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Identification techniques applied to a passive elasto-magnetic suspension

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Abstract *The paper presents an experimental passive elasto-magnetic suspension based on rare-earth permanent magnets, characterized by negligible dependence on mass of its natural frequency.*

The nonlinear behaviour of this system, equipped with a traditional linear elastic spring coupled to a magnetic spring, is analysed in time domain, for non-zero initial conditions, and in frequency domain, by applying sweep excitations to the test rig base. The dynamics of the system is very complex in dependence of the magnetic contribution, showing both hardening behaviour in the elasto-magnetic setup, and softening motion amplitude dependent behaviour in the purely magnetic case. Hence it is necessary to adopt nonlinear identification techniques, such as non-parametric restoring force mapping method and direct parametric estimation technique, in order to identify the system parameters in the different configurations.

Finally, it is discussed the ability of identified versus analytical models in reproducing the non-linear dependency of frequency on motion amplitude and the presence of jump phenomena.

1 Introduction

The paper presents the dynamic behaviour of a single degree of freedom system (sdof) equipped with a traditional linear elastic spring coupled to a magnetic spring.

This system was developed in order to achieve passive nonlinear suspensions whose natural frequency is independent of mass (Bonisoli and Vigliani, 2003). It is worth noting that linear traditional sdof elastic systems are characterized by natural frequency inversely proportional to the square root of mass, while systems equipped with purely magnetic repulsive springs show a direct dependence of frequency on a power of their mass in consequence of static configuration changes.

The need for passive magnetic forces suitable for the purpose implies the usage of rare-earth permanent magnets, sintered from Samarium-Cobalt or Neodymium-Iron-Boron; nowadays these materials permit to reach high residual magnetic induction and hysteresis energy (Coey, 2002) and are applied in passive mechanical field such as viscous-type dissipative elements in magnetic dampers and eddy current brakes (Nagaya et al., 1984), or as nonlinear magneto-elastic springs in magnetic bearings, suspensions and levitation devices (Yonnet, 1978).

The dynamics of magneto-mechanics interaction is analysed by use of nonlinear models and experimental data obtained from a sdof test bench. Nonlinear identification techniques are implemented on the experimental outcomes in order to detect the nonlinear effects, through the non-parametric restoring force mapping method (Masri and Caughey, 1979; Shin and Hammond, 1998; Ajjan Al-Hadid and Wright, 1989) and to compare numerical estimations, through the direct parametric estimation technique (Worden and Tomlinson, 2001; Chang and Tung, 1998). The restoring force method, that has already been applied to passive magnetic systems by Shin and Hammond (1998), allows to identify the system characteristics without any need of hypothesis on the elastic or dissipative nature of the forces, while the DPE method can give estimates of the nonlinear terms of the dynamic model in time domain, with a technique similar to the one proposed for frequency domain analysis by Worden and Tomlinson (2001).

Finally, the experimental and identified behaviour are compared with an analytical model, based on a nonlinear empirical formula for modelling the magnetic forces: both hardening and softening nonlinear behaviour of different elasto-magnetic setup configurations are reproduced, proving the effectiveness of the identification techniques and the performance of the analytical model.

2 Magneto-elastic adaptive suspension

The physical system under analysis is a single degree of freedom (sdof) elasto-magnetic suspension in which the elastic element consists of a traditional and a magnetic springs assembled in parallel.

Figure 1 shows the experimental test bench, consisting of three parallel plates: the top plexiglass plate is free to move in a vertical plane along four cylindrical bars (where the traditional elastic spring can be mounted) and it is arranged to carry the permanent magnets; the bottom plate (in aluminium) can be fixed to a reference plane or to a shaker; the intermediate plexiglass plate is the housing of the opposite permanent magnets, in order to generate the repulsive magnetic force.

The test rig is equipped with two accelerometers mounted on the top and bottom plates and with a resistive displacement sensor that measures the relative position between the two plexiglass surfaces. The signals from these transducers are used to analyse the system dynamic behaviour in the frequency range of interest (2-16 Hz). The bench is designed to directly measure the relative displacement ξ in order to apply nonlinear identification techniques, since the magnetic forces are nonlinear with respect to the relative displacement. Relative velocity $\dot{\xi}$ is computed as the numerical derivative of the relative displacement with suitable filtering of the reconstructed signal.

The bench allows to test different configuration for the elasto-magnetic suspension, varying the number of magnetic pairs and/or the suspended mass; hence it is possible to analyse both the influence of the magnetic contribution on the dynamic behaviour of the system and the adaptability of the suspension to different values of the mass.

2.1 Mathematical model

The dynamic behaviour of the adaptive suspension sketched in Fig.2 can be described with the following equation:

$$m\ddot{\xi} + \zeta\dot{\xi} + k\left(\xi - l_0\right) + F_m + mg = -m\ddot{y}, \tag{1}$$

where $\xi = x - y$ is the relative position of the suspended mass, ζ is the viscous damping, k is the linear spring stiffness, l_0 is the spring rest length, $g = 9.81$ m/s^2 is the gravity constant, m

is the mass of the system and the magnetic repulsive force F_m is modelled through an empirical formula, having form

$$F_m = -\frac{A}{(\xi + B)^n}, \tag{2}$$

where parameters A, B and n may be evaluated through magnetic models based on equivalent currents method (Nagaraj, 1988; Bonisoli and Vigliani, 2003) or, alternatively, A, B may be experimentally determined by a least square error approach and n is an integer, set equal to 3 (Bonisoli and Vigliani, 2003).

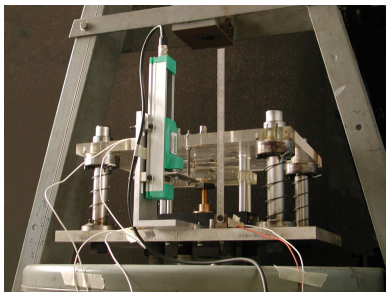

Figure 1: Experimental set-up of the elasto-magnetic suspension

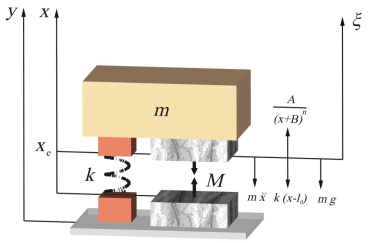

Figure 2: Model of the elasto-magnetic suspension

In order to test the dynamic behaviour of the system, the basis of the model is excited with a harmonic oscillation, i.e. the basis moves with law $y = y_0 \cos(\Omega t)$. Hence, the system dynamic equilibrium is described by

$$\ddot{u} + \lambda \dot{u} + \beta u - \gamma (u+1)^{-n} + \gamma = \delta \cos(\Omega t), \tag{3}$$

where $u = \xi/(x_e + B)$, $\lambda = \xi/m$, $\beta = k/m$, $\gamma = A(x_e + B)^{-(n+1)}/m$, $\delta = \Omega^2 y_0/(x_e + B)$ and x_e is the static equilibrium position.

Eq.(3) can be approximated with

$$\ddot{\xi} + \lambda \dot{\xi} + a\xi + b\xi^2 + c\xi^3 + d\xi^4 + e\xi^5 = \Omega^2 y_0 \cos(\Omega t), \tag{4}$$

where coefficients a, b, c, d and e can be analytically estimated or identified from experimental outcomes.

3 Nonlinear dynamic identification

In order to analyse the nonlinear characteristics of the system, two identification techniques are here applied: the non-parametric restoring force mapping (RF) (Masri and Caughey, 1979; Shin and Hammond, 1998; Ajjan Al-Hadid and Wright, 1989) and the direct parameter estimation (DPE) methods (Worden and Tomlinson, 2001). The first one is chosen because it does not need any a-priori knowledge of the nonlinear elasto-dissipative characteristics of the system and has already been applied to passive magnetic systems by Shin and Hammond (1998), while the DPE method in time domain is adopted to quantify and to compare the nonlinear terms of the dynamical model, with a technique similar to the one proposed for frequency domain analysis by Worden and Tomlinson (2001). With reference to the restoring force approach, a mechanical sdof system can be considered to undergo a generic force dependent on displacement and velocity, i.e. $F_r = f(z, \dot{z})$. The system equilibrium, given by equation

$$m\ddot{z}(t) + f(z, \dot{z}) = F(t), \tag{5}$$

can be written in the form

$$F_r(t) = F(t) - m\ddot{z}(t), \tag{6}$$

where it is evident that the time history of the restoring force F_r can be determined from the right hand side terms, that can be experimentally measured. Assuming that the nonlinear restoring

force can be divided in two parts, one dependent on displacement and the other on velocity, the interpolator surface is given by

$$f(z,\dot{z}) \cong \sum_{i=0}^{m} a_i T_i(z) + \sum_{j=0}^{n} b_j T_j(\dot{z}), \tag{7}$$

where $T_k(\nu) = \cos[k \arccos(\nu)]$, with $-1 \leq \nu \leq +1$ and $k = i$ or j, are Chebyshev polynomials, respectively of $k-$th order; coefficients a_i and b_j can be estimated by means of least square methods, respectively for null velocity and displacement:

$$f(z,0) \cong \sum_{i=0}^{m} a_i T_i(z) \qquad \text{and} \qquad f(0,\dot{z}) \cong \sum_{j=0}^{n} b_j T_j(\dot{z}). \tag{8}$$

The restoring force technique is applied when the lower plate of the experimental suspension undergoes a frequency sweep oscillations; hence, according to eq.(1), the restoring force is

$$F_r = f(\xi,\dot{\xi}) = -\ddot{\xi}(t) - \ddot{y}(t) = -\ddot{x}(t). \tag{9}$$

Experimental tests with frequency sweep highlight the nonlinear characteristics previously described: the elasto-magnetic suspension ($m = 11.4$ kg, $x_e = 10.2$ mm, $y_0 = 0.7$ mm, $k = 2098$ N/m, $l_0 = 19$ mm, $A = 2.26 \cdot 10^{-3}$ N m^3, $B = 18.8$ mm, $n = 3$) reveals a hardening behaviour, showing an evident jump phenomenon in time domain (Fig.3); on the contrary, in the purely magnetic setup ($m = 5.33$ kg, $x_e = 16.3$ mm, $y_0 = 0.7$ mm, $A = 2.26 \cdot 10^{-3}$ N m^3, $B = 18.8$ mm, $n = 3$), the system possess a softening dynamical behaviour, as visible in the time domain (Fig.4).

Numerical results obtained from analytic solutions are also compared with data from the elasto-magnetic third order model (eq. 4) in Fig.3 and with simulations of the fractional magnetic model described in eq. 1 (in the purely magnetic case, i.e. $k = 0$) in Fig.4. Results from the analytic model almost overlap with the nonlinear elastic characteristic identified through the polynomial interpolation of experimental values, both for elasto-magnetic case and for purely magnetic suspension.

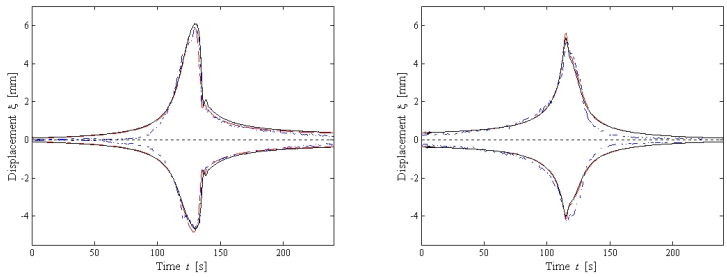

Figure 3: Displacement in time domain with increasing (left) and decreasing (right) frequency sweep of the elasto-magnetic suspension: experimental data (dash-dotted); identified cubic model (dashed); cubic analytical approximation (continuous)

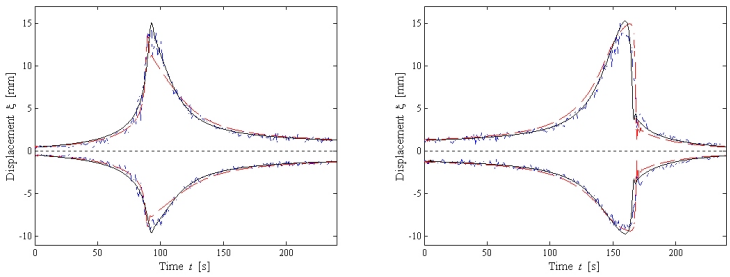

Figure 4: Displacement in time domain with increasing (left) and decreasing (right) frequency sweep of the purely magnetic suspension: experimental data (dash-dotted); identified 5^{th} order model (dashed); fractional analytical approximation (continuous)

The interpolator restoring force surface in the purely magnetic case is presented in Fig.5; assuming that stiffness and damping can be decoupled, through the analysis of two orthogonal

planes (for null velocity and null displacement) it is possible to determine the dissipative (see Fig.6) and nonlinear elastic characteristics for the elasto-magnetic suspension and for the purely magnetic suspension, as visible in Fig.7.

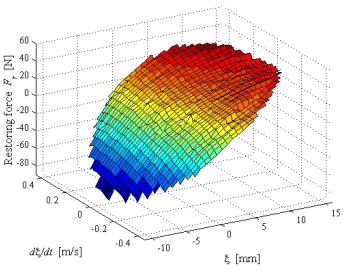

Figure 5: Experimental restoring force surface for purely magnetic suspension

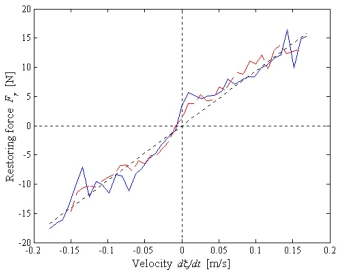

Figure 6: Dissipative characteristic: experimental data with increasing sweep (continuous); experimental data with decreasing sweep (dashed); identified linear model (dotted)

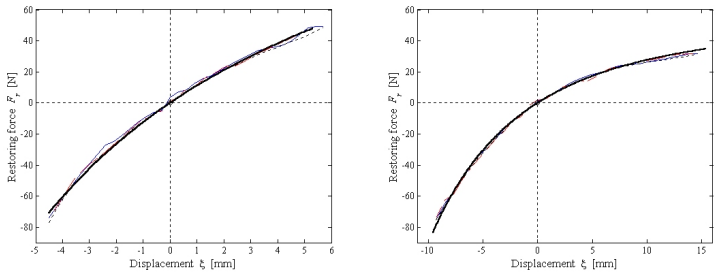

Figure 7: Elastic characteristic of the elasto-magnetic (left) and purely magnetic (right) suspension: experimental data with increasing sweep (continuous); experimental data with decreasing sweep (dashed); identified model (dotted), fractional model(thick continuous)

For the considered case studies, Fig.3 shows the time histories of the frequency sweep applied to the identified models with cubic polynomials, while in Fig.4 the dynamic behaviour with fifth order identified polynomials is presented (for the purely magnetic case).

The weights of the elastic characteristics polynomial terms demonstrate that terms having order higher than three are negligible for the elasto-magnetic system, while they are necessary to describe the softening behaviour of purely magnetic couplings. In fact in the softening case the polynomial approximation can effectively describe the real trend only in the neighbourhood of the static equilibrium condition, because, for large displacements, its behaviour is always hardening.

When applying the DPE method to the polynomial equation approximating the system dynamics, the system itself can be regarded to as time-discrete; hence the following vectorial representation holds:

$$\left\{\ddot{\xi}\right\} + \zeta\left\{\dot{\xi}\right\} + a\left\{\xi\right\} + b\left\{\xi^2\right\} + c\left\{\xi^3\right\} + d\left\{\xi^4\right\} + e\left\{\xi^5\right\} = -\left\{\ddot{y}\right\}, \tag{10}$$

where each element of the $(n \times 1)$ vectors represent a system state at time t (the system dynamical parameters are time invariant). Separating the unknown parameters, eq.(10) can be written in matrix form:

$$\left[\{\xi\} \;\; \{\xi^2\} \;\; \{\xi^3\} \;\; \{\xi^4\} \;\; \{\xi^5\} \;\; \left\{\dot{\xi}\right\}\right] \begin{Bmatrix} a \\ b \\ c \\ d \\ e \\ \zeta \end{Bmatrix} = -\left\{\ddot{\xi}\right\} - \{\ddot{y}\} = -\{\ddot{x}\} \tag{11}$$

that can be solved by means of generalized inversion techniques, such as singular value decomposition (SVD).

The DPE method is applied to the experimental free response to non-zero initial conditions (Fig.8-10) and proves particularly effective in estimating the nonlinear dynamical parameters. Furthermore results computed from the proposed analytical model are in good agreement with experimental measures and identified models, hence confirming that, within the considered amplitude range, third order polynomial models are suitable to describe the nonlinear behaviour of both elasto-magnetic and purely magnetic systems.

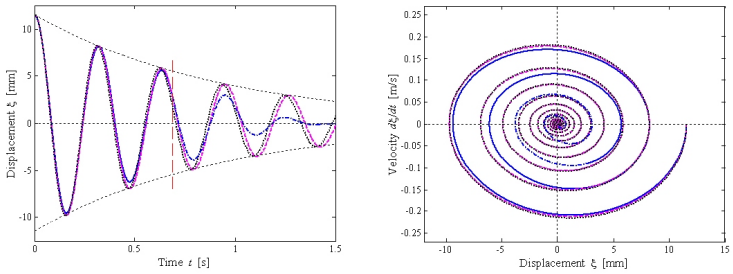

Figure 8: System free response time history (left) and phase plane (right) for traditional elastic suspension: experimental (continuous); identified linear (dashed); analytical linear (dotted)

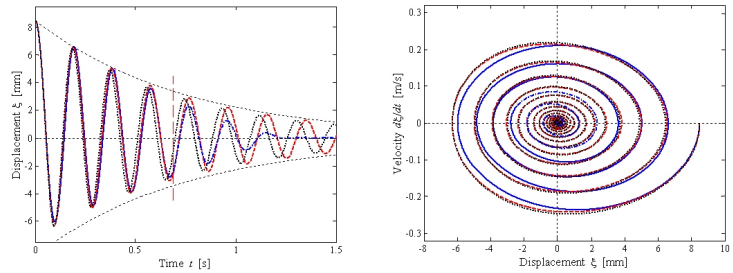

Figure 9: System free response time history (left) and phase plane (right) for elasto-magnetic suspension: experimental (continuous); identified cubic (dashed); analytical cubic (dotted)

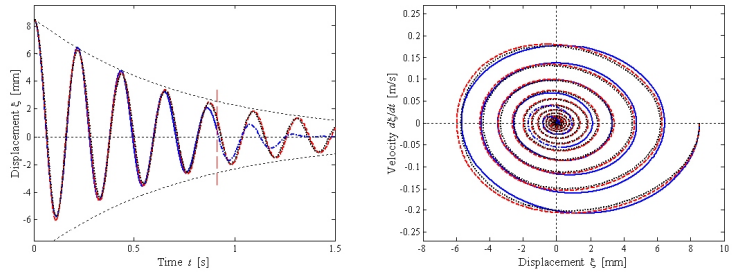

Figure 10: System free response time history (left) and phase plane (right) for purely magnetic suspension: experimental (continuous); identified cubic (dashed); analytical cubic (dotted)

As a conclusion, it can be stated that both RF and DPE prove effective tools, able to give satisfactory estimates that allow to simulate the system behaviour with results that are qualitatively and quantitatively close to experimental evidence.

Table 1 and 2 show the dynamical parameters identified during frequency sweeps and free response oscillations: in both cases, in accordance with the plotted comparison, it is evident an excellent agreement with the values predicted by analytical models up to third order polynomials (a, b, c terms), while the more relevant errors in the estimates relative to higher order terms (d, e) confirm their lower importance in describing the system dynamics.

To compare the results, the normalized mean square error (MSE) can be used; it holds:

$$\mathrm{MSE}(\xi) = \frac{100}{N\sigma_\xi^2} \sum_{i=1}^{N} \left(\xi_i - \tilde{\xi}_i\right)^2 \tag{12}$$

Setup	Model	a [s^{-2}]	b [m^{-1} s^{-2}]	c [m^{-2} s^{-2}]	d [m^{-3} s^{-2}]	e [m^{-4} s^{-2}]
Elasto-magnetic	Analytic	1002	$-5.765 \cdot 10^4$	$3.308 \cdot 10^6$	$-1.708 \cdot 10^8$	$8.233 \cdot 10^9$
	I.I. sweep	1000	$-5.736 \cdot 10^4$	$4.062 \cdot 10^6$	$-3.252 \cdot 10^8$	$2.153 \cdot 10^{10}$
	I.D. sweep	1004	$-6.35 \cdot 10^4$	$4.078 \cdot 10^6$	$-1.502 \cdot 10^8$	$1.222 \cdot 10^{10}$
Magnetic	Analytic	838.7	$-4.781 \cdot 10^4$	$2.271 \cdot 10^6$	$-9.708 \cdot 10^7$	$3.873 \cdot 10^9$
	I.I. sweep	799.4	$-5.0 \cdot 10^4$	$2.349 \cdot 10^6$	$-2.293 \cdot 10^7$	$6.713 \cdot 10^7$
	I.D. sweep	887.4	$-5.183 \cdot 10^4$	$2.347 \cdot 10^6$	$-3.952 \cdot 10^8$	$1.983 \cdot 10^{10}$

Table 1: Comparison of dynamic parameters from frequency sweep tests (I.I. → identified increasing frequency sweep, I.D. → identified decreasing frequency sweep)

Setup	Model	a [s^{-2}]	b [m^{-1} s^{-2}]	c [m^{-2} s^{-2}]
Elastic	Analytic	407.9	–	–
	Identified	394.3	–	–
Elasto-magnetic	Analytic	1135	$-4.125 \cdot 10^4$	$1.905 \cdot 10^6$
	Identified	1041	$-4.209 \cdot 10^4$	$2.499 \cdot 10^6$
Magnetic	Analytic	838.7	$-4.781 \cdot 10^4$	$2.271 \cdot 10^6$
	Identified	815.7	$-4.102 \cdot 10^4$	$2.676 \cdot 10^6$

Table 2: Comparison of dynamic parameters from free response tests

where ξ_i are the N experimental measures and $\tilde{\xi}_i$ are the correspondent values of the identified or analytic model and σ_ξ^2 is the signal variance that normalizes the error index to 100% when assuming for the model the mean value of the signal. Table 3 presents the MSE computed from the analysis of the experimental values relative to the free response oscillations compared with identified and analytic models: it is evident the excellent agreement of the results.

Setup	Comparison	MSE [%]
Elastic	Experimental/Identified	0.18
	Experimental/Analytic	0.44
Elasto-magnetic	Experimental/Identified	1.18
	Experimental/Analytic	1.40
Magnetic	Experimental/Identified	0.32
	Experimental/Analytic	0.42

Table 3: Normalized mean square errors

4 Conclusions

The experimental passive elasto-magnetic suspension, based on rare-earth permanent magnets coupled to traditional linear elastic spring, presents a negligible dependence on mass of its natural frequency for small amplitude vibration; moreover it shows relevant nonlinear motion amplitude dependent behaviour.

By varying the magnetic elastic contribution in the suspension, a wide range of nonlinear properties is analysed: in particular a transition between linear, hardening and softening behaviour with jump phenomena is evinced experimentally and reconstructed by means of identified and analytical models.

Both in time domain, for non-zero initial conditions, and in frequency domain, by applying sweep excitations to the test rig base, it can be stated that RF and DPE identification techniques prove effective tools, able to give satisfactory estimates that allow to simulate the system behaviour with results that are qualitatively and quantitatively close to experimental evidence.

References

M. Ajjan Al-Hadid, J.R. Wright, Developments in the force state mapping technique for non-linear systems and the extension to the location of non-linear elements in a lumped-parameter system, Mechanical Systems and Signal Processing, 3 (3), 1989, pp.269-290.

E. Bonisoli, A. Vigliani, Static behaviour of magneto-elastic forces, 16th AIMETA Congress of Theoretical and Applied Mechanics, Ferrara, Italy, 2003, (full text in CD Rom).

E. Bonisoli, A. Vigliani, Elastic suspension with magnetic springs, 16th AIMETA Congress of Theoretical and Applied Mechanics, Ferrara, Italy, 2003, (full text in CD Rom).

S.C. Chang, P.C. Tung, Identification of a non-linear electromagnetic system: an experimental study, Journal of Sound and Vibration, 214(5), 1998, pp. 853-871.

J.M.D. Coey, Permanent magnet applications, Journal of Magnetism and Magnetic Materials, 248, 2002, pp. 441-456.

S.F. Masri, T.K. Caughey, A nonparametric identification technique for nonlinear dynamic problems, Journal of Applied Mechanics, 46, 1979, pp. 433-447.

H.S. Nagaraj, Investigation of magnetic fields and forces arising in open-circuit-type magnetic bearings, Tribology Transactions, 31(2), 1988, pp. 192-201.

K. Nagaya, H. Kojima, Y. Karube, H. Kibayashi, Braking forces and damping coefficients of eddy current brakes consisting of cylindrical magnets and plate conductors of arbitry shape, IEEE Transactions on Magnetics, 20(6), 1984, pp. 2136-2145.

K. Shin, J.K. Hammond, Force-state mapping method of a chaotic dynamical system, Journal of Sound and Vibration, 218(3), 1998, pp. 405-418.

K. Worden, G.R. Tomlinson, Nonlinearity in structural dynamics: detection, identification and modelling, Institute of Physics Publishing, Bodmin, UK, 2001.

J.P. Yonnet, Passive magnetic bearings with permanent magnets, IEEE Transactions on Magnetics, 14, 1978, pp. 803-805.