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# Temporal dynamics of small perturbations for a 2D growing wake

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## 1 Introduction

A general three-dimensional initial-value perturbation problem is presented to study the linear stability of a two-dimensional growing wake. The base flow has been obtained by approximating it with an expansion solution for the longitudinal velocity component that considers the lateral entrainment process [1]. By imposing arbitrary three-dimensional perturbations in terms of the vorticity, the temporal behaviour, including both the early time transient as well as the long time asymptotics, is considered [2], [3], [4]. The approach has been to first perform a Laplace-Fourier transform of the governing viscous disturbance equations and then resolve them numerically by the method of lines. The base model is combined with a change of coordinate [5]. Base flow configurations corresponding to a  $R$  of 35, 50, 100 and various physical inputs are examined. In the case of longitudinal disturbances, a comparison with recent spatio-temporal multiscale Orr-Sommerfeld analysis [6], [7] is presented.

## 2 The initial-value problem

The base flow is viscous and incompressible. To define it, the longitudinal component of an approximated Navier-Stokes expansion for the two-dimensional steady bluff body wake [1], [8] has been used. The  $x$  coordinate is parallel to the free stream velocity, the  $y$  coordinate is normal. The coordinate  $x_0$  plays the role of parameter of the system together with the Reynolds number. The analytical expression for the wake profile is  $U(y; x_0, R) = 1 - a(R)x_0^{-1/2}e^{-(Ry^2)/(4x_0)}$ , where  $a(R)$  depends on the Reynolds number [8]. By changing  $x_0$ , the base flow profile locally approximates the behaviour of the actual wake generated by the body. The equations are

$$\nabla^2 \tilde{v} = \tilde{\Gamma} \quad (1)$$

$$\frac{\partial \tilde{\Gamma}}{\partial t} + U \frac{\partial \tilde{\Gamma}}{\partial x} - \frac{\partial \tilde{v}}{\partial x} \frac{d^2 U}{dy^2} = \frac{1}{R} \nabla^2 \tilde{\Gamma} \quad (2)$$

$$\frac{\partial \tilde{\omega}_y}{\partial t} + U \frac{\partial \tilde{\omega}_y}{\partial x} + \frac{\partial \tilde{v}}{\partial z} \frac{dU}{dy} = \frac{1}{R} \nabla^2 \tilde{\omega}_y \quad (3)$$

where  $\tilde{\omega}_y$  is the transversal component of the perturbation vorticity, while  $\tilde{\Gamma}$  is defined as  $\tilde{\Gamma} = \frac{\partial \tilde{\omega}_z}{\partial x} - \frac{\partial \tilde{\omega}_x}{\partial z}$ . All physical quantities are normalized with respect to the free stream velocity, the spatial scale of the flow  $D$  and the density. By introducing the moving coordinate transform  $\xi = x - U_0 t$  [5] and performing a combined Laplace-Fourier decomposition of the dependent variables in terms of  $\xi$  and  $z$ , the governing equations become

$$\frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i) \hat{v} = \hat{\Gamma} \quad (4)$$

$$\begin{aligned} \frac{\partial \hat{\Gamma}}{\partial t} = & -ik\cos(\phi)(U - U_0)\hat{\Gamma} + ik\cos(\phi)\frac{d^2 U}{dy^2} \hat{v} \\ & + \alpha_i(U - U_0)\hat{\Gamma} - \alpha_i\frac{d^2 U}{dy^2} \hat{v} + \frac{1}{R}\left[\frac{\partial^2 \hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i)\hat{\Gamma}\right] \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial \hat{\omega}_y}{\partial t} = & -ik\cos(\phi)(U - U_0)\hat{\omega}_y - ik\sin(\phi)\frac{dU}{dy} \hat{v} \\ & + \alpha_i(U - U_0)\hat{\omega}_y + \frac{1}{R}\left[\frac{\partial^2 \hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r \alpha_i)\hat{\omega}_y\right] \end{aligned} \quad (6)$$

where  $\hat{f}(y, t; \alpha, \gamma) = \int_{-\infty}^{+\infty} \int_0^{+\infty} \tilde{f}(\xi, y, z, t) e^{i\alpha\xi + i\gamma z} d\xi dz$  is the Laplace-Fourier transform of a general dependent variable,  $\phi = \tan^{-1}(\gamma/\alpha_r)$  is the perturbation angle of obliquity,  $k = \sqrt{\alpha_r^2 + \gamma^2}$  is the polar wavenumber and  $\alpha_r = k\cos(\phi)$ ,  $\gamma = k\sin(\phi)$  are the wavenumbers in  $\xi$  and  $z$  directions respectively. We choose periodic and bounded initial conditions:

CASE I (symmetric initial condition):  $\hat{v}(0, y) = e^{-y^2} \cos(\beta y)$ ,  $\hat{\omega}_y(0, y) = 0$

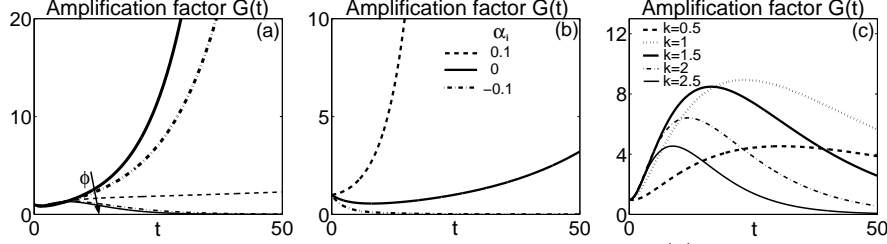
CASE II (asymmetric initial condition):  $\hat{v}(0, y) = e^{-y^2} \sin(\beta y)$ ,  $\hat{\omega}_y(0, y) = 0$

### 3 Results and Conclusions

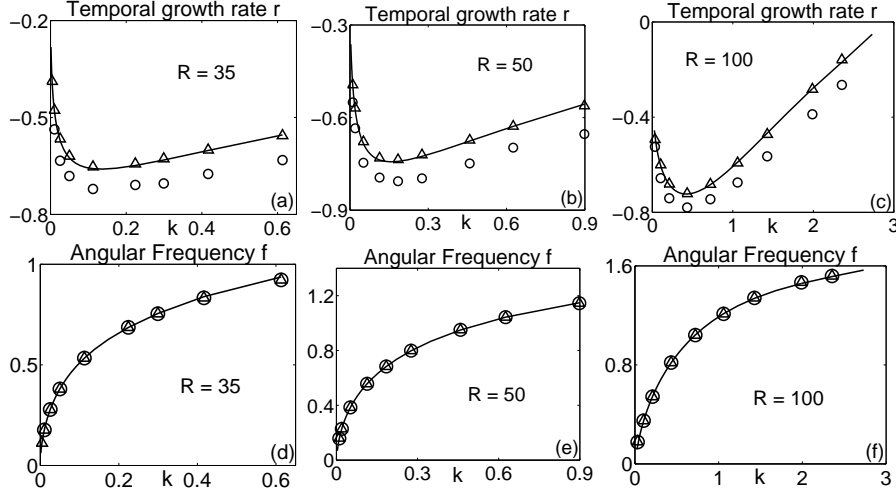
The amplification factor  $G$  is defined as the normalized energy density [3], namely  $G(t; k, \phi) = E(t; k, \phi)/E(t = 0; k, \phi)$ . It effectively measures the growth of the energy at time  $t$ , for a given initial condition at  $t = 0$  (fig. 1). By defining the temporal growth rate [4] as  $r = \log|E(t)|/(2t)$  ( $E(t)$  is the total perturbation energy) and the angular frequency  $f$  as the temporal derivative of disturbance phase, we can evaluate the initial stages of exponential growth and, in the case of 2D disturbances, compare them with the normal mode theory results [6] (fig. 2).

Figure 1 yields three differing examples of early transient periods. Case (a) shows that a growing wave becomes damped, increasing the obliquity angle beyond  $\pi/4$ . Case (b) corresponds to dispersion relation values far from the saddle point and shows that spatially damped/amplified waves can be temporally amplified/damped. Case (c) demonstrates that perturbations normal to the base flow are stable. Figure 2 presents the comparison between the initial value problem and the Orr-Sommerfeld problem. The results are parameterized with respect to the position  $x_0$  through the polar wavenumber  $k = k(x_0)$ . Equations are integrated in time beyond the transient until the

temporal growth rate asymptotes to a constant value. We observed a very good agreement with the stability characteristics given by the Orr-Sommerfeld theory for both the symmetric and asymmetric arbitrary disturbances considered.



**Fig. 1.** The amplification factor  $G$  as a function of time. (a):  $R = 100$ ,  $k = 1.2$ ,  $\alpha_i = -0.1$ ,  $\beta = 1$ ,  $x_0 = 10.15$ ,  $\phi = 0, \pi/8, \pi/4, (3/8)\pi, \pi/2$ , symmetric perturbation (case I). (b):  $R = 50$ ,  $k = 0.3$ ,  $\beta = 1$ ,  $\phi = 0$ ,  $x_0 = 5.20$ ,  $\alpha_i = -0.1, 0, 0.1$ , symmetric perturbation (case I). (c):  $R = 100$ ,  $\alpha_i = -0.01$ ,  $\beta = 1$ ,  $\phi = \pi/2$ ,  $x_0 = 7.40$ ,  $k = 0.5, 1, 1.5, 2, 2.5$ , symmetric perturbation (case I).



**Fig. 2.**  $\beta = 1$ ,  $\phi = 0$ . (a, b, c) Temporal growth rate and (d, e, f) angular frequency. Comparison between present results (triangles: symmetric perturbation, case I; circles: asymmetric perturbation, case II) and normal mode analysis by Tordella, Scarsoglio and Belan, 2006 Phys. Fluids (solid lines). The wavenumber  $\alpha = \alpha_r(x_0) + i\alpha_i(x_0)$ ,  $\alpha_r(x_0) = k(x_0)$  is the most unstable wavenumber in any section of the near-parallel wake (dominant saddle point in the local dispersion relation). The wake sections considered are in the interval  $3D \leq x_0 \leq 50D$ .

## References

1. D. Tordella, M. Belan: Phys. Fluids **15**, 7 (2003)
2. P. N. Blossy, W.O. Criminale, L.S. Fisher: J. Fluid Mech. submitted, (2006)
3. D.G. Lasseigne, et al.: J. Fluid Mech. **381**, (1999)
4. W. O. Criminale, et al.: J. Fluid Mech. **339**, (1997)
5. W.O. Criminale, P.G. Drazin: Stud. in Applied Math. **83**, (1990)
6. D. Tordella, S. Scarsoglio and M. Belan: Phys. Fluids **18**, 5 (2006)
7. M. Belan, D. Tordella: J. Fluid Mech. **552** (2006)
8. M. Belan, D. Tordella: Zamm **82**, 4 (2002)