Revisiting the partial-capacitance approach to the analysis of coplanar transmission lines on multilayered substrates

Revisiting the Partial-Capacitance Approach to the Analysis of Coplanar Transmission Lines on Multilayered Substrates

Giovanni Ghione, *Senior Member, IEEE,* and Michele Goano, *Member, IEEE*

***Abstract*—A theoretical justification is presented of the partial-capacitance (PC) approach, widely exploited in the modeling of coplanar waveguides on finite-thickness and multilayered substrates. The analysis is based on the static spectral-domain approach to the computation of the capacitance of a set of planar conductors embedded into a multilayered substrate. It is shown that the PC method can be derived from approximating the static Green's function of a multilayered dielectric as the sum of partial contributions; such a decomposition can be applied to the Green's function either in parallel (admittance) or series (impedance) form. The resulting (parallel or series) PC approaches are shown to be accurate with substrates having layers of decreasing or increasing permittivity, respectively. Lines backed by magnetic or electric walls are introduced as limiting cases of multilayered structures.**

***Index Terms*—Coplanar lines, partial-capacitance (PC) approach, planar transmission lines, spectral-domain static Green's function techniques.**

I. Introduction

SIMPLE computationally effective analytical techniques are still important for the evaluation of the quasi-static parameters of planar transmission lines. In the coplanar case, Schwarz–Christoffel conformal mapping (CM) has been extensively used to determine the line capacitance *in vacuo* and with dielectrics (see, e.g., [1]). Unfortunately, CM exactly applies only to homogeneous regions, while in most practical lines, the cross section is multilayered. The partial-capacitance (PC) method overcomes this problem since it allows the total line capacitance to be approximated as the series and/or parallel combination of the capacitances of homogeneous subregions; those, in turn, can be effectively evaluated through CM.

The PC approach was first proposed in 1967 by Kochanov [2], with application to coplanar striplines on a finite-thickness dielectric substrate. In [2], the line capacitance was expressed as the sum of the air capacitance, and the "capacitance between conductors due to the presence of the insulating plate" with relative permittivity $\epsilon_{r1} > 1$ [2, p. 129]. To evaluate the latter contribution, "the model assumes that the electric field is entirely concentrated in a strip of relative permittivity $\epsilon_{r1} - 1$" [2, p. 129–130], i.e., the slab surface is assumed as a magnetic wall (MW) outside the conducting strips. No proof of the approximation made is provided, only the comment: "The choice of $\epsilon_{r1} - 1$ as the permittivity of the insulating strip becomes evident if we use the formal analogy with the field of currents in a conducting medium" [2, p. 129, footnote].

In 1980, Fouad-Hanna and Veyres [3] applied the PC approach to finite-thickness substrate coplanar waveguides (CPWs). In [3, Sec. 3], the analysis of conductor-backed CPWs was addressed through an extension of the PC approach (denoted here as *upper–lower (UL) decomposition*): the line capacitance was expressed as the sum of an upper and a lower half-space contribution, evaluated by introducing MWs in the CPW slots. In the following years, the PC approach (combined with the UL decomposition) was applied to a number of coplanar structures, see, e.g., [4]–[9]; in all cases considered, the permittivity of the strip-supporting layer was larger than that of the layer underneath.

A CM-based generalization of the PC approximation to CPWs on multilayered substrates with an arbitrary number of layers and layer permittivities was presented in 1992 by Svačina [10]. Independent of [10], in 1994, Gevorgian [11] extended the PC method to lines supported by three dielectric layers. Reference [11] explicitly considers the possibility that $\epsilon_{r1} < \epsilon_{r2}$, where layer 1 is in contact with the metallic strips, and layer 2 is immediately underneath; this implies that one of the PCs can be negative. A generalization to three-layer structures with backing and shielding was further presented by Gevorgian *et al.* in [12]. Some partial validations of the PC approach in the case $\epsilon_{r1} < \epsilon_{r2}$ against other techniques can be found in [11] and [12], with seemingly good agreement; see also, the later paper by Chen and Chou [13].

The applicability of the conventional PC approach in the case $\epsilon_{r1} < \epsilon_{r2}$ was first questioned in 1995 by Zhu *et al.* [14], who suggested that this case should require a series rather than a parallel capacitance decomposition [14, p. 230]. The solution proposed was an empirical modification of the conventional (parallel) PC (PPC) approach obtained through fitting of numerical results. The PPC approach was also shown to provide inaccurate results in many examples (see [14, Fig. 2c and 2d]).

As a further application of the PC approach, in 1996 Zhu *et al.* [15] exploited the method to evaluate the field distribution (rather than the capacitance only) for a CPW on a two-layer substrate (conventional case $\epsilon_{r1} > \epsilon_{r2}$). A more accurate and extensive analysis of field evaluation through the PPC approach was presented in 1999 by Carlsson and Gevorgian [16], who

Manuscript received October 21, 2002; revised March 24, 2003. This work was supported in part by Corning–Optical Technologies Italy and by the Centro di Eccellenza per le Comunicazioni Multimediali under Project WP2.1 and Project WP2.3.

The authors are with the Dipartimento di Elettronica, Politecnico di Torino, 10129 Turin, Italy (e-mail: ghione@polito.it).

Digital Object Identifier 10.1109/TMTT.2003.815873

once again noticed that, in the case $\epsilon_{r1} < \epsilon_{r2}$, the conventional PC approach is inaccurate.

The idea of a modified or *series* PC (SPC) approach was naturally suggested by the analysis of microstrip lines on a multilayered substrate. Already in 1992, Svačina [10] had proposed a generalized form of the SPC as the result of approximate CM for a microstrip line on a stratified substrate; later, Sychev and Shevtsov [17] analyzed the same structure through a combination of the SPC and PPC approaches.

Concerning coplanar lines, the SPC approach was considered for the first time by Ghione *et al.* [18] to deal with the case $\epsilon_{r1} < \epsilon_{r2}$. For this case, SPC decomposition for the lower half-space capacitance was proposed, resulting in the series of two capacitances: a half-space contribution and a layer contribution; the latter being evaluated as the capacitance of a CPW backed by an electric wall (EW) and with an equivalent relative permittivity $\epsilon_{r1}\epsilon_{r2}/(\epsilon_{r2} - \epsilon_{r1})$. More recently, the extension of the PPC and SPC approaches to an arbitrary number of layers was proposed by Sychev [19] who also applied the technique to multiconductor CPWs with conductor backing and shielding [20].

Despite the fairly long record of the PC method, no satisfactory theoretical justification of the approach has been presented thus far; indeed, the CM strategy in [10] confines the SPC and PPC approaches to the microstrip and coplanar cases, respectively, and does not explain the PPC inaccuracy for $\epsilon_{r1} < \epsilon_{r2}$.

The purpose of this paper is to provide a comprehensive theoretical justification of the PC approach. The analysis is based on the spectral-domain Green's function quasi-static variational technique for evaluating the capacitance of infinitely thin lines on arbitrary multilayered dielectric substrates [21]. The spectral-domain technique enables to explain in which cases each method (SPC or SPC) works and why, and what is the source of the error introduced by the approximations. Finally, it helps in devising possible extensions and modifications of the technique.

This paper is structured as follows. Section II is a brief review of the quasi-static spectral-domain approach and of the related Green's function evaluation. The principle of UL decomposition is analyzed in Section III; starting from this discussion, Section IV provides a justification of the PPC and SPC approaches for a two-layer structure without conductor backing. The extension to an arbitrary number of layers and the treatment, within the PC context, of lines backed by EMs or MWs is dealt with in Section V. Results and validations based on other numerical techniques are presented throughout this paper.

II. Spectral-Domain Green's Function Formulation

Consider a planar stratified dielectric structure (with layer permittivities $\epsilon_i = \epsilon_{ri}\epsilon_0$) embedding a set of infinitely thin coplanar conducting strips (Fig. 1). The structure is closed (top and bottom) by a semi-infinite dielectric layer, or by an MW/EW. Notice that the EW can be grounded (i.e., connected with a set of metal strips serving as a ground electrode) or a floating electric wall (FEW). Suppose for the sake of simplicity that the strips define only one voltage difference (e.g., all strips are equipotential, but one, and the potential difference V is given); this situation can refer either to a single transmission line or to a modal configuration in a multiconductor transmission line.

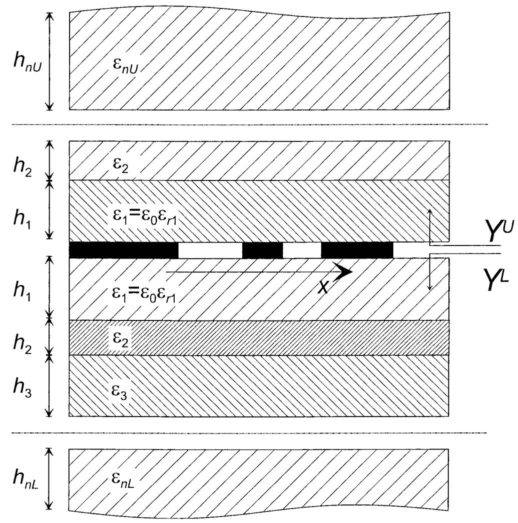

Fig. 1. Line in multilayered dielectric structure.

A. Variational Expression of the Per-Unit-Length (p.u.l.) Capacitance

Let us denote the charge distribution on the strips as $\rho(x)$, x being the transverse coordinate parallel to the stratification, and the electric-field distribution along x as $e(x)$; let α be the spectral variable along x, and $\widetilde{\rho}(\alpha)$, $\widetilde{e}(\alpha)$ be the Fourier transforms of the corresponding space functions, respectively,

$$\widetilde{f}(\alpha) = \int_{-\infty}^{\infty} f(x)e^{\mathrm{j}\alpha x}\mathrm{d}x, \qquad f = \rho, e. \tag{1}$$

The following dual spectral-domain variational expressions for the capacitance then hold [21]:

$$\frac{1}{C} = \frac{1}{\pi Q}\int_0^{+\infty} \left|\widetilde{\rho}(\alpha)\right|^2 g(\alpha)\mathrm{d}\alpha \tag{2}$$

$$C = \frac{1}{\pi V}\int_0^{+\infty} \left|\widetilde{e}(\alpha)\right|^2 G(\alpha)\mathrm{d}\alpha \tag{3}$$

where Q is the total charge on the set of electrodes kept at the positive potential and V is the total voltage between the set of positively biased electrodes and the reference electrode set. In the previous equations, we have taken into consideration that the integrand (and the spectral-domain Green's functions g and G) are even in α and can, therefore, be evaluated for $\alpha > 0$ only.

The above expressions give the exact value of the p.u.l. elastance and capacitance, respectively, if the exact charge or electric-field distribution is used. If a trial approximating function is exploited instead, a lower or upper bound estimate for the capacitance, respectively, is obtained. Careful numerical techniques are required to evaluate the semi-infinite spectral integrals, see, e.g., [22, Appendix II].

Functions $g(\alpha)$ and $G(\alpha)$ are spectral-domain Green's functions, which can be readily derived through an equivalent transmission-line formalism [21], [23] applied to the solution of Poisson equation. Moreover, one has $\alpha^2 g(\alpha)G(\alpha) = K$, a constant that can be set to one with proper normalization.

B. Expression of the Green's Function

For a dielectric structure made of n_L lower and n_U upper layers of thickness h_i and relative permittivity ϵ_{ri} (see Fig. 1), the following expressions hold [23]:

$$G(\alpha) = \frac{\epsilon_0}{\alpha} Y = \frac{\epsilon_0}{\alpha}\left\{Y^L + Y^U\right\} \tag{4}$$

$$g(\alpha) = \frac{1}{\epsilon_0 \alpha} Z = \frac{1}{\epsilon_0 \alpha}\left\{\frac{1}{Y^L + Y^U}\right\} \tag{5}$$

where Y^L is the equivalent normalized admittance seen from the strip interface looking down (lower layers), while Y^U is the equivalent normalized admittance seen from the strip interface looking up (upper layers). The admittances can be computed iteratively starting from the lowest (topmost) layer as the input admittance of a cascade of transmission lines with a real propagation constant (i.e., attenuation) α, normalized characteristic admittance ϵ_{ri}, and length h_i [23].[1] One has the iterative expressions

$$Y_i^\eta(\alpha) = \epsilon_{ri} \frac{Y_{i+1}^\eta(\alpha) + \epsilon_{ri} \tanh h_i\alpha}{\epsilon_{ri} + Y_{i+1}^\eta(\alpha) \tanh h_i \alpha}, \quad i = (n_\eta - 1), \ldots, 1; \quad \eta = U, L. \tag{6}$$

Admittances $Y_{nL}^L(\alpha)$ and $Y_{nU}^U(\alpha)$ can be zero (MW), infinity (EW), or equal to $\epsilon_{r\eta}$, $\eta = U, L$ (semi-infinite dielectric layer).

III. Decomposition of the Capacitance Into Upper and Lower Half-Space Contributions (UL Decomposition)

In the variational expression with field trial functions, the Green's function can be partitioned into upper and lower half-space contributions as

$$G(\alpha) = \frac{\epsilon_0}{\alpha}\left\{Y^L + Y^U\right\} = G_L + G_U \tag{7}$$

where

$$G_\eta(\alpha) = \frac{\epsilon_0}{\alpha} Y^\eta, \qquad \eta = L, U. \tag{8}$$

G_L and G_U are the Green's function of a structure closed (top or bottom, respectively) by an MW (lying on the interface plane). By exploiting this decomposition, we can write

$$\begin{aligned} C &= \frac{1}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 G(\alpha) \mathrm{d}\alpha \\ &= \frac{1}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 G_L(\alpha) \mathrm{d}\alpha + \frac{1}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 G_U(\alpha) \mathrm{d}\alpha \\ &\approx C_L + C_U \end{aligned} \tag{9}$$

[1]Notice that the equivalent admittances introduced according to the formalism in [23] are pure numbers.

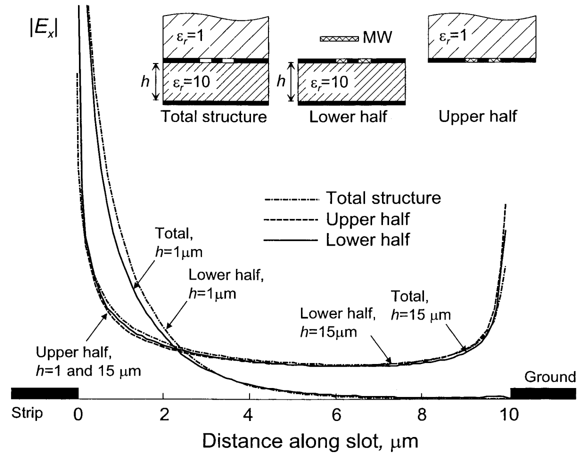

Fig. 2. Behavior of the horizontal (x) component of the electric field in the slot for a conductor-backed CPW for several values of the substrate thickness h; the upper and lower half-spaces are closed by an MW. All curves refer to the same applied potential difference.

where

$$C_\eta = \frac{1}{\pi V}\int_0^{+\infty} \left|\tilde{e}_\eta(\alpha)\right|^2 G_\eta(\alpha) \mathrm{d}\alpha, \qquad \eta = U, L. \tag{10}$$

The approximation sign in (9) has been introduced because C is exactly decomposed into the sum $C_L + C_U$ only if the field distribution of the complete structure is equal to the ones of the upper and lower half-spaces closed by an MW, i.e., if $\tilde{e}(\alpha) = \tilde{e}_L(\alpha) = \tilde{e}_U(\alpha)$. Owing to the variational properties of the capacitance expression, the decomposition is often accurate, and sometimes exact, as in lines on semi-infinite dielectric layers of permittivity ϵ_{rL} and ϵ_{rU}. In fact, in this case, one simply has

$$G(\alpha) = (\epsilon_{rL} + \epsilon_{rU})\frac{\epsilon_0}{\alpha} \tag{11}$$

i.e., the Green's function is proportional to the Green's function *in vacuo*. It follows that

$$\begin{aligned} C &= \frac{\epsilon_{rL} + \epsilon_{rU}}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 \frac{\epsilon_0}{\alpha} \mathrm{d}\alpha \\ &= \frac{\epsilon_{rL}}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 \frac{\epsilon_0}{\alpha} \mathrm{d}\alpha + \frac{\epsilon_{rU}}{\pi V}\int_0^{+\infty} \left|\tilde{e}(\alpha)\right|^2 \frac{\epsilon_0}{\alpha} \mathrm{d}\alpha \\ &= \epsilon_{rL}\frac{C_0}{2} + \epsilon_{rU}\frac{C_0}{2} = C_L + C_U \end{aligned} \tag{12}$$

where C_0 is the line capacitance *in vacuo*. This demonstrates that, as is well known [24], for infinitely thin electrodes, the effective permittivity of a coplanar line is exactly $(\epsilon_{rL} + \epsilon_{rU})/2$.

As an example of the accuracy achieved by the UL decomposition also in limiting cases, consider a conductor-backed CPW with substrate permittivity $\epsilon_{r1} = 10$; the upper half-space is air and the strip and slot widths are 10 μm. Three substrate thicknesses were considered, i.e., $h \to \infty$, $h = 15$ μm, and $h = 1$ μm. The behavior of the x-component of the electric field e [computed through the finite-element method (FEM)] in the right-hand slot is shown in Fig. 2 for the entire structure and for the upper and lower half-spaces closed by an MW in the cases

TABLE I
TOTAL AND PARTIAL (UPPER AND LOWER) CAPACITANCES FOR THE CONDUCTOR-BACKED CPW SHOWN IN FIG. 2. ALL CAPACITANCES ARE IN PICOFARADS PER METER

	C_L	C_U	$C_L + C_U$	$C_{\rm tot}$
$h \to \infty$	112.4094	11.2409	123.6504	123.6504
$h = 15\ \mu$m	120.7608	11.2409	132.0018	132.0181
$h = 1\ \mu$m	515.4187	11.2409	526.6596	532.6935

$h = 15\ \mu$m and $h = 1\ \mu$m. Notice the singular behavior on the strip edges. In the $h = 15\ \mu$m case, the lower, upper, and total field distributions, which exactly coincide for $h \to \infty$, are very close. For $h = 1\ \mu$m, the structure behaves more as a microstrip line, and while the lower and total field distributions are similar, the CPW-like upper field distribution is markedly different and, of course, equal to the one of the $h = 15\ \mu$m case. This suggests that the UL decomposition is exact for $h \to \infty$, very accurate for $h = 15\ \mu$m, and potentially inaccurate for $h = 1\ \mu$m. Indeed, inspection of the line capacitances (reported in Table I; $C_{\rm tot}$ is the reference total capacitance, computed through a spectral-domain moment-method technique exploiting $N = 6$ basis functions per slot) confirms the accuracy of the UL approach for $h \to \infty$, $h = 15\ \mu$m, but also reveals that this is reasonably accurate also in the case $h = 1\ \mu$m, mainly because C_L dominates $C_{\rm tot}$.

Finally, in the UL decomposition, we can further introduce the dual formulation (charge basis functions) and write

$$\frac{1}{C_\eta} = \frac{1}{\pi Q_\eta}\int_0^{+\infty} \left|\tilde{\rho}_\eta(\alpha)\right|^2 g_\eta(\alpha)\mathrm{d}\alpha, \qquad \eta = U, L \tag{13}$$

where $\alpha^2 g_L(\alpha)G_L(\alpha) = \alpha^2 g_U(\alpha)G_U(\alpha) = 1$, and

$$g_\eta(\alpha) = \frac{1}{\epsilon_0\alpha}\frac{1}{Y_\eta} = \frac{Z^\eta}{\epsilon_0\alpha}, \qquad \eta = U, L. \tag{14}$$

IV. PARTIAL CAPACITANCE APPROACH: TWO-LAYER CASE

Consider a CPW supported by two dielectric layers; the upper part being air. The first layer has thickness h_1 and permittivity ϵ_1, the second layer is semi-infinite and has permittivity ϵ_2. The (field) Green's function can be expressed as

$$G(\alpha) = \frac{1}{\alpha}\left\{\epsilon_0 + \epsilon_1\frac{\epsilon_1\tanh h_1\alpha + \epsilon_2}{\epsilon_2\tanh h_1\alpha + \epsilon_1}\right\}. \tag{15}$$

Applying the UL decomposition, one has for the lower half-space Green's functions

$$G_L(\alpha) = \frac{1}{\alpha}\frac{\tanh h_1\alpha + \xi}{\xi\tanh h_1\alpha + 1} \equiv \frac{F(\alpha)}{\alpha} \tag{16}$$

$$g_L(\alpha) = \frac{1}{\alpha}\frac{\xi\tanh h_1\alpha + 1}{\tanh h_1\alpha + \xi} \equiv \frac{f(\alpha)}{\alpha} \tag{17}$$

where $\xi = \epsilon^2/\epsilon^1$.

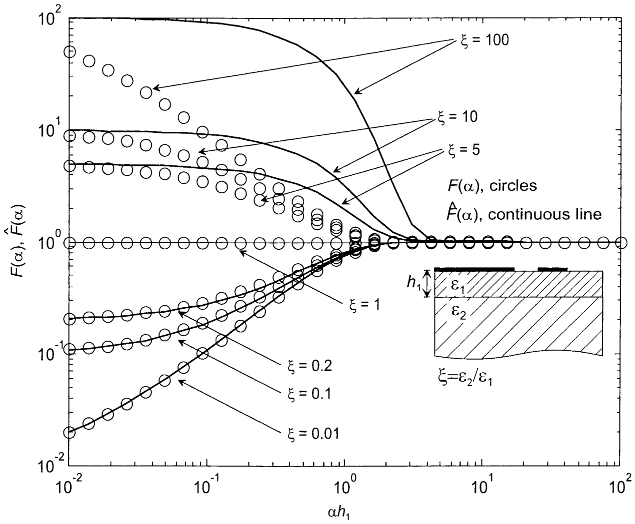

Fig. 3. Behavior of $F(\alpha)$ for different values of ξ (circles) and of the approximating function $\hat{F}(\alpha)$ (solid line); the approximation is accurate for $\xi < 1$ and very poor for $\xi > 1$.

A. Green's Function Approximation and PC Decomposition

We now explore the validity of the following approximations:

$$F(\alpha) \approx \widehat{F}(\alpha)$$
$$\widehat{F}(\alpha) = \xi + (1-\xi)\tanh h_1\alpha \tag{18}$$

$$f(\alpha) \approx \widehat{f}(\alpha)$$
$$\widehat{f}(\alpha) = \frac{1}{\xi} + \left(1 - \frac{1}{\xi}\right)\tanh h_1\alpha \tag{19}$$

which, in turn, imply

$$G_L(\alpha) \approx G_{L1}(\alpha) + G_{L2}(\alpha) \tag{20}$$

$$g_L(\alpha) \approx g_{L1}(\alpha) + g_{L2}(\alpha) \tag{21}$$

where

$$G_{L1}(\alpha) = \frac{\epsilon_2}{\alpha}$$
$$G_{L2}(\alpha) = \frac{1}{\alpha}(\epsilon_1 - \epsilon_2)\tanh h_1\alpha \tag{22}$$

$$g_{L1}(\alpha) = \frac{1}{\alpha\epsilon_2}$$
$$g_{L2}(\alpha) = \frac{1}{\alpha}\left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2\epsilon_1}\right)\tanh h_1\alpha. \tag{23}$$

The dual behavior of (18) and (19) with respect to ξ is clearly seen in Figs. 3 and 4. Fig. 3 shows that the approximation of $G_L(\alpha)$ is accurate when $\xi \le 1$ (i.e., when $\epsilon_1 \ge \epsilon_2$); in the second case (Green's function $g_L(\alpha)$), the behavior is complementary (see Fig. 4), i.e., the approximation is good when $\xi \ge 1$, ($\epsilon_2 \ge \epsilon_1$). It can be shown that the maximum relative error of the approximations within their ranges of validity is below 10%.

We now discuss the meaning of the decompositions $G_L(\alpha) \approx G_{L1}(\alpha) + G_{L2}(\alpha)$ ($\xi \le 1$) and $g_L(\alpha) \approx g_{L1}(\alpha) + g_{L2}(\alpha)$ ($\xi \ge 1$). The following identifications can be readily made by inspec-

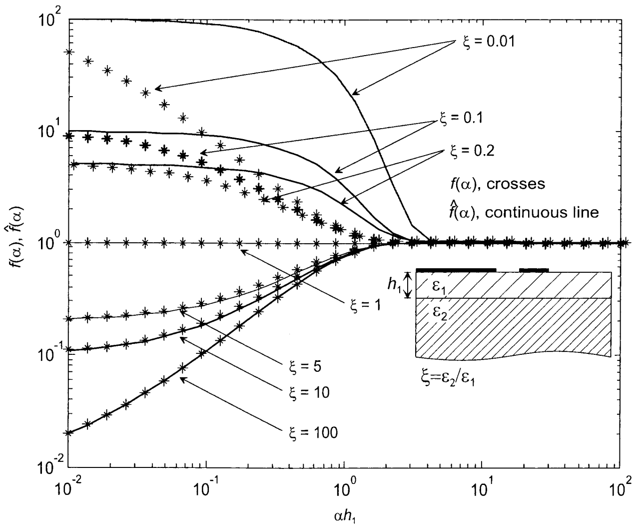

Fig. 4. Behavior of $f(\alpha)$ for different values of ξ (crosses) and of the approximating function $\hat{f}(\alpha)$ (solid line); the approximation is accurate for $\xi \geq 1$ and very poor for $\xi < 1$.

tion:

- $G_{L1}(\alpha)$ is the Green's function of a semi-infinite half space of permittivity ϵ_2 topped by an MW;
- $G_{L2}(\alpha)$ is the Green's function of a layer of thickness h_1 and effective permittivity $\epsilon_{\rm eqp} = \epsilon_1 - \epsilon_2$ closed by an MW;
- $g_{L1}(\alpha)$ is the Green's function of a semi-infinite half space of permittivity ϵ_2 topped by an MW;
- $g_{L2}(\alpha)$ is the Green's function of a layer of thickness h_1 and effective permittivity $\epsilon_{\rm eqs} = (\epsilon_2\epsilon_1)/(\epsilon_2 - \epsilon_1)$ closed by an EW.

The partial Green's functions G_{Li} and g_{Li}, $i = 1, 2$ can be related to a set of PCs. With the field–trial–function formulation, we have

$$\begin{aligned} C_L &= \frac{1}{\pi V}\int_0^{+\infty} |\tilde{e}|^2 G_L \mathrm{d}\alpha \\ &\approx \frac{1}{\pi V}\int_0^{+\infty} |\tilde{e}|^2 G_{L1} \mathrm{d}\alpha + \frac{1}{\pi V}\int_0^{+\infty} |\tilde{e}|^2 G_{L2} \mathrm{d}\alpha \\ &\approx \frac{1}{\pi V}\int_0^{+\infty} |\tilde{e}_{L1}|^2 G_{L1} \mathrm{d}\alpha + \frac{1}{\pi V}\int_0^{+\infty} |\tilde{e}_{L2}|^2 G_{L2} \mathrm{d}\alpha \\ &= C_{L1a} + C_{L2a} \end{aligned} \tag{24}$$

where

- C_{L1a} is the capacitance of a semi-infinite half space of permittivity ϵ_2 topped by an MW;
- C_{L2a} is the capacitance of a layer of thickness h_1 and effective permittivity $\epsilon_{\rm eqp} = \epsilon_1 - \epsilon_2$ closed by an MW.

The accuracy of the approximation $C_L \approx C_{L1a} + C_{L2a}$ depends on two conditions. First, the Green's function should be accurately reproduced as the sum of the two partial contributions. Second, the field distribution of the complete structure should be close enough to the ones of the partial structures, i.e., $\tilde{e}(\alpha) \approx \tilde{e}_{L1}(\alpha) \approx \tilde{e}_{L2}(\alpha)$.

Consider now, the dual case, in which the elastance is given a variational formulation. In this case, we have

$$\begin{aligned} \frac{1}{C_L} &= \frac{1}{\pi Q_L}\int_0^{+\infty} |\tilde{\rho}|^2 g_L \mathrm{d}\alpha \\ &\approx \frac{1}{\pi Q_L}\int_0^{+\infty} |\tilde{\rho}|^2 g_{L1} \mathrm{d}\alpha + \frac{1}{\pi Q_L}\int_0^{+\infty} |\tilde{\rho}|^2 g_{L2} \mathrm{d}\alpha \\ &\approx \frac{1}{\pi Q_L}\int_0^{+\infty} |\tilde{\rho}_{L1}|^2 g_{L1} \mathrm{d}\alpha + \frac{1}{\pi Q_L}\int_0^{+\infty} |\tilde{\rho}_{L2}|^2 g_{L2} \mathrm{d}\alpha \\ &= \frac{1}{C_{L1b}} + \frac{1}{C_{L2b}} \end{aligned} \tag{25}$$

where

- C_{L1b} is the capacitance of a semi-infinite half space of permittivity ϵ_2 topped by an MW;
- C_{L2b} is the capacitance of a layer of thickness h_1 and effective permittivity $\epsilon_{\rm eqs} = (\epsilon_2\epsilon_1)/(\epsilon_2 - \epsilon_1)$ closed by an EW.

Regarding the accuracy of the approximation, the same remarks hold as in the previous case, with one more difficulty since the condition $\tilde{\rho}(\alpha) \approx \tilde{\rho}_{L1}(\alpha) \approx \tilde{\rho}_{L2}(\alpha)$ can be critical to satisfy as far as $\tilde{\rho}_{L2}$ is concerned. In fact, notice that, in this partial structure, an EW is introduced, which was not present in the original one; it follows that ρ and, therefore, ρ_{L2}, must satisfy the charge neutrality condition. This implies that the EW introduced should be floating rather than grounded; it also implies that ρ_{L2} will be fairly different from ρ if the EW is strongly coupled. Thus, the series decomposition is potentially less accurate than the parallel one, even though the Green's function approximations in the two cases are equivalent.

In conclusion, notice that, in the first case (PPC), accurate for $\epsilon_1 \geq \epsilon_2$, the total capacitance is expressed as the *parallel* of PCs C_{L1a} and C_{L2a}, in the second case (SPC, $\epsilon_1 \leq \epsilon_2$) as the *series* of C_{L1b} and C_{L2b}. Considering also the UL decomposition, the total capacitance will be finally expressed as

$$C \approx C_U + C_{L1a} + C_{L2a} \quad \epsilon_1 \geq \epsilon_2 \tag{26}$$

$$C \approx C_U + \frac{C_{L1b}C_{L2b}}{C_{L1b} + C_{L2b}} \quad \epsilon_1 \geq \epsilon_2. \tag{27}$$

B. Results

In this section, we present some results and validations. As a reference solution, we use, unless otherwise stated, a static spectral-domain method of moments with edge-singular (field or charge) basis functions; in all computations shown, we use $N = 6$ basis functions per slot/strip, which is enough to achieve numerical convergence (the result is excellent, already with one basis function in most cases). In the PC technique, all relevant capacitances are exactly computed through CM.

Fig. 5 shows the lower half-space capacitance of a CPW on a two-layer substrate (second layer air) as a function of the substrate thickness for several values of the dielectric-layer permittivity. The agreement between PPC and the exact value is good

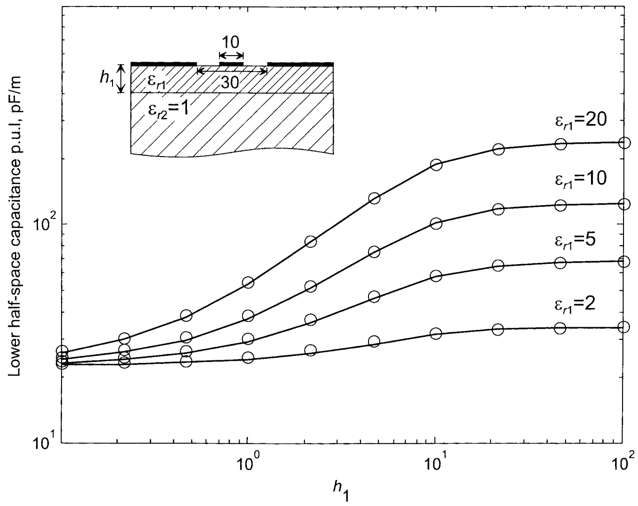

Fig. 5. Lower half-space capacitance as a function of h_1 for different values of ϵ_1; the supporting layer is air. Solid lines: PPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

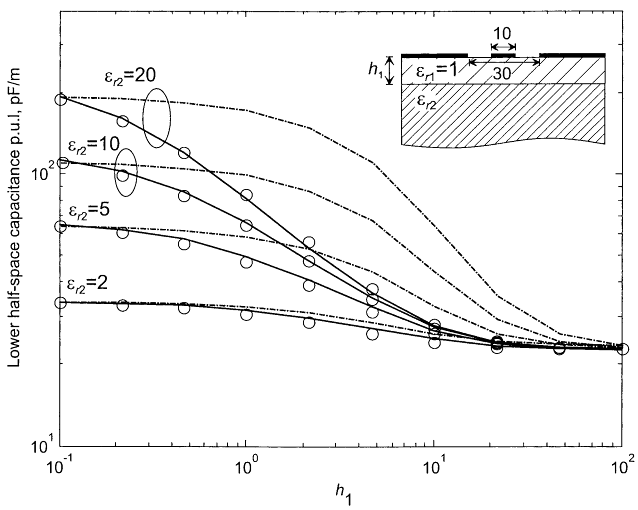

Fig. 6. Lower half-space capacitance as a function of h_1 for different values of ϵ_2; the first layer is air. Solid lines: SPC approach; dashed–dotted lines: PPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

in all cases, although the error somewhat increases when the permittivity ratio becomes less than two. In Fig. 6, we show results concerning the dual case, i.e., the lower half-space capacitance of a suspended CPW as a function of the air substrate thickness and of the underlying dielectric permittivity. While the SPC approach is accurate, the PPC result is (as expected) very different from the reference solution, unless $h_1 \to 0$ or $h_1 \to \infty$. However, it is interesting to note that the SPC solution, though generally satisfactory, is less accurate than the PPC solution in its own validity domain. In particular, when the permittivity ratio becomes less than 0.5, the error increases and the SPC solution is still better, but not much better, than the PPC solution. The same behavior can be seen in Fig. 7, where the permittivity of the supporting layer is varied on a wide range (the example and FEM reference solution are taken from [16, Fig. 11]); also in this case, the PPC and SPC approximations exhibit clear domains of

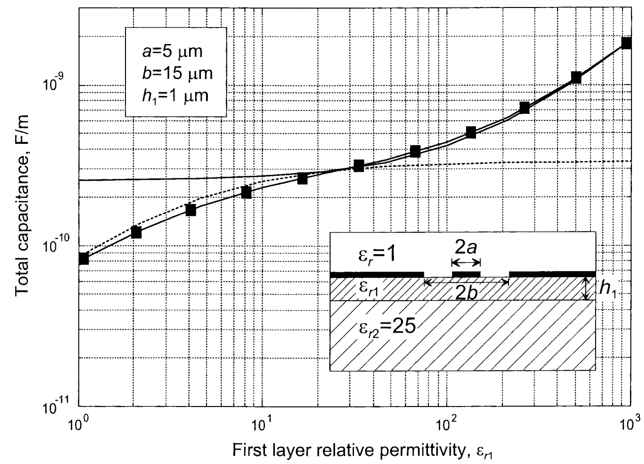

Fig. 7. Total capacitance as a function of the first layer permittivity. Solid line: PPC approach; dashed line: SPC approach; squares: exact (PBM and spectral domain).

validity, but the PPC is more accurate in its own domain than the SPC.

V. Extension of the PPC and SPC Approaches to an Arbitrary Number of Layers

The SPC and PPC approaches can, in principle, be extended to an arbitrary number of layers by approximating the lower half-space Green's function (G_L or g_L) as the sum of n_L terms. Namely, the lower half-space admittance Y^L (PPC) or impedance Z^L (SPC) will be approximated as

$$Y^L \approx \sum_{i=1}^{i=n_L-1} \left(\epsilon_{ri} - \epsilon_{r(i+1)}\right) \tanh\left[\left(\sum_{j=1}^{j=i} h_j\right)\alpha\right] + \epsilon_{rn_L} \tag{28}$$

$$Z^L \approx \sum_{i=1}^{i=n_L-1} \left(\frac{1}{\epsilon_{ri}} - \frac{1}{\epsilon_{r(i+1)}}\right) \tanh\left[\left(\sum_{j=1}^{j=i} h_j\right)\alpha\right] + \frac{1}{\epsilon_{rn_L}}. \tag{29}$$

Concerning the range of validity of such approximations, direct inspection reveals that if $\epsilon_{r1} \geq \epsilon_{r2} \ldots \geq \epsilon_{rn_L}$ or if $\epsilon_{r1} \leq \epsilon_{r2} \ldots \leq \epsilon_{rn_L}$, the Green's function is monotonic as a function of the spectral variable α and the PPC (28) or SPC (29) decomposition, respectively, turns out to be accurate. A Monte Carlo analysis suggests that the maximum error is around 10%, as in the two-layer case; however, the approximation is very accurate only when $\epsilon_{ri}/\epsilon_{r(i+1)} \gg 1$ (PPC case) or $\epsilon_{ri}/\epsilon_{r(i+1)} \ll 1$ (SPC case) $\forall i$ or, in both cases, when $\epsilon_{ri}/\epsilon_{r(i+1)} \to 1$. Since these conditions are more restrictive than in the two-layer case, the Green's function approximation slightly deteriorates with an increasing number of layers.

For generic (i.e., not increasing or decreasing) layer permittivities, the Green's function is not monotonic, and neither the PPC, nor the SPC, provide, in general, a satisfactory approximation.

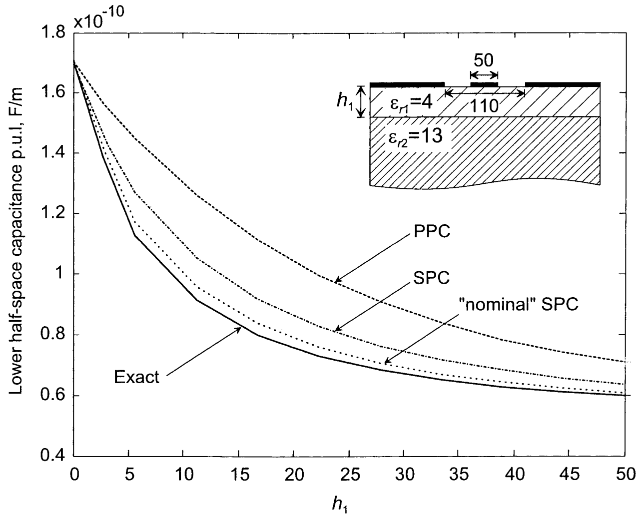

Fig. 8. Lower half-space capacitance of a CPW supported by a three-layer substrate with decreasing permittivity as a function of h_1 for different values of h_2. Solid lines: PPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

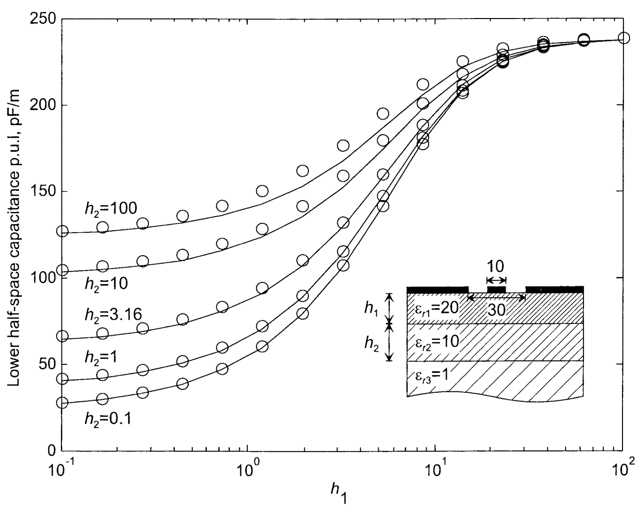

Fig. 9. Lower half-space capacitance of a CPW supported by a three-layer substrate with increasing permittivity as a function of h_1 for different values of h_2. Solid lines: SPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

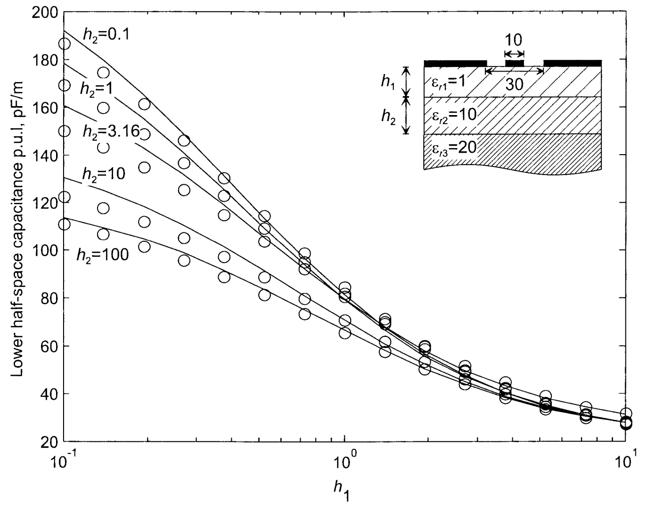

Fig. 10. Lower half-space capacitance of a CPW supported by a two-layer substrate with decreasing permittivity and backed by an MW as a function of h_1 for different values of h_2. Solid lines: PPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

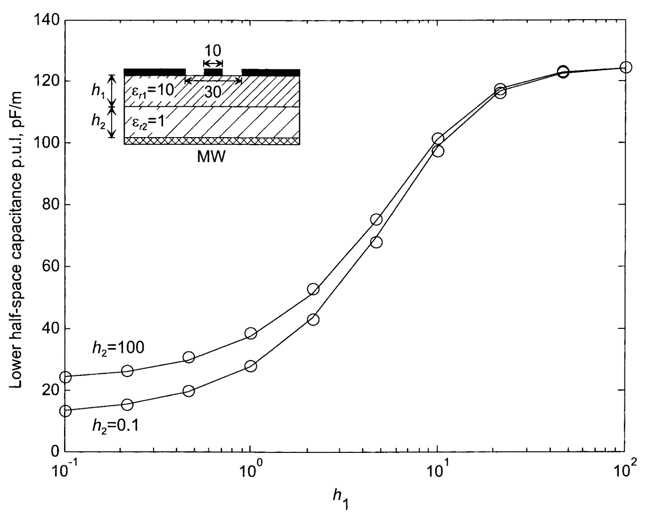

Fig. 11. Lower half-space capacitance of a CPW supported by a two-layer substrate with increasing permittivity and backed by an EW as a function of h_1 for different values of h_2. Solid lines: SPC approach; circles: exact (spectral domain). All dimensions are in micrometers.

The Green's function decomposition leads, in the allowed cases, to a parallel or series decomposition of the capacitance

$$\text{PPC}: \; C_\eta \approx \sum_{i=1}^{i=n_\eta-1} \left(\epsilon_{ri} - \epsilon_{r(i+1)}\right) C_{\text{MW}} \left(\sum_{j=1}^{j=i} h_j\right) + \epsilon_{rn_\eta} C_{\text{MW}}(\infty), \qquad \eta = L, U \tag{30}$$

$$\text{SPC}: \; \frac{1}{C_\eta} \approx \sum_{i=1}^{i=n_\eta-1} \left(\frac{1}{\epsilon_{ri}} - \frac{1}{\epsilon_{r(i+1)}}\right) \frac{1}{C_{\text{EW}}\left(\sum_{j=1}^{j=i} h_j\right)} + \frac{1}{\epsilon_{rn_\eta}} \frac{1}{C_{\text{EW}}(\infty)}, \qquad \eta = L, U \tag{31}$$

where $C_{\text{MW}}(h)$ is the capacitance *in vacuo* of a layer with thickness h closed by an MW, and $C_{\text{EW}}(h)$ is the capacitance *in vacuo* of a layer with thickness h closed by an EW (which is floating if the original structures does not include any conductor backing). The total capacitance results from UL decomposition as $C = C_L + C_U$.

As a numerical example, we present some comparisons concerning the PC approach applied to CPWs on three-layer substrates and on two-layer substrates backed by an MW or EW. The line dimensions a and b and the substrate permittivities are kept constant, while the substrate thicknesses h_1 and h_2 are varied. The refractive index ratio is large for one interface, but small for the other one considered. Figs. 8 and 9 concern substrates with decreasing and increasing permittivity, respectively; in the first case, the PPC is applied with fairly good agreement while, as already noticed for the two-layer case, the SPC approach leads to a slightly larger error. The same trend can be detected in Fig. 10, concerning a line backed by an MW (PPC approach), and in Fig. 11 (CPW backed by an EW SPC approach).

VI. Conclusions

The PC approach to the analysis of transmission lines (in particular, coplanar lines) on multilayered dielectric substrates has been reviewed, and a theoretical justification has been provided, for the first time, on the basis of a spectral-domain Green's function approach. Two general cases of capacitance decomposition have been addressed, the SPC and PPC approaches, which are appropriate for multilayered dielectrics with decreasing and increasing permittivity, respectively. Throughout this paper, the approximations involved both in the PPC and SPC methods have been stressed. Further work will concern a generalization of the approach, making use of series, parallel, or mixed decomposition.

Acknowledgment

The authors are indebted to S. Gevorgian, Chalmers University, Göteborg, Sweden, A. Sychev, Tomsk University, Tomsk, Russia, and S. Nightingale, ERA Technology, Surrey, U.K., for helpful discussions, and to F. Bertazzi, Politecnico di Torino, Turin, Italy, for providing the FEM computations presented in Fig. 2.

References

[1] B. C. Wadell, *Transmission Line Design Handbook*. Boston, MA: Artech House, 1991.

[2] E. S. Kochanov, "Parasitic capacitances in printed wiring of radio equipment," *Telecommun. Radio Eng.*, vol. 22, no. 7, pp. 129–132, 1967.

[3] V. Fouad-Hanna and C. Veyres, "Extension of the application of conformal mapping techniques to coplanar lines with finite dimensions," *Int. J. Electron.*, no. 48, pp. 47–56, Jan. 1980.

[4] G. Ghione and C. Naldi, "Parameters of coplanar waveguides with lower ground plane," *Electron. Lett.*, vol. 19, no. 18, pp. 734–735, Sept. 1983.

[5] ——, "Analytical formulas for coplanar lines in hybrid and monolithic MICs," *Electron. Lett.*, vol. 20, no. 4, pp. 179–181, Feb. 1984.

[6] R. Hoffmann, *Integrierte Mikrowellenschaltungen*. Berlin, Germany: Sringer-Verlag, 1983.

[7] G. Ghione and C. Naldi, "Coplanar waveguides for MMIC applications: Effect of upper shielding, conductor backing, finite-extent ground planes, and line-to-line coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 260–267, Mar. 1987.

[8] S. S. Bedair and I. Wolff, "Fast and accurate analytic formulas for calculating the parameters of a general broadside-coupled coplanar waveguide for (M)MIC applications," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 843–850, May 1989.

[9] ——, "Fast, accurate and simple approximate analytic formulas for calculating the parameters of supported coplanar waveguides for (M)MIC's," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 41–48, Jan. 1992.

[10] J. Svačina, "A simple quasistatic determination of basic parameters of multilayer microstrip and coplanar waveguide," *IEEE Microwave Guided Wave Lett.*, vol. 2, pp. 385–387, Oct. 1992.

[11] S. S. Gevorgian, "Basic characteristics of two-layered substrate coplanar waveguides," *Electron. Lett.*, vol. 30, no. 15, pp. 1236–1237, July 1994.

[12] S. S. Gevorgian, P. L. J. Linner, and E. L. Kollberg, "CAD models for shielded multilayered CPW," *IEEE Trans. Microwave Theory Tech.*, vol. 3, pp. 772–779, Apr. 1995.

[13] E. Chen and S. Y. Chou, "Characteristics of coplanar transmission lines on multilayer substrates: Modeling and experiments," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 939–945, June 1997.

[14] N. H. Zhu, E. Y. B. Pun, and J. X. Li, "Analytical formulas for calculating the effective dielectric constant of coplanar lines for OIC applications," *Microwave Opt. Technol. Lett.*, vol. 9, no. 4, pp. 229–232, July 1995.

[15] N. H. Zhu, Y. B. Pun, and P. S. Chung, "Field distribution in supported coplanar lines using conformal mapping techniques," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 1493–1496, Aug. 1996.

[16] E. Carlsson and S. S. Gevorgian, "Conformal mapping of the field and charge distributions in multilayered substrate CPW's," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1544–1552, Aug. 1999.

[17] A. N. Sychev and A. Y.A. Y. Shevtsov, "Design of slablines on multilayer dielectrics," in *Int. Microwave and Millimeter-Wave Technology Conf.*. Beijing, China, 1998, pp. 581–586.

[18] G. Ghione, M. Goano, G. L. Madonna, G. Omegna, M. Pirola, S. Bosso, D. Frassati, and A. Perasso, "Microwave modeling and characterization of thick coplanar waveguides on oxide-coated lithium niobate substrates for electrooptical applications," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2287–2293, Dec. 1999.

[19] A. N. Sychev, *Controllable Microwave Circuits Based on Multimode Stripline Structures* (in Russian). Tomsk, Russia: Tomsk State Univ., 2001.

[20] ——, "A model of the shielded multiconductor microstrip lines on double-layer substrate—A novel approach," in *Microwave Electronics: Measurements, Identification, Application Conf.*, Novosibirsk, Russia, 2001, pp. 77–81.

[21] E. Yamashita, "Variational method for the analysis of microstrip like transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 251–256, Apr. 1968.

[22] M. Horno, F. L. Mesa, F. Medina, and R. Marques, "Quasi-TEM analysis of multilayered, multiconductor coplanar structures with dielectric and magnetic anisotropy including substrate losses," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1059–1068, Aug. 1990.

[23] R. Crampagne, M. Ahmadpanah, and J. L. Guiraud, "A simple method for determining the Green's function for a large class of MIC lines having multilayered dielectric structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 82–87, Feb. 1978.

[24] C. P. Wen, "Coplanar waveguide: A surface strip transmission line suitable for nonreciprocal gyromagnetic device applications," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 1087–1090, Dec. 1969.

Giovanni Ghione (M'87–SM'94) received the Laurea degree in electronics engineering from the Politecnico di Torino, Turin, Italy, in 1981.

From 1983 to 1987, he was a Research Assistant with the Politecnico di Torino. From 1987 to 1990, he was an Associate Professor with the Politecnico di Milano, Milan, Italy. In 1990, he joined the University of Catania, Catania, Italy, as Full Professor of electronics. Since 1991, he has been a Full Professor with the II Faculty of Engineering, Dipartimento di Elettronica, Politecnico di Torino. Since 1981, he has been engaged in Italian and European research projects (ESPRIT 255, COSMIC, and MANPOWER) in the field of active and passive microwave computer-aided design (CAD). His current research interests concern the physics-based simulation of active microwave and opto-electronic devices, with particular attention to noise modeling, thermal modeling, and active device optimization. His research interests also include several topics in computational electromagnetics, including coplanar component analysis. He has authored or coauthored over 150 papers and book chapters in the above fields.

Prof. Ghione is member of the Associazione Elettrotecnica Italiana (AEI). He is an Editorial Board member of the IEEE Transactions on Microwave Theory and Techniques.

Michele Goano (M'98) received the Laurea and Ph.D. degrees in electronics engineering from the Politecnico di Torino, Turin, Italy, in 1989 and 1993, respectively.

In 1994 and 1995, he was a Post-Doctoral Fellow with the Département de Génie Physique, École Polytechnique de Montréal, Montréal, QC, Canada. In 1996, he joined the faculty of the Dipartimento di Elettronica, Politecnico di Torino. He was a Visiting Scholar with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, and with the Department of Electrical and Computer Engineering, Boston University, Boston, MA. He is currently involved in research on coplanar components, optical modulators, and wide-bandgap semiconductor devices.