictly stable, thus allowing the algebraic characterization of the norm using the (positive-definite) Gramian $\mathbf{P}_{11}$.

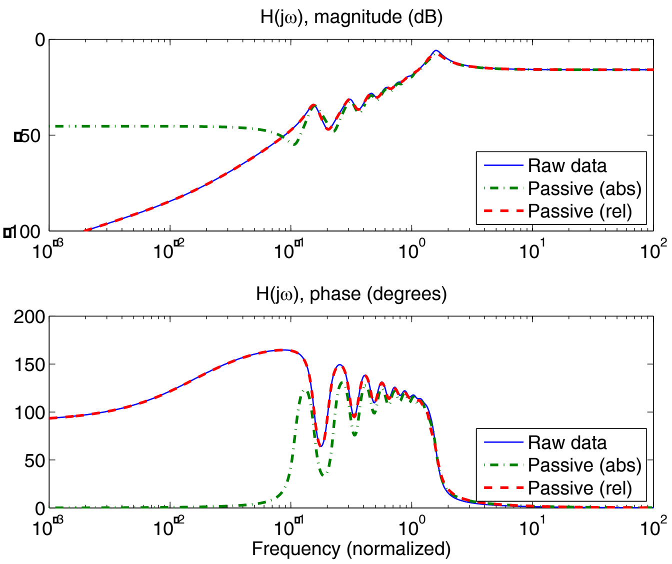

Figure 1: Validation example. The responses of two passive models obtained by enforcing absolute and relative error are compared to raw frequency data.

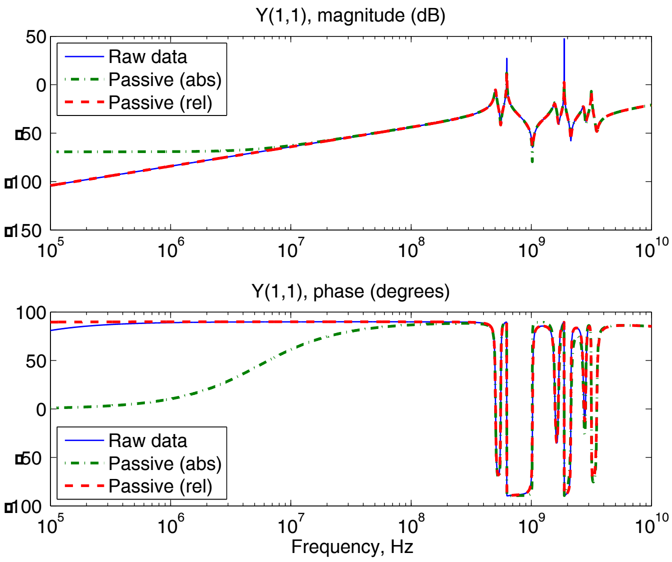

Figure 2: AC coupled interconnect. The responses of two passive models obtained by enforcing absolute and relative error are compared to raw frequency data.

To summarize, the passivity enforcement with relative error minimization is achieved by iteratively solving the system

$$\min_{\boldsymbol{\delta}} \boldsymbol{\delta}\,\mathbf{P}_{11}\,\boldsymbol{\delta}^T\,, \quad \mathbf{Z}\,\boldsymbol{\delta} = \boldsymbol{\mu}\,. \tag{17}$$

The solution of this system in least squares sense is straightforward.

A few remarks on the extension of the above technique to multi-port macromodels. Two alternative approaches can be devised. A first approach is to define a matrix-valued relative perturbation as

$$\boldsymbol{\Theta}(s) = (\mathbf{H}_p(s) - \mathbf{H}(s))\,\mathbf{H}^{-1}(s)\,. \tag{18}$$

This definition, however, does not allow the minimization of the relative perturbation on each individual response of the model. This may be crucial for Signal and Power Integrity applications, since small crosstalk waveforms may be very sensitive to model perturbations, unless their individual accuracy is preserved. This is possible by defining an element-wise relative perturbation

$$\Theta_{ik}(s) = (H_{p,ik}(s) - H_{ik}(s))\,H_{ik}^{-1}(s)\,. \tag{19}$$

Implementation of (19) requires special care in the derivation of the modified Gramian matrix to be used in the perturbation norm. All details are postponed to a forthcoming report. However, the numerical results of next section are indeed multi-port cases.

Examples

The first example we consider is a synthetic 6-port lumped structure that was specifically designed to compare the performance of the various passivity enforcement schemes. A rational scattering matrix $\mathbf{S}(s)$ was generated by randomly selecting poles and zeros of each element. Then, the maximum singular value of $\mathbf{S}(j\omega)$ was constrained via appropriate scaling to $\sigma_{\max} = 1.2$. Therefore, the raw frequency responses exploit a non-passive behavior.

The standard passivity enforcement scheme with absolute error control was applied to correct the model residues and recover passivity. The results for one of the model responses are depicted in Fig. 1. Significant accuracy degradation is observed at low frequencies. The proposed scheme based on relative error control was also applied to this example. Fig. 1 shows that accuracy is indeed preserved over a large dynamic range.

The second example is an AC-coupled transmission link. The interconnect is formed by two stripline segments separated by a series capacitance. The 2×2 short-circuit admittance matrix of the structure was computed, and a rational macromodel was generated using VF iterations with inverse magnitude weighting [2, 7]. This strategy was necessary since the series capacitance induces a wide dynamic range in the responses, as depicted in Fig. 2 and 3. These plots show that application of the standard passivity enforcement with absolute error control degrades the low-frequency accuracy, whereas the proposed scheme provides a much better performance.

Conclusions

We have presented some preliminary results on the preservation of relative errors during the passivity enforcement of linear lumped macromodels. The algebraic characterization of the relative error norm allows the implementation of this constraint into existing passivity enforcement schemes with minimal modifications. The results show that the proposed technique results in accurate modeling of responses characterized by large dynamic range.

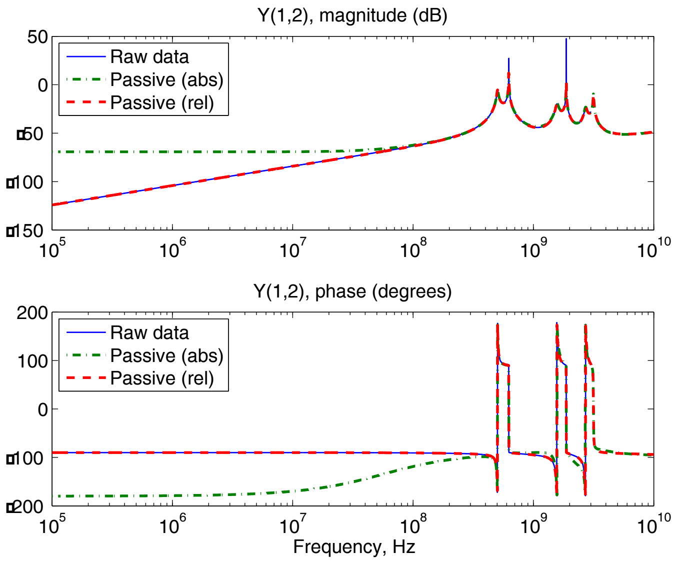

Figure 3: AC coupled interconnect. The responses of two passive models obtained by enforcing absolute and relative error are compared to raw frequency data.

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