

**LINKING OVER CONES AND NONTRIVIAL SOLUTIONS FOR  
 $p$ -LAPLACE EQUATIONS WITH  $p$ -SUPERLINEAR NONLINEARITY**

MARCO DEGIOVANNI AND SERGIO LANCELOTTI

ABSTRACT. We prove that the quasilinear equation  $-\Delta_p u = \lambda V|u|^{p-2}u + g(x, u)$ , with  $g$  subcritical and  $p$ -superlinear at 0 and at infinity, admits a nontrivial weak solution  $u \in W_0^{1,p}(\Omega)$  for any  $\lambda \in \mathbb{R}$ . A minimax approach, allowing also an estimate of the corresponding critical level, is used. New linking structures, associated to certain variational eigenvalues of  $-\Delta_p u = \lambda V|u|^{p-2}u$ , are recognized, even in absence of any direct sum decomposition of  $W_0^{1,p}(\Omega)$  related to the eigenvalue itself.