LOAD DEPENDENT COULOMB FRICTION: A MATHEMATICAL AND COMPUTATIONAL MODEL FOR DYNAMIC SIMULATION IN MECHANICAL AND AERONAUTICAL FIELDS

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ABSTRACT

The proper evaluation of the friction forces and torques acting on a typical mechanical transmission is usually necessary when an accurate simulation of its dynamic behaviour is requested. For example, the authors consider the flap actuation systems of most commercial and military aircraft consisting of a centrally located Power Drive Unit (PDU), a shaft system and a certain number of reversible or irreversible actuators assembled in several different configurations. The dynamic behaviour of the flap actuation system is strongly dependent on the actuator dynamics; so an appropriate actuator simulation model is necessary in order to evaluate the system behaviour with a high degree of accuracy, both in failure and in normal operating conditions. A high compactness of the simulation model is recommended, nevertheless the requested marked computational accuracy.
Aims of the work are:
- *the proposal of a general purpose dynamic model concerning any transmission gear, equipped or not with ends of travel (such as the flap control system actuators), characterised by an innovative dry friction physical model having a component proportional to the load acting on the driven element (efficiency < 1) and a component independent on it;*
- *the development of the corresponding mathematical model and computational algorithm;*
- *the implementation of a Matlab – Simulink numerical model able to simulate the dynamic behaviour of a typical flap control system equipped with the above mentioned actuators;*
- *the simulation of some typical actuations, in order to validate the proper accuracy of the actuator dynamic model (by means of comparison with the results coming from a suitable and reliable FORTRAN numerical model), and the analysis of the results.*

The algorithm developed by the authors in Matlab – Simulink supplies an effective answer to such problems and, by means of a self-contained Simulink subsystem, can describe the effects produced by friction forces on the dynamic behavior of a generic mechanical actuator and simulate many of typical coulomb friction's effects interacting with its mechanical ends of travel. The first author has previously developed the physical-mathematical model shown here, performing and optimizing the Fortran algorithm used for validation of the simulations produced by the present computing method; the second author, on the basis of these results, has instead developed the Matlab-Simulink algorithm devising a self-contained Simulink subsystem of absolutely general validity and easily employable in many various applications

Keywords: coulomb, friction, actuator

1 INTRODUCTION

The primary and secondary flight control actuation systems of most commercial and military aircrafts consist of a certain number of actuators fed by two or more hydraulic lines; the actuation devices are normally linear and based upon jacks (usually adopted for spoilers, airbrakes, flaps, etc. and realized by means of irreversible ACME screws or reversible ballscrews) or rotary actuator (often based upon epicyclical gear). The behaviour of this kind of system and the role that coulomb friction forces play in their dynamics, equipped with reversible or irreversible actuators, are analyzed and evaluated by means of numerical models. The flap actuation systems of most aircrafts consist of a centrally located Power Drive Unit (PDU), a shaft system and a certain number of actuators (normally two for each flap surface). Depending on the performance requirements and on the specified interface with the other aircraft systems and structure, several different configurations have been used in the design of such actuation systems. PDU's can be either hydromechanical or electromechanical and be either of a single or dual motor type. In the last case, the outputs of the two motors can be either torque summed or speed summed. The shaft system generally consists of torque tubes connecting the PDU output with the right and the left wing actuators; however, the flap actuation systems of small commercial aircrafts often use flexible drive shafts rotating at high speed in place of the low speed rigid shafts.

The actuators are normally linear and are based on ballscrews (usually of reversible type), though some flap actuators use an ACME screw (irreversible); some flap actuators are of a rotary type (usually reversible).

The system must be able to prevent asymmetries between the left and right wing flaps in case of a shaft failure (detected by a proper asymmetry monitoring system) and to hold the surfaces in the commanded position following the shutoff command given when no actuation is required.

If the actuators use an irreversible ACME screw, the abovementioned requirements are intrinsically accomplished; if the actuators are reversible (in order to obtain higher efficiency), a brake system is necessary:

- controlled wingtip brakes (one for each wing) located at the end of the motion transmission, close to the position transducers, that become engaged and brake the system after a failure has been positively recognised;
- self-acting irreversibility brakes within each actuator, which self engage when the actuator output overruns the input shaft.

The relative merits of the three solutions (non-reversible actuators, reversible actuators with wingtip brakes or reversible actuators with irreversibility brakes) and which of the three is better is a long debated matter: the maximum asymmetry in failure conditions is greater with the wingtip brake solution, the solution with non-reversible actuators requires higher hydraulic power owing to its lower efficiency and the irreversibility brake solution, that overcomes the shortcomings of the two previous solutions, is more expensive. Therefore the most commonly used architecture employs the reversible actuators with wingtip brakes and centrally located PDU (of a dual motor type for operational reliability) because it is cheaper and more efficient, nevertheless the associated high asymmetries in case of failure. Whichever the actual configuration of the flap actuation system is, its dynamic behaviour is strongly dependent on the actuator dynamics; so an appropriate actuator simulation model is necessary in order to evaluate the system behaviour with a high degree of accuracy both in failure and in normal operating conditions. A high compactness of the simulation model is recommended, nevertheless the high computational accuracy requested.

2 AIMS OF THE WORK

Aims of the work are:

- the proposal of a general purpose dynamic model concerning any transmission gear equipped or not with ends of travel characterised by an innovative dry friction model having a component proportional to the load acting on the driven element (efficiency < 1) and a component independent on the load;
- the development of the corresponding mathematical model and computational algorithm;
- the implementation of a Matlab – Simulink numerical model able to simulate the dynamic behaviour of a typical flap control system equipped with the above mentioned actuators;
- the simulation of some typical actuations in order to validate the proper accuracy of the actuator dynamic model (by means of comparison with the results obtained by means of a suitable and reliable FORTRAN numerical model) and the analysis of the results.

3 PHYSICAL - MATHEMATICAL MODEL

The dry friction acting on a movable mechanical element consists of a force opposing the motion having a value variable as a function of the speed.

In the most of the applications, however the following model (Coulomb friction) can represent the relationship between the friction force and the speed:

- in standstill conditions the friction force can have any value lower or equal in module to the so said static friction value;
- otherwise, the force module has a constant value equal to the so said dynamic friction value.

The corresponding mathematical model must be able to describe the behaviour of a mechanical element subjected to the friction, dividing between the four possible conditions as follows:

- mechanical element initially stopped which must stay in a standstill condition;
- mechanical element initially stopped which must break away;
- mechanical element initially moving which must stay in movement;

- mechanical element initially moving which must stop.

This ability can be important in order to point out some undesired behaviours characterising the position servo-mechanisms as the powered flight controls. Like previously said, the nature of the phenomenon of the coulomb friction cannot completely be described by linear models (even though more favourable for the possible analytical solutions of the related dynamic equations); therefore whichever attempt of modellization not too simplified needs the use of nonlinearities of such complexity to advise the employment of techniques of numerical solution based on the dynamic simulation in the dominion of the time. The techniques of numerical solution mainly employed (and generally proposed in literature) are based however on mathematical models that are affected by severe shortcomings. The proposed model overcomes the above-mentioned shortcomings and correctly simulates the behaviour of the mechanical device, as follows:

- selects the correct friction torque sign as a function of the actuation rate sense;
- computes the friction torques according to the actual load value on the mechanical element;
- distinguishes between aiding and opposing load conditions;
- selects either the adhesion condition or the dynamic one;
- evaluates the eventual stop of the previously running mechanical element;
- keeps correctly in a standstill or moving condition the previously still or running mechanical element respectively;
- evaluates the eventual break away of the previously still mechanical element;
- is able to simulate the dynamics of both reversible and irreversible actuators;
- simulates correctly the mechanical ends of travel of the landing flap actuators.

The proposed model concerns the complete dynamics of the actuator-surface assembly, considered as a rigid mechanical element characterised by a single degree of freedom; the eventual actuator compliance and backlash are considered within the mathematical model of the motion transmission.

3.1 DYNAMIC EQUILIBRIUM EQUATION

The equation of dynamic equilibrium is

$$Act - T_{FR} = J_S \ddot{\vartheta} \qquad (1)$$

where

$$Act = T_{TR} - T_{LD} - C_S \dot{\vartheta}_S \qquad (2)$$

represents the sum of the active torques, whose previous acquaintance is necessary to evaluate the friction torque.

3.2 FRICTION TORQUE EVALUATION

The Coulomb Friction models available in literature are usually characterized by extremely simplified structures and limited performances; their shortcomings, easily verifiable by means of opportune numerical simulations, are particularly emphasized if "integrated" dynamical models are employed, that not only describe the performances of the actuator in Matlab – Simulink taking into account the frictions forces but estimating also the possible presence of mechanical ends of travel and their eventual interactions. The algorithm developed by the authors in Matlab – Simulink environment supplies an effective answer to such problems and, by means of a self-contained subsystem, can describe the effects produced by friction forces on the dynamic behavior of a generic mechanical actuator; authors' computational routine can correctly describe many of typical coulomb friction's effects as well as their interactions with mechanical position limitation on actuator. The true potentialities of the proposed algorithm for the calculation of the coulomb friction come from the implementation of a relatively simple but reliable and effective mathematical model in the greatly versatile Matlab – Simulink environment; the result is a self-contained, general-purpose Matlab – Simulink subsystem that can be used directly in a lot of different applications (aeronautical, mechanical, etc). In the proposed model, the friction torque is considered as the sum of a component not depending on the load and a further one related to the load through a defined value of efficiency. The former can assume two alternate values, static ($T_{FR0,stat}$) and dynamic ($T_{FR0,din}$). About the latter, in order to simulate both reversible and irreversible actuators, the proposed model introduces four suitable definitions of efficiency. Usually the efficiency is the ratio between the output and the input power (dynamic conditions) of a mechanical device; if it is characterised by a constant gear ratio (as the considered actuators are), the efficiency can be intended as the ratio between the output and the input torque (related to the same shaft), even in static conditions. The efficiency of the actuator depends on the motion (static or dynamic) and load (aiding or opposing) conditions. Therefore, if the Coulomb friction model is employed, whatever actuator is characterised by the following four different types of efficiency:

$\eta_{din,opp}$= dynamic out/in torque ratio, opposing load

$\eta_{din,aid}$ = dynamic out/in torque ratio, aiding load

$\eta_{stat,opp}$= static out/in torque ratio, load opposing the eventual motion

$\eta_{stat,aid}$ = static out/in torque ratio, load aiding the eventual motion

In the opposing conditions the output torque is essentially represented by the aerodynamic load and the input one by the transmission torque; vice versa in the aiding conditions (in aiding conditions the mobile surface is seen like motor element or input and the PDU like user element or output). By an adequate selection of the values of the above reported efficiencies, it is possible to simulate the behaviour of both reversible and irreversible actuators. Generally, the efficiencies of the irreversible actuators are lower than the reversible ones; particularly the aiding efficiencies of the irreversible arrangement must be intended as negative. The model computes the friction torque as the above-mentioned sum of a component not depending on the load and a further one related to the load through the efficiency as:

$$T_{FR} = T_{FR0} + \left(\frac{1}{\eta_{opp}} - 1 \right) \cdot T_{LD} \tag{3}$$

in opposing conditions (both static and dynamic)

$$T_{FR} = T_{FR0} + (1 - \eta_{aid}) \cdot T_{LD} \tag{4}$$

in aiding conditions (both static and dynamic).

When the actuation speed is not null and the load opposing, T_{FR} is obtained by the relationship (3) in which $\eta_{opp} = \eta_{din,opp}$ and $T_{FR0} = T_{FR0,din}$; when the load is aiding, T_{FR} is obtained by the relationship (4) in which $\eta_{aid} = \eta_{din,aid}$ and $T_{FR0} = T_{FR0,din}$; in both cases the sign of T_{FR} is assumed the same of the actuation speed, owing to its sign in equation (1). Else, when the actuation speed is null, T_{FR} is equal to *Act* if it lays within two limit values; when the load is positive, the lower value, negative, is obtained by the relationship (4) in which $\eta_{aid} = \eta_{stat,aid}$ and $T_{FR0} = T_{FR0,stat}$ and the upper one, positive, is obtained by the equation (3) in which $\eta_{opp} = \eta_{stat,opp}$ and $T_{FR0} = T_{FR0,stat}$. If the load is negative, the lower value, negative, is obtained by the relationship (3) in which $\eta_{opp} = \eta_{stat,opp}$ and $T_{FR0} = T_{FR0,stat}$ and the upper one, positive, is obtained by the relationship (4) in which $\eta_{aid} = \eta_{stat,aid}$ and $T_{FR0} = T_{FR0,stat}$.

3.3 DYNAMIC EQUATION INTEGRATION

By means of the equation (1) it is possible to compute the acceleration of the actuator-surface assembly

$$\ddot{\vartheta} = (Act - T_{FR}) / J_S \tag{5}$$

Through two following numerical integrations is possible to obtain the angular rate and, subsequently, the displacement of the actuator-surface assembly; if, within a generic integration step, a sign inversion of the angular rate occurs, the model determines the assembly stop as a possible consequence of the friction. The following integration step is able to evaluate either the eventual uninterrupted standstill condition or the break away, according to the actual value of the active torques related to the friction. Fig. 1 represents the above-mentioned model.

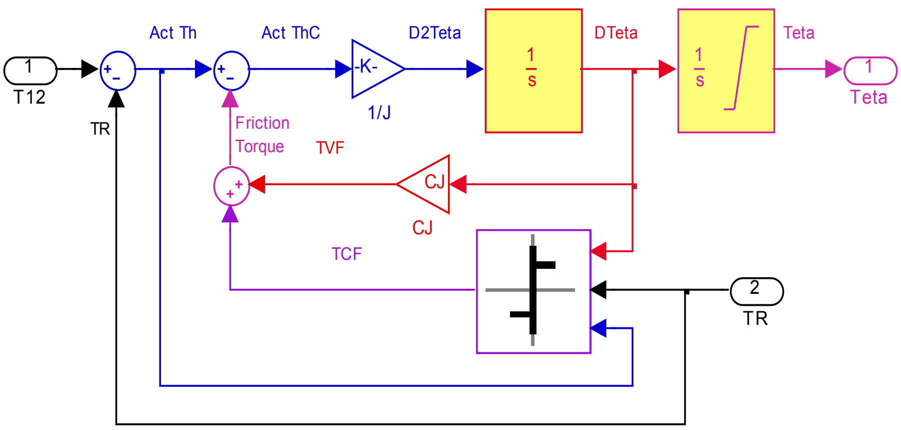


Figure 1 Theoretical actuator dynamics block diagram schematic

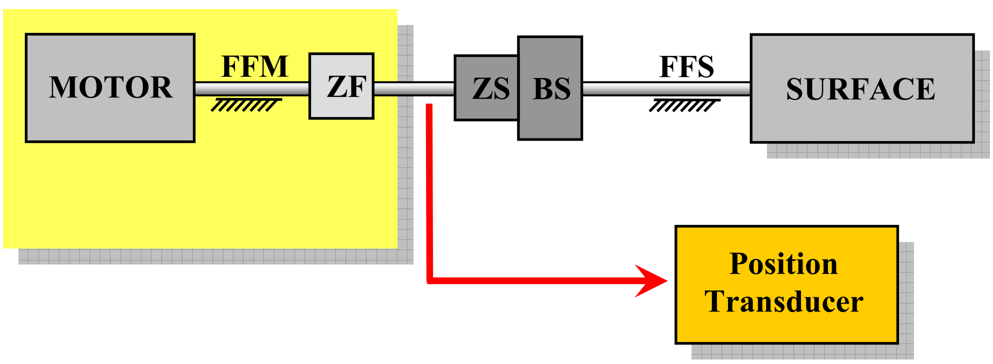


Figure 2 Passive subsystem schematic of control system dynamic model

4 ACTUATION SYSTEM MODELLING

In order to validate the actuator dynamic model characterised by the load depending Coulomb friction, its behaviour is studied as a part of a hydraulic position servomechanism, typical of those currently used for flap control systems. Therefore, a control system dynamic model was prepared, whose passive subsystem is schematically represented in Fig. 2. The system model, which obviously represents only the essential elements of the real control flap system, consists of a Power Control and Drive Unit (PDU) that, through a drive shaft system and ballscrew actuators (BS), drives the flaps. Each actuator is an assembly containing a gear reducer (ZS) and a ballscrew. At the outer end of the shaft system is located the position transducers (PT) and the tachometers, if present. In order to put in evidence the eventual actuator reversibility, the considered system is never equipped with wingtip or irreversibility brakes. An Electronic Control Unit (ECU), not shown in Fig. 2, which closes the position control loop, performs the system control. The PDU contains the hydraulic motor, the gear reducer (ZM) and control valve; the system considered for this work was assumed to also contain tachometers for a continuous actuation speed control. Fig. 2 shows the mechanical model of the actuation system.

The model takes into account the hydraulic and mechanical characteristics of all system components, including their friction.

In particular, the model takes into account the following:

- Coulomb friction (FF) altogether generated in the PDU (FFM) and in the actuators (FFS),
- third order electromechanical dynamic model of the servovalve with limitations on the maximum excursion of flapper (XFM) and spool (XSM) and simplified fluid-dynamic model sensitive to the internal leakage (Clk) and the differential supply – return pressure (PSR),
- dynamic and fluid-dynamic hydraulic motor and high speed gear reducer model taking into account, beside the above mentioned Coulomb friction, viscous friction, internal leakage and mechanical ends of travel of the actuator and flap assembly.

5 SIMULATION RESULTS

The above-described model of the actuation system has been used to build a mathematical model of the whole system and a dedicated computer code written in Matlab – Simulink has been prepared.

In order to validate the computer code, some simulations have been run for the cases of irreversible and reversible actuator, with eventually reduced hydraulic system supply pressure and opposing or aiding loads; such simulations have been then compared with those obtained from an equivalent FORTRAN 90 model previously validated. In the following figures ***Com*** is the input command, ***XF*** the servovalve's first stage flapper position, ***XS*** the second stage spool position, ***TR*** the aerodynamic load acting on the surface, ***DTeta*** the surface angular rate and ***Teta*** the surface position. Figures 6, 7 and 8 show the simulations results for the cases of irreversible actuators. Fig. 3 represents the case of flaps deployment (0 deg to 17 deg) and retraction (17 deg to 0 deg) in loaded condition (35000 N*m, constant value).

As it is typical for the flap controls, the deployment travel is characterised by an opposing load, so having a reduced actuation rate (the commanded position is reached in approximately 6 seconds); in the retraction travel the aiding load produces a higher actuation rate (the commanded position is reached in approximately 2 seconds).

It must be noted that, owing to the actuator irreversibility, the aiding aerodynamic load produces a slightly higher friction load, which results in a net small opposing load.

Fig. 4 represents the case of flaps deployment (0 deg to 17 deg) under opposing load (35000 N*m, constant value) and temporarily reduced supply pressure PSR.

At time = 1 s the supply pressure starts to drop at a rate of 12 MPa/s, reaching, two seconds later, the value of 2 MPa, kept constant up to time = 4 s; as a consequence, the actuation rate drops till to a complete stop, without any back movement owing to the irreversibility of the actuators. At time = 4 s the supply pressure starts to grow at a rate of 24 MPa/s, restoring, one second later, the design value of 26 MPa; thus, the actuation system starts again, reaching the commanded position approximately at time = 10 s.

Fig. 5 represents the case of flaps retraction (35 deg to 0 deg) under aiding load (35000 Nm, constant value) and temporarily reduced supply pressure with the same time history of the previous case; the pressure drop causes the actuation rate drops till to a complete stop, without any forward movement owing to the irreversibility of the actuators; the following pressure growth causes the actuation system starts again, reaching the commanded position approximately at time = 6.5 s. Figures 6, 7 and 8 show the simulations results for the cases of reversible actuators.

Fig. 6 represents the case of flaps deployment (0 deg to 17 deg) and retraction (17 deg to 0 deg) in loaded condition (35000 Nm, constant value) as in the previous case of Fig 3. The same considerations can be done, except for the reduced actuation times (deployment travel in approximately 3.5 s and retraction travel in approximately 1.6 s) in consequence of the higher efficiencies of the reversible actuators. It must be noted that, owing to the actuator reversibility, the aiding aerodynamic load produces a lower friction load, which results in a net aiding load.

Fig. 7 represents the case of flaps deployment (0 deg to 17 deg) under opposing load (35000 Nm, constant value) and temporarily reduced supply pressure with the same time history of Fig. 4. The pressure drop causes the actuation rate drops till to a complete stop, followed by a back movement owing to the reversibility of the actuators; the following pressure growth causes the actuation system starts forward again (after a temporary stop caused by the friction), reaching the commanded position approximately at time = 7.2 s. Fig. 8 represents the case of flaps retraction (17 deg to 0 deg) under aiding load (35000 Nm, constant value) and temporarily reduced supply pressure with the same time history of the previous case; the pressure drop causes the actuation rate drops till to a reduced value without any stop condition owing to the reversibility of the actuators; the following pressure growth causes the actuation system attempts to restore the previous rate, only limited by the achievement of the commanded position approximately at time = 4.4 s.

6 CONCLUSIONS

The simulations performed show the proper accuracy of the proposed algorithm taking into account the effects of the dry friction and of the ends of travel on the behaviour of the actuators. It must be noted the ability of the proposed model to describe correctly the dynamic/static behaviour of both reversible and irreversible types, employing the proper values of the respective efficiencies.

So, the algorithm developed by the authors supplies an effective answer to the necessity of accurate tools in evaluating the effects produced by friction forces or torques and ends of travel acting on a generic mechanical actuator by means of a self-contained Simulink computational routine.

7 LIST OF SYMBOLS

Act sum of the active torques
Com input command
C_S viscous damping coefficient of the surface
PSR differential supply-return pressure
T_{LD} aerodynamic load acting on the surface
T_{FR} friction torque
$T_{FR0,stat}$ static friction torque component no load depending
$T_{FR0,din}$ dynamic friction torque component no load depending
T_{TR} torque acting on the transmission line
J_S moment of inertia of the aerodynamic surface
X_F servovalve first stage flapper position
X_S servovalve second stage spool position
$\eta_{din,aid}$ out/in torque ratio, aiding load, dynamic conditions
$\eta_{din,opp}$ out/in torque ratio, opposing load, dynamic conditions
η_{opp} out/in torque ratio, opposing load
η_{aid} out/in torque ratio, aiding load
$\eta_{stat,aid}$ out/in torque ratio, aiding load, static conditions
$\eta_{stat,opp}$ out/in torque ratio, opposing load static conditions
Teta surface position
DTeta surface angular rate
D2Teta surface angular acceleration

Figure 3

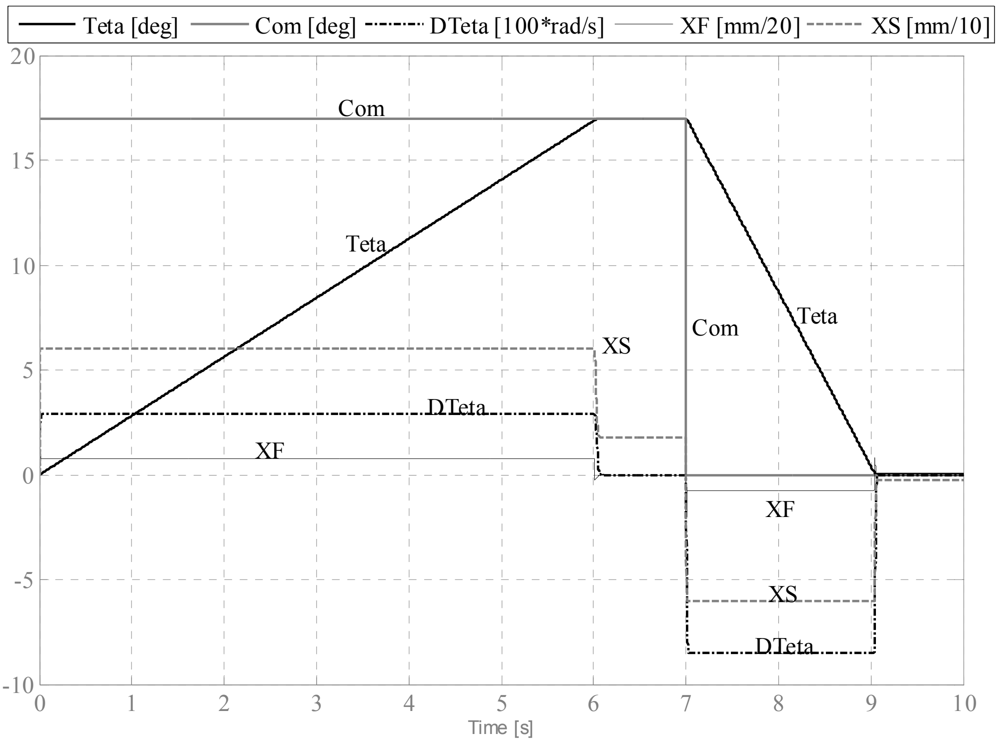


Figure 4

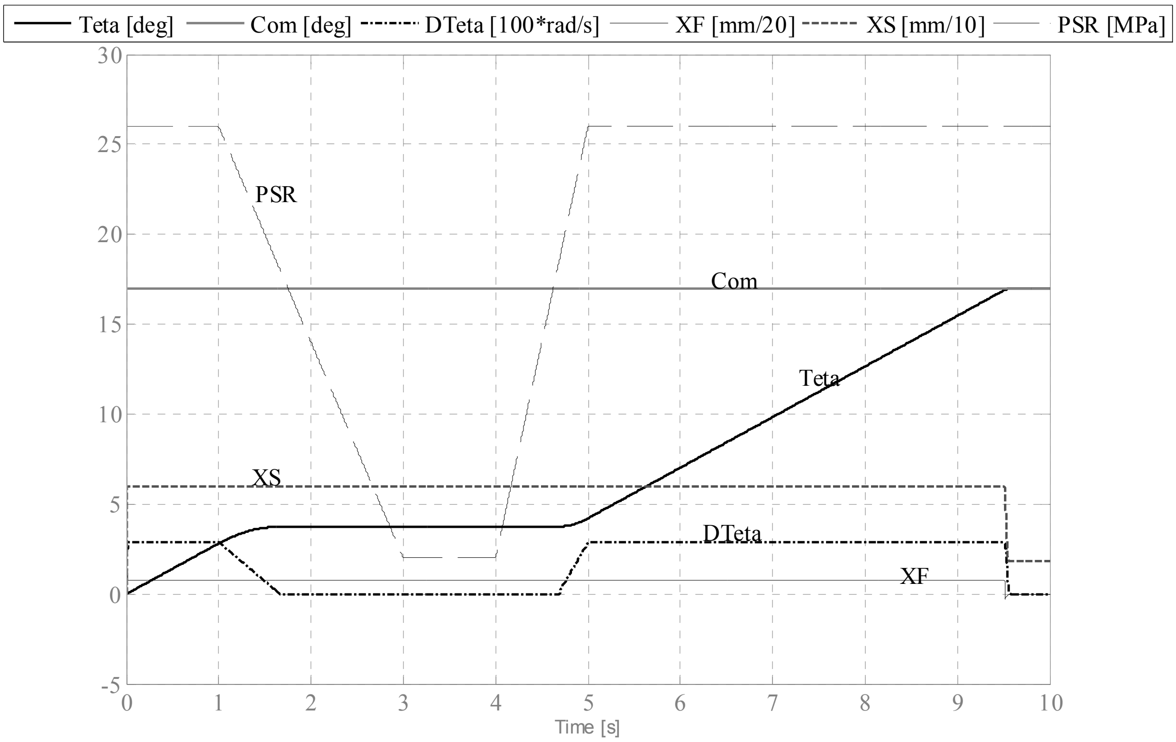


Figure 5

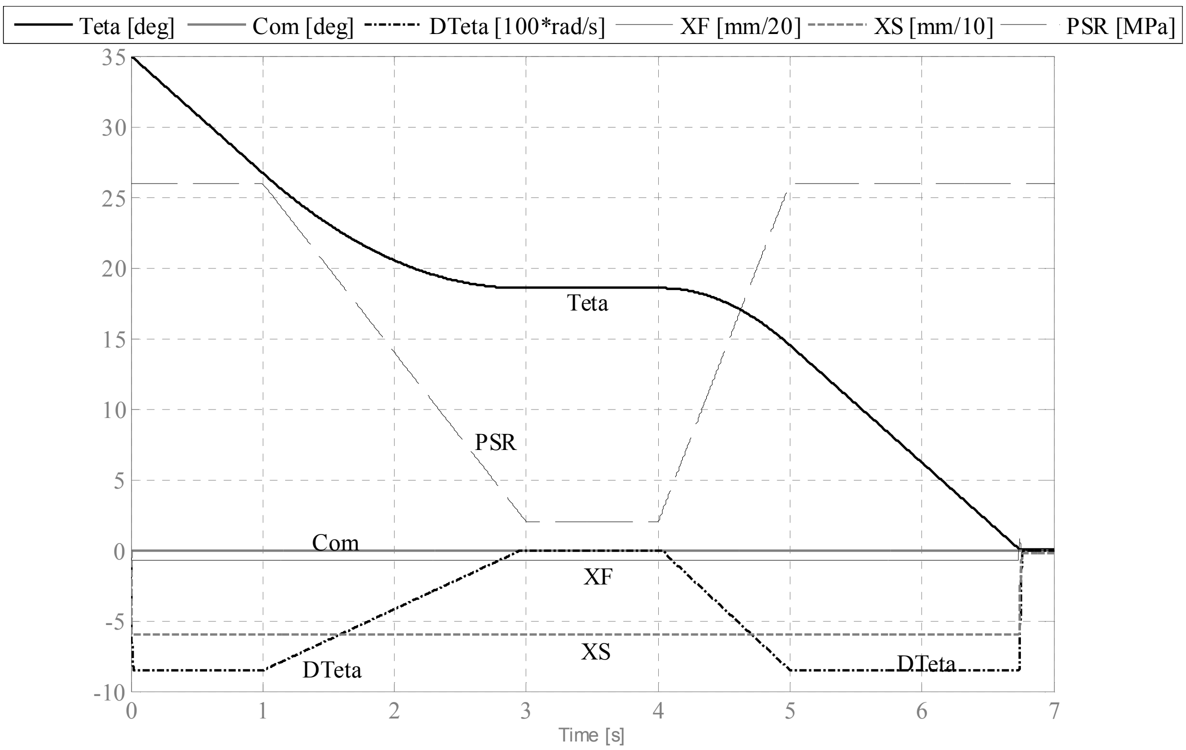


Figure 6

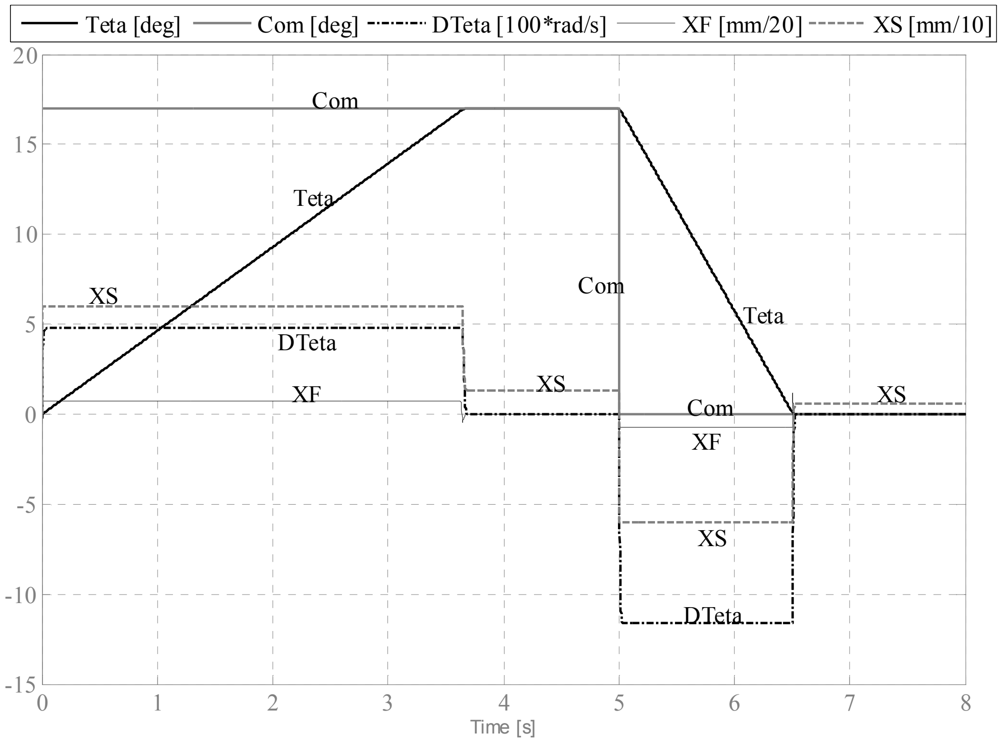


Figure 7

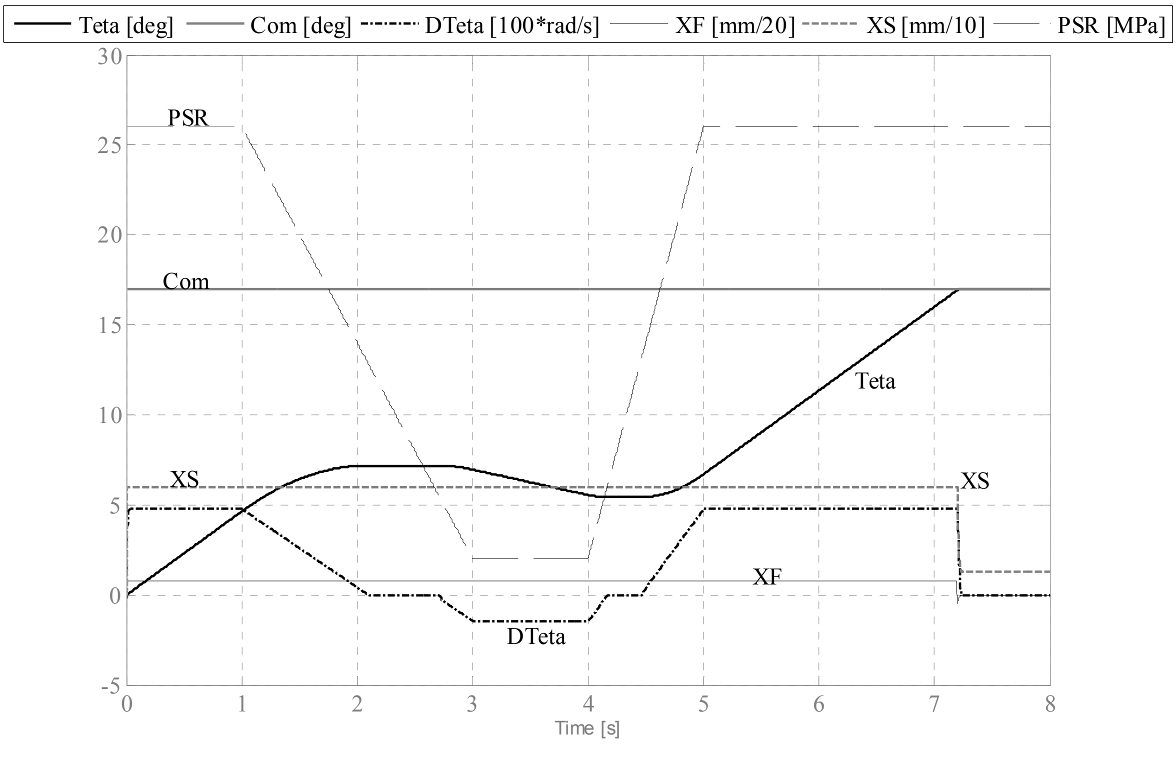
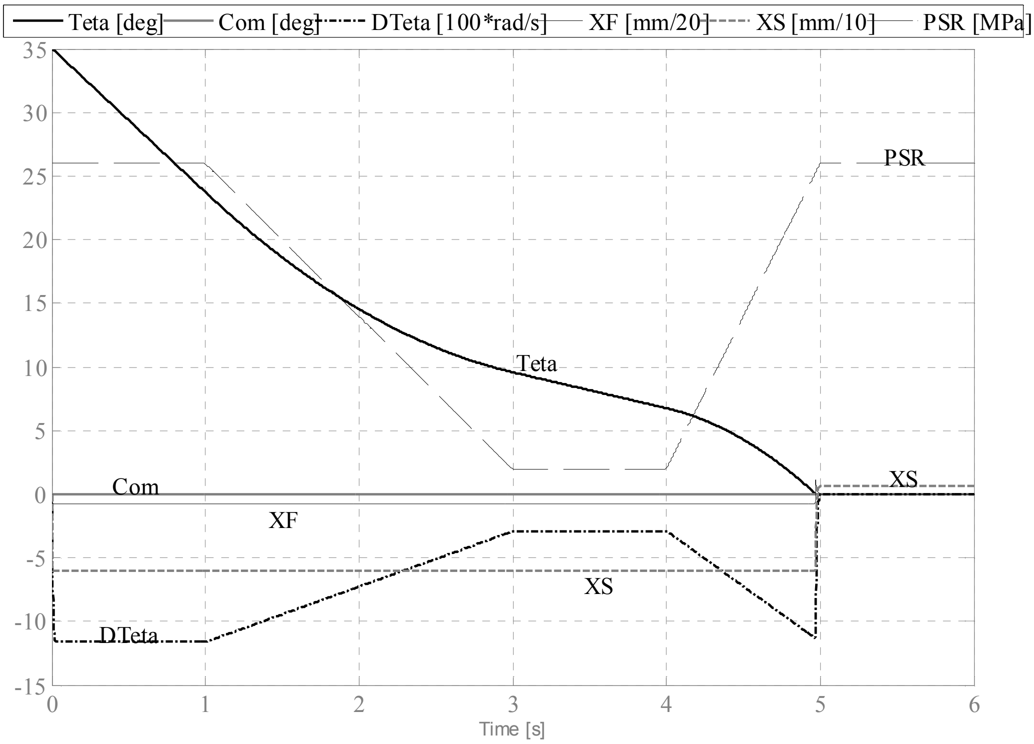


Figure 8



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