Fast analysis of large finite arrays with a combined multiresolution-SM/AIM approach

Fast Analysis of Large Finite Arrays With a Combined Multiresolution—SM/AIM Approach

Paolo De Vita, Angelo Freni, *Senior Member, IEEE*, Francesca Vipiana, Paola Pirinoli, *Member, IEEE*, and Giuseppe Vecchi, *Member, IEEE*

***Abstract*—We present a synthesis of the sparse matrix/adaptive integral method (SM/AIM) and the multiresolution (MR) approach for the analysis of electrically large finite arrays, with planar or 3-D radiating elements; the two methods were separately introduced previously. The use of the MR has the effect of a preconditioner and speeds up the convergence rate of the SM/AIM of almost two orders of magnitude, with a total reduction of the numerical complexity with respect to the standard MoM of almost three orders of magnitude.**

***Index Terms*—Antenna arrays, method of moments (MoM), multiresolution, numerical methods, wavelets.**

I. Introduction

As well-known, an efficient cure to the limitation of the standard method of moments (MoM) is represented by the so-called "fast methods", that have been essentially developed for the analysis of scattering problems (e.g., [1], [2]), or for planar structures, as single-layer printed antennas [3]. Its efficient extension to antenna problems involving 3-D radiating elements, as stacked patches or radiators connected to the ground plane (e.g., Fig. 1, bottom) is, however, not straightforward.

For this reason part of these authors have improved the BMIA/AIM formulation introduced in [5] for the analysis of stacked patch antennas to the sparse matrix—adaptive integral method (SM/AIM), able to handle efficiently 3-D structures, including also vertical connection, by introducing an *ad hoc* version of the AIM [2]. As the other fast (iterative) methods (e.g., see [1]–[4]), the SM/AIM acts on the operation count of matrix-vector multiplication: the number of iterations remains essentially the same as with the standard MoM. To accelerate convergence, one needs a suitable preconditioner, that has to satisfy three constraints; low memory storage, low computational cost for its generation and application, compatibility with a fast method, in which the MoM matrix is never fully computed and stored. Results presented in literature (e.g., [6]) show that techniques as the sparse LU or the sparse iterative method can be adopted to generate a preconditioner suitable also for fast methods. However they are not able to both reduce the number of iterations and require a low memory space, and this may undo the computational advantages of the use of the preconditioner itself.

Manuscript received December 23, 2004; revised September 8, 2006.

P. De Vita and A. Freni are with the Department of Electronics and Telecommunications, University of Florence, I-50139 Firenze, Italy.

F. Vipiana, P. Pirinoli, and G. Vecchi are with the Department of Electronics, Politecnico di Torino, I-10129 Torino, Italy (e-mail: paola.pirinoli@polito.it).

Digital Object Identifier 10.1109/TAP.2006.886476

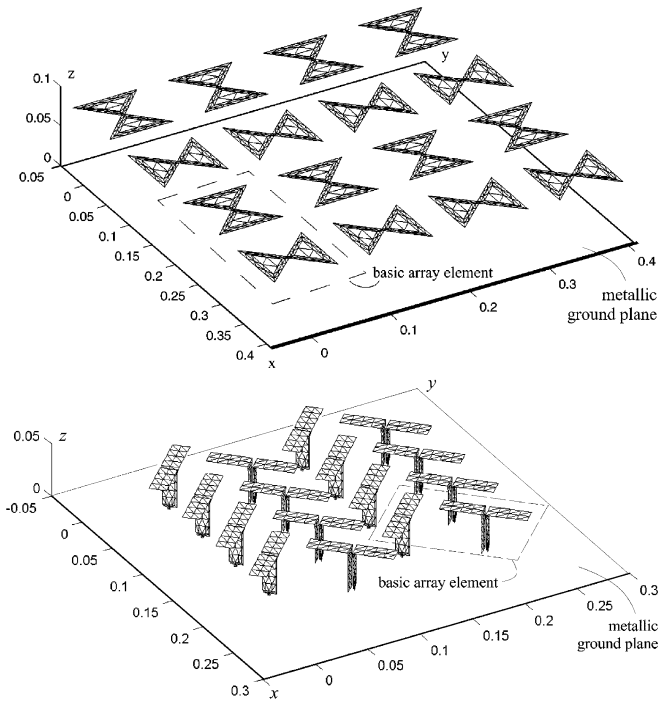

Fig. 1. Geometry of the test problems.

In this paper, we demonstrate that the multiresolution (MR) [8] technique produces an efficient MoM preconditioner for antenna problems, that is fully compatible with the SM/AIM algorithm. The MR approach was first introduced in [7] (for a rectangular grid) then extended in [8] to the generation of wavelet-like vector functions that can be adopted for the analysis of arbitrary-shape 2.5-D or 3-D antenna problems. The employed approach allows expressing the MR functions as linear combination of RWG [10] functions, and the MR basis is represented by a matrix $\mathbf{T}$ whose rows represent the MR functions in the RWG basis. For the reader interested in more theoretical MR issues, [9] shows that the approach in [8] produces indeed a MR analysis.

Unlike other preconditioners (see e.g., [6], [11]), the results in Section IV will show that the MR allows a strong reduction of the iterative solver number of iterations (more than one order of magnitude) with a negligible increase of memory storage and of computational effort for its construction and application. This is related to two aspects of the MR preconditioner. The first is that the matrix $\mathbf{T}$ is very sparse ($\boldsymbol{O}(2N \log_2 N)$ non-zero elements) and frequency independent; this is important in antenna problems, where the resulting linear system has to be solved for

several frequencies. The second is that the eigenvalue distribution of the MoM matrix in the MR basis can be controlled by application of a simple diagonal preconditioner.

The present work can be considered a synthesis of the SM/AIM and MR techniques. A preliminary stage of this work was reported in the conference paper [12]. To the best of authors' knowledge, this is the first work reporting how the MR properties in [8] can be implemented in an AIM like formulation resulting in a preconditioner that is direct, i.e., that does not require any inversion, and with storage and operational count compatible with the intrinsic structure of AIM.

II. The SM/AIM Formulation

Let us consider a metallic structure, located over an infinite ground plane extending along the x, y directions, two examples of which are shown in Fig. 1. The EFIE-MoM linear system $\mathbf{ZI} = \mathbf{V}$ is set up for the RWG basis [10].

In the classical SMCG [13], computational efficiency is achieved by defining a distance r_d which separates two regions: a near-interaction region and a weak-interaction region. One can then write the matrix $\mathbf{Z}$ as a superposition of a strong interaction—matrix $\mathbf{Z}^s$ and a weak-interaction matrix $\mathbf{Z}^w$ (i.e., $\mathbf{Z} = \mathbf{Z}^s + \mathbf{Z}^w$). The elements of $\mathbf{Z}^s$ are related to weighting and basis functions having distances $d_{xy} = \sqrt{(x-x')^2+(y-y')^2} < r_d$. This matrix is evaluated using the standard MoM and results very sparse. The weak-interaction region is defined as that in which $h = |z-z'| = |f(x,y) - f(x',y')| \ll d_{xy}$ for all x, y, x', y'; in it, we can approximate the Green's function in a Taylor series with respect to height h; keeping terms up to $h^{\widetilde{M}}$ one obtains

$$G(d_{xy}, z, z') = \sum_{m=0}^{M} A_m(d_{xy}) R_m(z) Q_m(z') \tag{1}$$

where $M = (\widetilde{M}+1)^2 - 1$. Therefore, by assuming the Galerkin discretization scheme, each element of the weak-interaction matrix $\mathbf{Z}^w$ can be expressed as a linear combination of four elements of the form [5]

$$Z_{i,j}^{\alpha} = \sum_{m=0}^{M} \int\int_S R_m(z)\psi_i^{\alpha}(x,y,z) \cdot \int\int_S A_m(x-x', y-y') Q_m(z') \psi_j^{\alpha}(x',y',z') dS' dS. \tag{2}$$

where $\alpha = x, y, z$ refers to the components of the vector potentials, and $\alpha = \phi$ to the scalar potential, and ψ_i^{α} is the pertinent basis or weighting function. Each term (2) has a two-dimensional (2-D) convolution form and can be efficiently evaluated by means of a 2-D fast Fourier transform (FFT). This formulation, however, cannot be directly applied to the analysis of non planar antenna structures: it either requires a regular grid to describe the structure or does not allow the use of basis functions along the z direction.

To overcome this limitation, we consider the products $Q_m(z')\psi_j^{\alpha}(x',y',z')$ or $R_m(z')\psi_j^{\alpha}(x',y',z')$ and we approximate them with equivalent sets of point-like currents. We choose the point-like current elements located at the nodes of a regular Cartesian 2-D grid, parallel to the x, y plane. Both the basis and weighting functions are approximated as a linear combination of Dirac delta functions; denoting by $P_m(z) = Q_m(z)$ or $P_m(z) = R_m(z)$, the generic term of the expansion is then

$$P_m(z)\psi_n^{\alpha}(x,y,z) \simeq \sum_{i=0}^{L} \Lambda_{n,i}^{\alpha,m} \delta(x-x_i)\delta(y-y_i)\delta(z-z_c) \tag{3}$$

with $(x_i, y_i) \in C_n$, where C_n is the set of $L+1$ grid nodes closest to the center (x_c, y_c, z_c) of the basis or weighting function support. This procedure is similar to the classical AIM approach [2]; however, by approximating not only the basis (weighting) functions but their product with the part of the factorized Green's function (1) depending on $z'(z)$, we require that point-like current elements belong only to a regular Cartesian 2-D grid having N_g nodes. Hence, we enforce the equality

$$\int\int_S P_m(z)\psi_n^{\alpha}(x,y,z)(x-x_c)^{m_1}(y-y_c)^{m_2} dS = \sum_{i=0}^{L} \Lambda_{n,i}^{\alpha,m} (x_i-x_c)^{m1}(y_i-y_c)^{m2} \tag{4}$$

for $0 \le m_1, m_2 \le L$. By inserting (3) into (2), the weak-interaction matrix can readily be written as

$$\mathbf{Z}^w \boldsymbol{I} = \sum_{m=0}^{M} \sum_{\alpha=x,y,z,\phi} \Lambda_{\alpha,m}^{R} A_m \Lambda_{\alpha,m}^{Q} \boldsymbol{I} \tag{5}$$

where $\mathbf{A}_m = \{A_m(x_i - x_j, y_i - y_j)\}$, and $\mathbf{\Lambda}_{\alpha,m}^{R,Q} = \{\Lambda_{n,i}^{\alpha,m}\}$. Furthermore, the matrix $\mathbf{A}_m$ is a block Toeplitz matrix, while the matrices $\mathbf{\Lambda}_{\alpha,m}^{R,Q}$ are extremely sparse. While the $\mathbf{Z}^w$ matrix is a full matrix with $\boldsymbol{O}(N^2)$ elements and usually cannot be stored for large-scale problems, we now have to store only $\boldsymbol{O}(4N_g)$ elements for each matrix $\mathbf{A}_m$, where N_g is the number of nodes on the regular 2-D grid, and a few other matrices that need the overall storage of $\boldsymbol{O}(4(L+1)N_g)$ elements. Furthermore, when a conjugate gradient is used to solve the matrix equation, based on the discrete convolution theorem, the product $\mathbf{A}_m \cdot (\Lambda_{\alpha,m}^{Q}\boldsymbol{I})$ can be suitably evaluated by means of two 2-D-FFTs, since $\mathbf{A}_m$ is a block Toeplitz matrix. This allows us to calculate the $\mathbf{Z}^w\boldsymbol{I}$ product using $\boldsymbol{O}(16N_g(M+1)[2\log_2(4N_g)+1])$ operations instead of $\boldsymbol{O}(N^2)$.

III. Multiresolution Preconditioner

The effect of the MR system [8] on the MoM matrix spectrum is a complex topic, presently under investigation and beyond the scope of this paper; a preliminary account is presented in [14] and briefly outlined here. A key point is that the MR functions of [8] possess both spatial (as the RWG) and spectral resolution [7], since they are defined on meshes of different levels, and have different spectral occupations. Because of the spectral resolution, the evaluation of the matrix entries in the spectral domain results in sampling the spectral Green function in correspondence of the peaks of the Fourier transform of the basis

TABLE I
TEST ARRAY SIMULATION FEATURES FOR DUAL-POLARIZATION ARRAYS OF BOWTIE DIPOLES SHOWN IN FIG. 1, TOP. ($\mathrm{Freq.} = 1$ GHz, TAYLOR GREEN FUNCTION EXPANSION ORDER $= 2$, BASIS FUNCTIONS MULTIPOLE EXPANSION ORDER $= 1, r_d = 0.288\,\mathrm{m} = 0.96\lambda, \Delta_{\mathrm{FFT}}^{\max} = \lambda/20$, CG Tol. $= 10^{-3}$, CPU TIME IS RELEVANT TO A PENTIUM III 1 GHz)

# of bow-tie pairs	antenna surface	Nr. of unknowns	$\mathbf{Z}^s$ non zero elements	$\mathbf{Z}^s$ filling time	MR basis			RWG basis		
					CG iter.time	# iter.	Ram Mb	CG iter.time	# iter.	Ram Mb
2×4	$2.16\,\lambda^2$	$1,968$	$2,540,932$	$43''$	$0.55''$	32	62	$0.42''$	2925	62
4×8	$9.61\,\lambda^2$	$7,872$	$14,084,712$	$239''$	$2.82''$	89	248	$2.20''$	3388	248
5×10	$15.34\,\lambda^2$	$12,300$	$23,329,258$	$398''$	$5.59''$	114	409	$4.60''$	2157	409
6×12	$22.09\,\lambda^2$	$17,712$	$38,529,538$	$604''$	$7.69''$	132	593	$6.21''$	3681	592
7×14	$29.88\,\lambda^2$	$24,108$	$48,763,662$	$854''$	$10.20''$	112	810	$8.20''$	3354	809
8×16	$39.27\,\lambda^2$	$31,488$	$64,953,520$	$1156''$	$13.46''$	155	1066	$10.34''$	3190	1064

TABLE II
TEST ARRAY SIMULATION FEATURES FOR ARRAYS OF DUAL-POL. DIPOLES SHOWN IN FIG. 1, BOTTOM. ($\mathrm{Freq.} = 2.1$ GHz) : TAYLOR GREEN FUNCTION EXPANSION ORDER$= 2$, BASIS FUNCTIONS MULTIPOLE EXPANSION ORDER $= 1, r_d = 0.2\,m = 1.4\lambda, \Delta_{\mathrm{FFT}} = 0.0025\,m = 0.028\lambda$, CG Tol. $= 1.\mathrm{e}-4$; (CPU TIME IS RELEVANT TO A PENTIUM III 1 GHz).$^{(*)}$: RESIDUE $= 5.9 \times 10^{-4}$ (STAGNATION)

# of dipo-les pairs	Unk-nowns	$\mathbf{Z}^s$ non zero elements	FFT–2D points	$\mathbf{Z}^s$ filling time	MR basis - Prec MR		Ram Mb	RWG basis - No Prec		Ram Mb
					CG iter. time	# iter.		CG iter. time	# iter.	
4×8	$8,000$	$21,119,018$	256×1024	$392''$	$6.73''$	53	269	$7.75''$	$5641^{(*)}$	269
6×10	$15,000$	$49,413,620$	512×1024	$938''$	$19.16''$	55	622	$17.51''$	12291	621
8×12	$24,000$	$87,363,560$	512×2048	$1689''$	$36.17''$	69	1057	$32.31''$	13524	1055
10×14	$35,000$	$134,968,764$	512×2048	$2567''$	$39.16''$	77	1659	$34.74''$	13543	1656

and test functions. As a result of all of the above, the MoM matrix in the MR basis, $\mathbf{Z}_{\mathrm{MR}}$, presents a peaked diagonal, whose entries are very close to the matrix eigenvalues [7, Sec. VII]. Thanks to this, the application of a simple diagonal preconditioner improves the distribution of the eigenvalues [14], that is the main responsible of the convergence rate of iterative solvers. In this sense, the MR scheme induces an efficient physics-based preconditioner.

The construction and application of this preconditioner can be summarized in the following three steps:

— construction of the matrix $\mathbf{T}$ whose rows represent the MR functions in the RWG basis, i.e., it is the basis change matrix. Note that $\mathbf{T}$ is very sparse and frequency independent;
— extraction of the symmetric diagonal preconditioner $\mathbf{D} = \mathrm{diag}(\mathbf{Z}_{\mathrm{MR}}) = \mathrm{diag}(\mathbf{T}\mathbf{Z}\mathbf{T}^T)$;
— construction of the preconditioning matrix $\mathbf{S} = \mathbf{T}\mathbf{D}^{-1/2}$, that share the same memory space of matrix $\mathbf{T}$.

Note that, unlike other commonly used preconditioner (see for instance [11], [6] and reference therein), this scheme presents the advantage of requiring a low memory occupation, since only the matrix $\mathbf{S}$, having $\boldsymbol{O}(N \log_2 N)$ non-zero elements, has to be stored.

The procedure described above is general; however, the construction of the diagonal matrix $\mathbf{D}$ requires the knowledge of the entire MoM matrix $\mathbf{Z}$, which is undesirable in fast methods. One then conveniently resorts to the approximate diagonal matrix $\widetilde{\mathbf{D}}$ generated via the strong-interaction matrix $\mathbf{Z}^s$, i.e., $\widetilde{\mathbf{D}} = \mathrm{diag}(\mathbf{T}\mathbf{Z}^s\mathbf{T}^T)$, and uses $\widetilde{\mathbf{S}} = \mathbf{T}\widetilde{\mathbf{D}}$ instead of $\mathbf{S}$.

For the specific array problem at hand, however, the radiating elements are not connected to one another, and the matrix $\mathbf{T}$ is a block diagonal matrix, since in this case there are no functions covering more than one array element, not even those belonging to the $j = 1$ mesh; therefore to construct the *exact* preconditioner $\mathbf{S}$ we only need the diagonal blocks of $\mathbf{Z}$, that represent the interactions between RWG basis functions within the same radiating element. As a result, this exact MR preconditioner can be computed and stored at a very low cost.

Once the matrix $\mathbf{S}$ has been evaluated, the following equivalent linear system:

$$\mathbf{S}^T(\mathbf{Z}^s + \mathbf{Z}^w)\mathbf{S}\widetilde{\mathbf{I}} = \mathbf{S}^T\mathbf{V} \tag{6}$$

can be solved adopting the scheme described in Section II. The solution $\mathbf{I} = \mathbf{S}\widetilde{\mathbf{I}}$ can be calculated, where $\widetilde{\mathbf{I}}$ and $\mathbf{I}$ are the unknown currents in the preconditioned MR basis and in the RWG basis, respectively.

IV. RESULTS

As a first example, we considered dual-polarization arrays of bowtie dipoles of the type shown at the top of Fig. 1. Each element has been discretized with the mesh generated by the MR procedure [8], on which 125 unknowns are defined.

The efficiency of the MR-SM/AIM has been investigated for arrays with an increasing number of elements; its performance and those of the SM/AIM with the standard RWG functions are summarized in Table I.

From these data, it appears that the introduction of the MR functions slightly increases the time taken to complete one CG iteration compared to that taken when the RWG basis is used, since it introduces two additional products with the basis-change and preconditioning matrices (columns 6 and 9 in Table I), but strongly reduces the number of iterations required to achieve the desired tolerance, as it appears comparing columns 7 and 10. Therefore, if for instance we consider the largest array, corresponding to about 30 000 unknowns, the *total* time needed to solve the linear system using the SM/AIM with the RWG functions is almost 9 hr., 45 min., while, using the MR functions, it is reduced to almost 54 min., with a gain of almost a factor of eight.

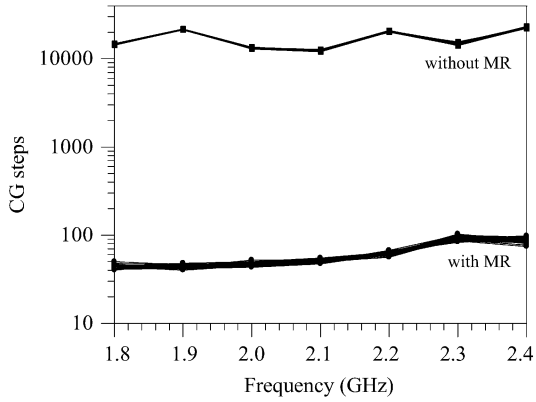

Fig. 2. Number of iterations for reaching the tolerance of 10^{-4} versus the frequency for an array of $6 \times 10 \times 2$ dipoles of the type in Fig. 1, bottom.

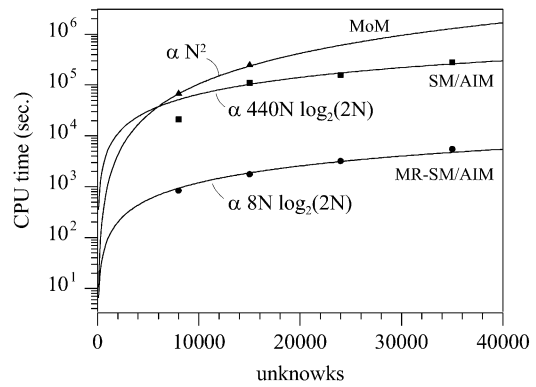

Fig. 3. CPU time for the whole analysis of arrays of dipoles of the type in Fig. 1, bottom.

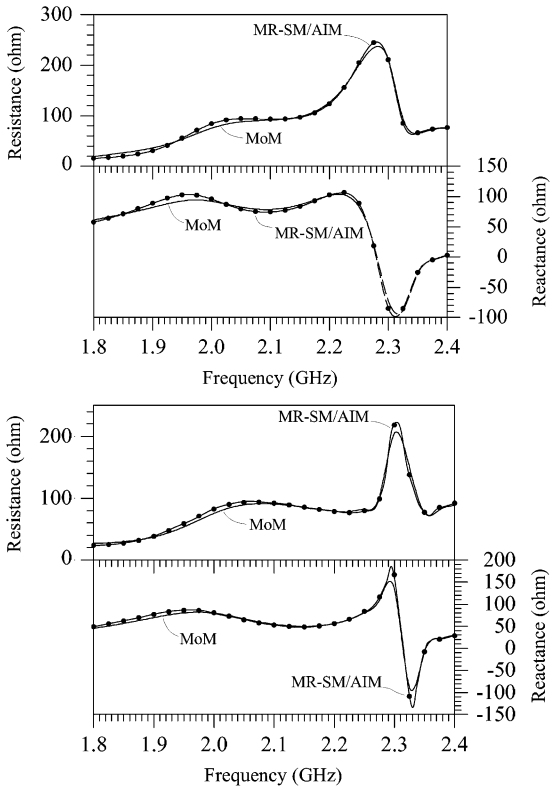

Fig. 4. Real and imaginary part of the input impedance of the 50th (top) and of the 59th (bottom) element of the 6×10 dipoles pairs array of the type sketched in Fig. 1 bottom. Continuous line: standard MoM; marked line: MR-SM/AIM.

Like Table I, Table II shows the relevant characteristics of the simulations carried out at a frequency of 2.1 GHz on arrays of the type sketched at the bottom of Fig. 1. Also, in this case, columns 6 and 9 in Table II show that it takes slightly more time to complete one CG iteration when the MR preconditioning is used than it does when the standard RWG basis is used; however, this increase is always less than 10%, and the total gain is very high, since the number of iterations required to achieve the set CG tolerance value $(\mathrm{tol} = 10^{-4})$ is reduced by almost two orders of magnitude when the MR scheme with the diagonal preconditioner is introduced, as shown in columns 7 and 10 of Table II. Fig. 2 shows the number of iterations necessary for the SM/AIM method to reach the set tolerance on the frequency interval $1.8 \div 2.4$ GHz, with and without the MR preconditioner. These results refer to an array of 6×10 dipole pairs (15 000 unknowns), feeding one dipole at a time as required to obtain the array admittance matrix. It is also clear from Table II, that the use of MR reduces the number of iterations of about a factor of 200. The slight increase in the number of iterations around 2.3 GHz can be connected to the strong variation of the impedance around this frequency, as evident in Fig. 4.

Fig. 3 reports the total CPU time needed to solve the MoM linear system for increasing problem dimensions (number N of unknowns). The three curves refer to the use of the SM/AIM approach with the MR (circle) or the RWG (square) functions and of the standard MoM (triangle). The marks correspond to computed values, while the continuous curves are the interpolating functions, which give the dependence of the CPU time on N, whose expression is also reported in the figure. The use of the SM/AIM instead of the classical MoM reduces the numerical complexity from $\alpha \cdot N^2$ to $\alpha \cdot 440N \log_2 2N$; in this case, $\alpha = 1.05 \cdot 10^{-3}$ sec. This means that, starting from medium size problems $(N \geq 25\,000)$, the CPU time reduction achieved using the SM/AIM is almost one order of magnitude. The introduction of the MR functions within the SM/AIM frame reduces the solution time by about two orders of magnitude, so that the MR-SM/AIM always reduces the CPU time compared with the standard MoM, even for small size problems. In case of $N \geq 25\,000$ the gain is of almost three orders of magnitude. Finally, it is worth noting that the increase of the dynamic memory in the MR formulation is negligible (see columns 8 and 11 in Tables I and II).

In addition to the numerical efficiency of the proposed method, we checked its accuracy, via the active (embedded) impedance for broadside scan. Fig. 4 shows the frequency performance of the real and imaginary part of the active impedance of two array elements on the frequency band $[1.8 \div 2.4]$ GHz for the array consisting of the 6×10 dipole pairs in Table II, when the standard MoM or the MR-SM/AIM are used. We intentionally placed the elements of the array very close one each other $(\sim 0.4\lambda)$ to point out the critical behavior of the active input impedance around 2.3 GHz: despite the sharp variation, the agreement between the two sets of results is very good.

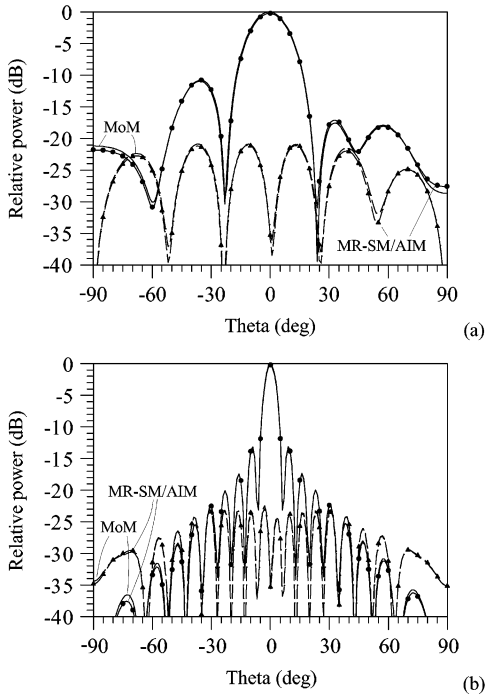

Fig. 5. Amplitude pattern at 2.1 GHz for the 6×10 dipole pairs array of the type sketched in Fig. 1 bottom: (a) xz plane, (b) yz plane. Solid line: co-polar component, standard MoM; Dashed line: cross-polar component, standard MoM; •: co-polar component, MR-SM/AIM ▲: cross-polar component, MR-SM/AIM.

Moreover, Fig. 5(a) and (b) shows the co-polar (solid) and the cross-polar (dashed) radiation patterns at the frequency of 2.1 GHz in the (xz) and the (yz) planes, respectively; the continuous lines refer to the radiated field evaluated with the MoM, while the marked lines to the standard MR-SM/AIM solution: the plots show the very good agreement between the results obtained with both approaches.

V. Conclusion

We proposed and tested a combined application of the SM/AIM formulation and of the MR approach to antenna arrays of medium/large or large dimensions. It reduces the SM/AIM total solution time by about two orders of magnitude for medium-size problems, without affecting storage requirements. The total gain of the MR-SM/AIM over the standard MoM is of almost three orders of magnitude; the computational gain increases for larger problems. Extension of this work to other fast methods is presently underway [15].

Acknowledgment

This work was developed in the Sixth Framework Programme of the European Community within the Antenna Centre of Excellence (ACE).

References

[1] J. M. Song and W. C. Chew, "Fast multipole method solution of three dimensional integral equation," in *IEEE Antennas and Propagation Society Int. Symp. Digest*, 1995, pp. 1528–1531.

[2] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: Adaptive integral method for solving large-scale electromagnetic scattering and radiation problems," *Radio Sci.*, vol. 5, pp. 1225–1251, 1996.

[3] F. Ling, C.-F. Wang, and J.-M. Jin, "An efficient algorithm for analyzing large-scale microstrip structures using adaptive integral method combined with discrete complex-image method," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-48, pp. 832–839, 2000.

[4] K. Zhao and J. F. Lee, "A single-level dual rank IE-QR algorithm to model large microstrip antenna arrays," *IEEE Trans. Antennas Propag.*, vol. AP-52, pp. 2580–2585, 2004.

[5] P. De Vita, A. Mori, and A. Freni, "A BMIA/AIM formulation for the analysis of large stacked patch antennas," in *Proc. MMET*02-Int. Conf. on Math. Methods in Electromagnetic Theory*, Kiev, Ukraine, Sep. 10–13, 2002, vol. 1, pp. 48–53.

[6] C. C. Huang, L. Tsang, C. H. Chan, and K. H. Ding, "Multiple scattering among vias in planar waveguides using preconditioned SMCG method," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-52, pp. 20–28, 2004.

[7] P. Pirinoli, G. Vecchi, and L. Matekovits, "Multiresolution analysis of printed antennas and circuits: A dual-isoscalar approach," *IEEE Trans. Antennas Propag.*, vol. AP-49, pp. 858–874, Jun. 2001.

[8] F. Vipiana, P. Pirinoli, and G. Vecchi, "A multiresolution method of moments for triangular meshes," *IEEE Trans. Antennas Propag.*, vol. AP-53, pp. 2247–2258, 2005.

[9] F. P. Andriulli, A. Tabacco, and G. Vecchi, "A multiresolution approach to the electric field integral equation in antenna problems," in *SIAM, J. Scientific Comput.*, to be published.

[10] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propag.*, vol. AP-30, pp. 409–418, 1982.

[11] J. F. Lee, R. Lee, and R. J. Burkholder, "Loop-star basis functions and a robust preconditioner for EFIE scattering problems," *IEEE Trans. Antennas Propag.*, vol. AP-51, pp. 1855–1863, 2003.

[12] P. De Vita, A. Mori, A. Freni, F. Vipiana, P. Pirinoli, and G. Vecchi, "A BMIA/AIM multi-resolution approach for the analysis of large printed array," in *IEEE Antennas and Propagation Society Int. Symp. Digest*, Jul. 2003, pp. 803–806.

[13] L. Tsang, C. H. Chan, and K. Pak, "Monte-Carlo simulations of two-dimensional random rough surfaces using the sparse-matrix flat-surface iterative approach," *Electron. Lett.*, vol. 29, pp. 1153–1154, 1993.

[14] F. Vipiana, P. Pirinoli, and G. Vecchi, "Regularization effect of a mutiresolution basis on the EFIE-MoM matrix," in *IEEE Antennas and Propagation Society Int. Symp. Digest*, Washington, DC, Jul. 2005, vol. 3b, , pp. 192–195.

[15] P. De Vita, A. Freni, P. Pirinoli, F. Vipiana, and G. Vecchi, "A combined MR-FMM approach," presented at the URSI Nat. Radio Science Meeting, Jul. 2005.

Paolo De Vita received the M.S. degree in electronic engineering and the Ph.D. degree in information and telecommunication engineering from the University of Florence, Florence, Italy, in 1999 and 2004, respectively .

Since 1999, he has served as a Research Assistant at the Electronics and Communications Department of University of Florence. His main research interests are on numerical techniques for electromagnetic radiation and scattering problems.

Angelo Freni (S'90–M'91–SM'03) received the Laurea (Doctors) degree in electronics engineering from the University of Florence, Florence, Italy, in 1987.

Since 1990, he has been with the Department of Electronic Engineering of the University of Florence, first as an Assistant Professor and since 2003 as an Associate Professor of electromagnetism. During 1994, he was involved in research in the Engineering Department at the University of Cambridge, Cambridge, U.K. From 1995 to 1999, he was an Adjunct Professor at the University of Pisa. His research interests include meteorological radar systems, radiowave propagation, numerical and asymptotic methods in electromagnetic scattering and antenna problems, remote sensing. In particular, part of his research concerned the extension and the application of the finite element method to the electromagnetic scattering from periodic structures and to the electromagnetic interaction with moving media.

Francesca Vipiana received the Laurea and Ph.D. (Dottorato di Ricerca) degrees in electronic engineering from the Politecnico di Torino, Torino, Italy, in 2000 and 2004, respectively, with doctoral research carried out partly at the European Space Research Technology Centre (ESTEC, Noordwijk, The Netherlands).

Since 2004, she has been with the Electronics Department, Politecnico di Torino, as a Research Assistant. Her main research activities concern numerical techniques for antennas and circuits analysis, and synthesis techniques for multi-beam antennas.

Dr. Vipiana received the Young Scientist Award at the Union of Radio Science (URSI) General Assembly in 2005.

Paola Pirinoli (M'96) received the Laurea and Ph.D. (Dottorato di Ricerca) degrees in electronic engineering, from the Politecnico di Torino, Italy, in 1989 and 1993, respectively.

In October 1994, she joined the Department of Electronics of the Politecnico di Torino as an Assistant Professor where, since December 2003, she has been an Associate Professor. From November 1996 to February 1997, she was a Visiting Research Fellow at the Laboratoire d'Electronique, Antennes et Telécomunication of the University of Nice (F), through a Fellowship of the Communauté de Travail des Alpes Occidentales (COTRAO). Her main research activities include the development of analytically based numerical techniques, essentially devoted to the fast analysis of printed structures on planar or curved substrates, the modelling of nonconventional substrates as bi-isotropic ones, the design and analysis of antennas for wireless communications.

Dr. Pirinoli received the Young Scientist Award in 1998, the "Barzilai" prize for the best paper at the National Italian Congress of Electromagnetic (XII RiNEm) in 1998 and the prize for the best oral paper on antennas at the Millennium Conference on Antennas and Propagation (AP2000) in 2000.

Giuseppe Vecchi (M'90) received the Laurea and Ph.D. (Dottorato di Ricerca) degrees in electronic engineering from the Politecnico di Torino, Torino, Italy, in 1985 and 1989, respectively, with doctoral research carried out in part at Polytechnic University, Farmingdale, NY.

He was a Visiting Scientist at Polytechnic University from August 1989 to February 1990, when he joined the Department of Electronics at Politecnico di Torino as an Assistant Professor (Ricercatore). From 1992 to 2000, he was an Associate Professor at Politecnico, and a Professor since 2000. He was a Visiting Scientist at the University of Helsinki, Espoo, Finland, in 1992 and an Adjunct Faculty at the University of Illinois at Chicago, Department of Electrical and Computer Engineering since 1997. His main research activities concern analytical and numerical techniques for antennas and circuits analysis, RF plasma heating, antenna design for wireless communications, and electromagnetic compatibility.