Decoherence-free emergence of macroscopic local realism for entangled photons in a cavity

PHYSICAL REVIEW A **73**, 020101(R) (2006)

Decoherence-free emergence of macroscopic local realism for entangled photons in a cavity

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(Received 21 December 2004; published 21 February 2006)

We investigate the influence of environmental noise on polarization entangled light generated by parametric emission in a cavity. By adopting a recent separability criterion, we show that (i) self-stimulation may suppress the detrimental influence of noise on entanglement, but (ii) once it becomes effective, a noise-equipped classical model of parametric emission provides the same results of quantum theory with respect to the separability criterion. More generally we also show that, in the macroscopic limit, it is not possible to observe violations of local realism with measurements of finite order n-particle correlations only. These results provide a prototypical case of the emergence of macroscopic local realism in the presence of strong entanglement even in the absence of decoherence.

DOI: 10.1103/PhysRevA.73.020101 PACS number(s): 03.65.Ud, 42.65.Lm, 03.65.Ta

Entanglement is one of the most profound features of quantum mechanics. It plays an essential role in all branches of quantum information theory [1]. Bell theorem [2], which is derived from the Einstein-Podolsky-Rosen (EPR) notion of local realism [3], quantifies how measurements on entangled quantum mechanical systems may invalidate local classical models of reality. While all bipartite pure entangled states violate some Bell inequality [4], the relationship between entanglement and nonlocality for mixed quantum states is not well understood yet [5,6]. Moreover, recent proposals [7] and realizations [8–12] of many-particle entangled quantum states require a better understanding of the domain of validity of quantum behavior. A relevant point is whether the conflict between classical elements of reality and quantum mechanics may persist at a macroscopic level [13,14]. Indeed continuous-variable entanglement of intense-light sources has been recently demonstrated in [8,10] and polarization entanglement of macroscopic beams in [11]. It has been recently shown in Ref. [7] that a source of strongly entangled states with photon numbers up to a million seems achievable. In these works entanglement has been tested and quantified by means of specific separability criteria that are inequalities among expectation values of experimentally measurable quantities, violated by entangled quantum states [15]. The behavior of entanglement towards a macroscopic situation (even close to classical everyday life phenomena) and its robustness versus noise and decoherence are not well understood and the quantum-to-classical transition is usually associated with decoherence [14].

In this paper we shall address this crucial problem focusing on a particular promising source of macroscopic entanglement: parametric downconversion of photons inside an optical cavity. On the one hand, we shall quantify the detrimental influence of such environment channels and show how self-stimulation may suppress them efficiently. On the other hand, we shall tackle the problem of the macroscopic limit and of the emergence of classical elements of reality within a quantum framework. We shall illustrate a counterexample where the emergence of macroscopic local realism (MLR) may be seen as an intrinsic feature of quantum systems, endogenous in the quantum theory itself (even in the presence of strong entanglement that is the quintessential of nonclassicality). In such a case there is no need at all to rely on environment ingredients (like noise and decoherence). Our results, of course, do not imply that macroscopic entangled systems cannot display violations of local realism [16,17], but that there is a large class of quantum correlation measurements that cannot be used to show them. We shall first deal with a specific situation that is the Heisenberg steady-state Eq. (2); this way we are able to focus on the most important physical ingredients in a very neat way. It is worth noting that this does not restrict our conclusions at all, indeed we prove that in the more realistic t-dependent Langevin approach of Eq. (8) all our results continue to hold (see Fig. 1). The results presented here indicate that MLR may result from the inability of the observer, practically unavoidable for macroscopic systems, to catch the quantized structure of the system.

We consider polarization entangled light from parametric downconversion driven by an intense pump field inside a cavity. The multiphoton states produced are close approximations to singlet states of two very large spins [7]. The interaction Hamiltonian describing the process is given by

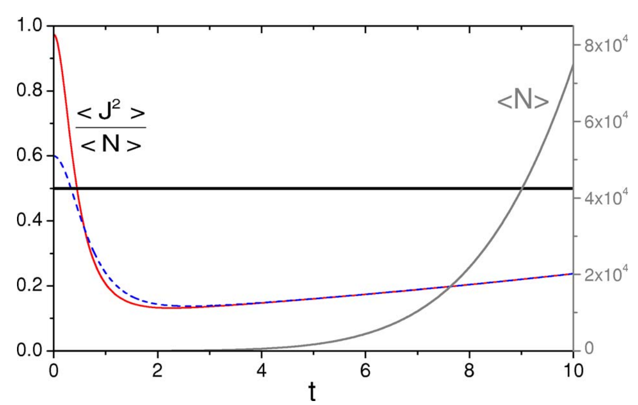

FIG. 1. (Color online) Time development of $\langle \mathbf{J}^2\rangle/\langle N\rangle$ according to classical (dashed line) and quantum (continuous line) mechanics, and quantum calculation of of the total mean photon number $\langle N\rangle$. Parameters are given in the text.

$$\hat{H} = i\hbar\Omega(\hat{a}_h^\dagger \hat{b}_v^\dagger - \hat{a}_v^\dagger \hat{b}_h^\dagger) + \text{H.c.}, \tag{1}$$

where a and b refer to the two conjugate directions the frequency-degenerate photon pairs are emitted along. h and v denote horizontal and vertical polarization and $\hbar\Omega$ is a coupling constant whose magnitude depends on the nonlinear coefficient of the crystal and on the intensity of the pump pulse. In the absence of losses, within the Heisenberg picture, the interaction Hamiltonian in (1) dictates the following steady-state solution for photon operators

$$\hat{a}_{h,v} = \hat{a}_{h,v}(0)\cosh(r) \pm \hat{b}_{v,h}^\dagger(0)\sinh(r),$$

$$\hat{b}_{v,h} = \hat{b}_{v,h}(0)\cosh(r) \pm \hat{a}_{h,v}^\dagger(0)\sinh(r), \tag{2}$$

where the interaction parameter r is $\Omega\tau$ being $\tau=L/v$ the interaction time interval, i.e., the time spent by the photons with velocity v inside a crystal of length L. In the absence of losses and considering the photon vacuum as initial state, the Hamiltonian (1) produces a multiphoton quantum state $|\psi\rangle$ that is the polarization equivalent of a spin-singlet state, where the spin components correspond to the Stokes polarization parameters, $\hat{J}_z^A=(\hat{a}_h^\dagger\hat{a}_h-\hat{a}_v^\dagger\hat{a}_v)/2$, $\hat{J}_x^A=(\hat{a}_+^\dagger\hat{a}_+-\hat{a}_-^\dagger\hat{a}_-)/2$, and $\hat{J}_y^A=(\hat{a}_l^\dagger\hat{a}_l-\hat{a}_r^\dagger\hat{a}_r)/2$, where $\hat{a}_{+,-}=(\hat{a}_h\pm\hat{a}_v)/\sqrt{2}$ corresponds to linearly polarized light at $\pm 45^\circ$, and $\hat{a}_{l,r}=(\hat{a}_h\pm i\hat{a}_v)/\sqrt{2}$ to left- and right-ended circularly polarized light. The label A refers to the a modes. Analogous expressions can be obtained for $\hat{\mathbf{J}}^B$ in terms of the b modes. It has been shown [7] that the state $|\psi\rangle$ is a singlet state of the total angular momentum operator $\hat{\mathbf{J}}=\hat{\mathbf{J}}^A+\hat{\mathbf{J}}^B$. As a consequence $\langle\psi|\hat{\mathbf{J}}^2|\psi\rangle=0$. Losses, fluctuations, and imperfections may lead to nonzero values for the total angular momentum, corresponding to nonperfect correlations between the Stokes parameters in the a and b beams. Within this picture it is straightforward to include the presence of noise in the system assuming that, before the pump is switched on, the system is in an incoherent thermal-like state described by a diagonal density matrix. Dealing with such systems the first analysis one may perform is an intensity measurement

$$n_{ah(v)}(r) \equiv \langle \hat{a}_{h,v}^\dagger \hat{a}_{h,v}\rangle = \sinh^2 r + n_0(1+2\sinh^2 r), \tag{3}$$

where $n_0\equiv\langle\hat{a}_{h,v}^\dagger(0)\hat{a}_{h,v}(0)\rangle=\langle\hat{b}_{h,v}^\dagger(0)\hat{b}_{h,v}(0)\rangle$ is the noise present in the system. There are two different contributions: The first term arises from vacuum fluctuations and describes true (eventually self-stimulated) spontaneous emission, vanishing in the absence of parametric interaction; the latter describes a classical-like amplification of the input thermal noise n_0. It is worth noting that the solution of the corresponding classical model of an optical parametric amplifier has the same structure of Eq. (2) with a and b being, of course, replaced by classical amplitudes [19],

$$n^C(r) \equiv \langle a_{h,v}^* a_{h,v}\rangle = n_0^C(1+2\sinh^2 r), \tag{4}$$

where $\langle\cdots\rangle$ denotes statistical average and n_0^C is, as before, statistical noise. For small r values and for negligible n_0 ($n_0 \ll r^2$), quantum and classical descriptions lead to distinct behavior in r, being $n(r)\simeq r^2$ and $n^C(r)\simeq n_C(0)$. It can be related to the fact that vacuum fluctuations (in contrast with classical ones) do not produce work and hence, while they can stimulate pump scattering, they cannot be directly evidenced by photodetection. In contrast, when r increases, it is no more possible to identify quantum behavior by means of simple intensity measurements. In particular, for $r\geqslant 2$ a classical model with $n_0^C=n_0+1/2$ (in order to properly include vacuum fluctuations) is able to give an intensity description that cannot be distinguished from the quantum one. It is worth noting that this behavior can also be found in intriguing second-order interference effects [18,20] and it agrees with the old idea that many quanta in a system give rise to a classical-like behavior. Other relevant second-order correlations are given by the following anomalous correlators:

$$A_{hv(vh)} = \langle \hat{a}_{h(v)}\hat{b}_{v(h)}\rangle = (n_0+1/2)\sinh(2r),$$

$$A_{hv(vh)}^C = \langle a_{h(v)} b_{v(h)}\rangle = n_0^C \sinh(2r), \tag{5}$$

which quantify the pair correlation induced by the parametric process. Equation (5) shows that replacing again $n_0^C=n_0+1/2$, the classical description coincides with the quantum one.

If the above hold for second-order correlations, now we want to focus our attention on what we can say about entanglement measurements on this system. In order to test the degree of entanglement, a simple inseparability criterion has been derived [7]: If $\langle\hat{\mathbf{J}}^2\rangle/\langle\hat{N}\rangle$ (where $\hat{N}=\hat{N}_A+\hat{N}_B$ is the total photon number) is smaller than $1/2$, then the state is entangled. We now consider the system at $r\leqslant 0$ (before switching on the pump) to be in thermal equilibrium, i.e., in a completely incoherent (mixed) state described by a diagonal density matrix. The only input for the system are thermal noise (if $T\neq 0$) and vacuum fluctuations. From Eq. (2) we obtain

$$\frac{\langle\hat{\mathbf{J}}^2\rangle}{\langle\hat{N}\rangle} = \frac{3n_0(n_0+1)}{4n_0+(1+5n_0)\sinh^2 r}. \tag{6}$$

At zero temperature $n_0=0$ and the system is maximally entangled ($\langle\hat{\mathbf{J}}^2\rangle=0$) independently of the magnitude of r. As Eq. (6) shows, even when thermal noise largely exceeds vacuum fluctuations ($n_0\gg 1$), $\langle\hat{\mathbf{J}}^2\rangle/\langle\hat{N}\rangle$ goes below $1/2$ providing r is large enough. Moreover, $\langle\hat{\mathbf{J}}^2\rangle/\langle N\rangle\to 0$ for $r\to\infty$. Thus macroscopic entanglement may in principle be achieved even in the presence of strong fluctuations, provided that self-stimulation of the emitted pairs takes place. In particular the system becomes entangled when $\sinh^2 r > 2n_0(3n_0+1)/(5n_0+1)$. Nevertheless, according to the criterion, in order to beat the detrimental effect of strong fluctuations on entanglement we need to rely on self-stimulation. It is known that entanglement as well as violations of Bell inequalities have limited resistance to noise. Here a small amount of noise is enough to completely destroy entanglement, e.g., for $r=10^{-3}$, $n_0=2\times 10^{-6}$ is sufficient to wash out entanglement according to Eq. (6); nevertheless switching on self-stimulation (by increasing r) we shall restore it. In order to get a deeper insight we seek some additional information wondering if, from the criterion viewpoint (this time), we can distinguish between classical and quantum findings. To this end we put the two descriptions (classical and quantum) on equal footing and compute the entanglement criterion evaluating their differences and similarities. A classical calculation, computed according to the above described prescriptions, gives $\langle \mathbf{J}^2\rangle/\langle N\rangle = 3n_0^C/(4+5\sinh^2 r)$. Of course classical optics does not require a minimum amount of fluctuations, thus within a classical model it is possible to obtain $\langle \mathbf{J}^2\rangle/\langle N\rangle < 0.5$. In the low excitation regime ($r \ll 1$) and n_0 lower than r^2 classical and quantum calculations of $\langle \hat{\mathbf{J}}^2\rangle/\langle \hat{N}\rangle$ display very different variations with r as it happens for simpler intensity cases. As r increases they tend to coincide. This means that experiments eventually demonstrating macroscopic entanglement for this system can be accounted for in terms of purely classical correlations, with no need for a quantum-mechanical explanation. Analogous conclusions can be reached for different experimentally tested criteria [8,10–12]. This does not mean at all that the entanglement criterion is wrong or contradictory. In contrast to Bells' inequalities, these kind of criteria are derived exploiting the fact that involved operators do not commute (hence they do not hold for classical descriptions). Also these results do not imply that macroscopic entangled systems cannot display quantum nonlocality effects. It has been shown in [16] that quantum states of a nondegenerate optical parametric amplifier display violations of the Bell inequality due to Clauser, Horne, Shimony, and Holt in the macroscopic limit ($r \to \infty$). However the above analysis suggests that there is a large class of quantum correlation functions that cannot differ from the corresponding classical ones in the macroscopic limit. Indeed we can define the following set of correlation functions $\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger}\hat{\mathcal{B}}_{\alpha'}^{(n')}\rangle$, where $\hat{\mathcal{B}}_{m,l,k}^{(n)} = (\hat{b}_v)^{n-m}(\hat{b}_h)^{m-l}(\hat{a}_v)^{l-k}(\hat{a}_h)^k$ is a generic n-particle destruction operator. These correlation functions (and also their classical counterparts $\langle \mathcal{B}_\alpha^{(n)*}\mathcal{B}_{\alpha'}^{(n')}\rangle$) are different from zero only if $n = n'$ and $\alpha \equiv (k,l,m) = \alpha'$. Since we are dealing with a Gaussian system [21], such correlators (quantum and classical) can be decomposed in a sum of products of second-order correlation functions only [$n(r)$ and $A(r)$]. Since $A(r) = A^C(r)$ (for $n_0^C = n_0 + 1/2$), and $n(r)/n^C(r) \to 1$ for $r \to \infty$, we obtain that

$$\lim_{r\to\infty} \frac{\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger}\hat{\mathcal{B}}_\alpha^{(n)}\rangle}{\langle \mathcal{B}_\alpha^{(n)*}\mathcal{B}_\alpha^{(n)}\rangle} = 1. \tag{7}$$

This result implies that it is not possible to observe violations of macroscopic local realism (e.g., violations of Bell inequalities, including those which are not yet known) by measurements of any finite set of expectation values that can be expanded as a finite sum of these correlation functions. It is easy to verify via explicit calculations that convergence of limit (7) is very fast even for large values of n. Bell inequalities can be schematically expressed as $\mathcal{F}[\langle \mathcal{B}_\alpha^{(n)*}\mathcal{B}_\alpha^{(n)}\rangle(r)] \le \mathcal{L}(n)$, where $\mathcal{L}(n)$ is a bound imposed by local realism that cannot be violated by classical correlations, and $\mathcal{F}$ is a generic continuous function of $\langle \mathcal{B}_\alpha^{(n)*}\mathcal{B}_\alpha^{(n)}\rangle$ depending also on the different settings choosen by the observers. From Eq. (7): $\mathcal{F}[\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger}\hat{\mathcal{B}}_{\alpha'}^{(n')}\rangle(r)] \to \mathcal{F}[\langle \mathcal{B}_\alpha^{(n)*}\mathcal{B}_{\alpha'}^{(n')}\rangle(r)]$ when $r \to \infty$, thus any bound $\mathcal{L}$ cannot be violated (in the limit). One example of these wide class of Bell inequalities can be found in [22]. We stress that Eq. (7) has been obtained simply by exploiting Gaussian factorization and comparing the macrolimit for classical and quantum second-order correlations; it is thus clear that it can be easily generalized to a wide variety of Gaussian systems with larger degrees of freedom, e.g., to multipartite situations obtained by inserting in the setup a number of beam splitters [23].

So far we have considered a situation where the system is initially in thermal equilibrium, but Eq. (2) has been obtained under the hypothesis that the system has no losses, hence it is assumed that for the steady-state calculations at any value of the interaction parameter r the system is disconnected from the environment. However, in real systems amplification, losses, and noise disturbances actually take place simultaneously. Thus (steady-state) Heisenberg equations have to be replaced by (t-dependent) Langevin equations with noise sources. In the symmetric case (equal losses for all the four modes), we obtain

$$\begin{aligned}\hat{a}_h(t) &= \hat{a}_h(0)e^{-\lambda t}\cosh\Delta(t,0) + \hat{b}_v^\dagger(0)e^{-\lambda t}\sinh\Delta(t,0)\\ &\quad + \int_0^t e^{-\lambda(t-t')}\cosh\Delta(t,t')\hat{f}_{ah}(t')dt'\\ &\quad + \int_0^t e^{-\lambda(t-t')}\sinh\Delta(t,t')\hat{f}_{bv}^\dagger(t')dt',\end{aligned} \tag{8}$$

where $\Delta(t,t') = \int_{t'}^t \kappa(t'')dt'' = \kappa_0/\Lambda(e^{-\Lambda t'} - e^{-\Lambda t})$; $\hat{f}_\alpha(t)$ are Bose-quantum noise operators associated with the losses [24] [α denotes the specific mode, e.g., $\alpha \equiv (a,h)$]. In the following we will assume $\langle \hat{f}_\alpha^\dagger(t)\hat{f}_{\alpha'}(t')\rangle = 2\lambda n_0 \delta_{\alpha,\alpha'}\delta(t-t')$. Analogous expressions can be obtained for the other three modes. Figure 1 displays the quantum and the classical calculation of $\langle \hat{\mathbf{J}}^2\rangle/\langle N\rangle$ for $\lambda = \Lambda = 0.1$, $\kappa_0 = 1$. For the quantum (continuous line) and the classical (dashed line) calculations we adopted $n_0 = 0.3$ and $n_0^{(cl)} = 0.8$, respectively. The figure clearly shows that for $r > 3$ classical and quantum results cannot be distinguished. Thus (i) also in this more realistic case self-stimulation can suppress the detrimental effect of noise, (ii) coincidence between classical and quantum results can be obtained without requiring $n \gg 1$ if we choose $n_0^{(cl)} = n_0 + 1/2$; (iii) decoherence due to losses and noise seems to affect equally quantum entanglement and classical correlations, hence it cannot be invoked in the present case for the emergence of a classical behavior. In order to interpret our results, we distinguish between two situations: When $r \ll 1$, the probability to deal with states with more than two photons is negligible, so measurements of $\langle \hat{\mathbf{J}}^2\rangle$ and of $\langle \hat{N}\rangle$ can probe the system at a microscopic level, but with a lot of particles (when r increases) they both become macroscopic observables unable to probe the system fluctuations with precision at a few quanta level. In this case the information recovered by observations is a coarse grained quantity missing the underlying quantum structure. This lack of microscopic information seems able to introduce elements of local realism even in the presence of strong entanglement and in the absence of decoherence. As shown in [25], the partition of a quantum system into subsystems and hence the entanglement structure, is dictated by the set of operationally accessible interactions and measurements. A given set can hide a multipartite structure. Our results suggest that, analogously, the set of operationally accessible measurements and their ability to catch the quantized structure of the system determine the quantum or classical nature of the observed correlations. As pointed out above, our findings [included Eq. (7)] do not imply that macroscopic entangled systems cannot display quantum nonlocality effects. As already mentioned, it has been shown that these kind of systems do violate the Clauser Horne Shimony Holt (CHSH) Bell inequality in the macroscopic limit ($r \to \infty$) [16]. However in that case the Bell operator is constructed by means of operators with single quantum sensitivity independently of the number of particles in the system in contrast to operators $\hat{\mathcal{B}}_\alpha^{(n)\dagger}\hat{\mathcal{B}}_\alpha^{(n)}$. Of course these operators cannot be expanded in a finite sum of operators $\hat{\mathcal{B}}_\alpha^{(n)\dagger}\hat{\mathcal{B}}_\alpha^{(n)}$. Analogous conclusions can be drawn for the violations of macroscopic local realism shown in [13].

The emergence of macroscopic local realism in the presence of strong entanglement, shown here, provides insight into the boundary between classical and quantum worlds. These results, with the care that they have been obtained for a Gaussian system, suggest that, despite the feasible realization of systems with a huge amount of entangled particles, the lack of information gathered by coarse-grained observations may lead to the introduction of elements of local realism even in the presence of strong entanglement and in the absence of decoherence. In particular, Eq. (7) shows that by using apparata able to measure finite-order correlations only, it is not possible to detect violations of local realism for macroscopic Gaussian states. Further investigations are needed to understand if these results can be extended to more general quantum systems.

We thank P. Zanardi for helpful discussions and comments.

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