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Abstract

We present an instability study that exploits, as basic-flow to be perturbed, a new asymptotic expansion solution for the intermediate and far 2D bluff body wake (Belan & Tordella, 2002; Tordella & Belan, 2003). The solution comes from the matching between an inner Navier-Stokes flow and an outer Navier-Stokes flow. The adoption of the Navier-Stokes model also in the outer region is a new approach, which takes into account the fact that the nonlinear convection and the diffusion are comparable in the lateral far field. The matching criteria are based on the vorticity, entrainment and pressure gradient matching. This solution yields an accurate representation of the cross momentum and pressure distributions. These last are properties of the mean-flow never included – to our knowledge – into a linear analysis of the convective instability that is framed on the parametric Orr-Sommerfeld problem. Here, through a spatio-temporal multiscaling which yields to an inhomogeneous Orr-Sommerfeld equation, the hypothesis of near parallelism is explored. The variation, with the streamwise coordinate and the Reynolds number, R, of the instability characteristics is obtained. This allows to discuss the limits of the near parallel theory, see, e.g., Tordella & Belan, 2004, where the quasi-steady analysis of the instabilty was performed through a multiscaling limited to the space variable, and the review by Huerre & Monkewitz, 1990).

In the wake flow solution here considered as basic flow (Belan & Tordella, 2002; Tordella & Belan, 2003), the lateral decay results to be algebraic at high orders in the inner expansion solution. Using the quasi-similar transformation $x = x$, $\eta = x^{-1/2}y$ the velocity components, up to $O(x^{-3/2})$, can be written as

$$u = 1 + x^{-1/2}\phi_1(\eta) + x^{-1}\phi_2(\eta) + x^{-3/2}\phi_3(\eta) \quad (1)$$

$$v = x^{-1}\chi_1(\eta) + x^{-3/2}\chi_2(\eta) \quad (2)$$

The multiscale approach, that introduces the slow spatial and temporal variables, $x_1 = \varepsilon x$, $t_1 = \varepsilon t$ with $\varepsilon = 1/R$, yields $\eta = (Rx_1)^{-1/2}y$, $u = \partial_y\Psi = u_0(x_1, y) + \varepsilon u_1(x_1, y) + \cdots$, $v = -\partial_x\Psi = -\varepsilon\partial_{x_1}\Psi = \varepsilon v_1(x_1, y) + \cdots$, where Ψ is the stream function of the mean flow. This leads to the the perturbation hypothesis

$$\psi = [\varphi_0(x_1, y, t_1) + \varepsilon\varphi_1(x_1, y, t_1) + \cdots]\, \mathrm{e}^{i\theta(x,t;\varepsilon)}. \quad (3)$$

According to the Whitham theory (Whitham, 1974), $\partial_x\theta = h_0 = k_0 + is_0$, $\partial_t\theta = -\sigma = -(\omega + ir)$, where h_0 is the complex dimensionless wave number and σ is the complex dimensionless pulsation. The imaginary parts of h_0 and σ are the spatial and temporal growth rates, respectively. In terms of x_1 and θ the spatial and temporal derivatives transform like

$$\partial_x \to h_0\partial_\theta + \varepsilon\partial_{x_1}, \quad \partial_y \to \partial_y, \quad \partial_t \to -\sigma_0\partial_\theta + \varepsilon\partial_{t_1}. \quad (4)$$

Following Bouthier (1972), by applying the previous derivative transformations to the linearized equations for the perturbation of the stream function, a hierarchy of ODE is obtained. The zero order equation is the parametric Orr-Sommerfeld equation, where x_1 and the Reynolds number$R$ are parameters,

$$\mathcal{A}\varphi_0 = \sigma\mathcal{B}\varphi_0 \ , \quad (5)$$

with $\mathcal{A} = \left\{(\partial_y^2 - h_0^2)^2 - ih_0R\left[u_0(\partial_y^2 - h_0^2) - u_0''\right]\right\}$, $\mathcal{B} = -iR(\partial_y^2 - h_0^2)$. By writing $\varphi_0(x_1, t_1, y) = A(x_1, t_1)\zeta_0(x_1, y)$, (A: slow spatio-temporal modulation), (5) remains parametric in x_1. The unknown amplitude A describes the transmission of the instability wave from one near parallel region to the next and is determined at the next order. The first order inhomogenous multiscaling equation is:

$$\mathcal{A}\varphi_1 = \sigma\mathcal{B}\varphi_1 + \mathcal{M}\varphi_0. \quad (6)$$

The operator $\mathcal{M}$ is

$$\mathcal{M} = \Big\{ \left[R(2h_0\sigma - 3h_0^2u_0 - u_0'') + 4ih_0^3\right]\partial_{x_1} + (Ru_0 - 4ih_0)\partial^3_{x_1yy} - Rv_1(\partial_y^3 - h_0^2\partial_y) + Rv_1''\partial_y + ih_0R\left[u_1(\partial_y^2 - h_0^2) - u_1''\right] + R(\partial_y^2 - h_0^2)\partial_{t_1}\Big\}. \quad (7)$$

Notice that $\mathcal{M}$ is function of the basic flow, in particular of the transversal momentum v_1 and thus of the entrainment, as well of the zero order dispersion relation and eigenfunction. To avoid secular terms in the solution of (6), a solvability condition has to be satisfied. This condition requires that the inhomogeneous term is orthogonal to every solution of the adjoint homogenous problem. In short form, by means of the substitution $A(x_1, t_1) = e^{a(x_1,t_1)}$, Bouthier (1972), and going back to the original coordinates:

$$\partial_t a + K_2 \partial_x a + \varepsilon K_1 = 0 \tag{8}$$

where coefficients K_1 e K_2 are not singular. This modulation equation is solved numerically, specifying the accessory conditions on the domain of validity of the perturbative theory (from a few body-scales d downstream of the body, $x = x_b$, to the far field, $x_f \gg d$. Having one boundary condition only, it is better to use it to impose the far field uniformity to the solution. With regards to the initial condition, it is opportune to write $a = \mathrm{e}\theta_1$. By recalling that $\varphi_1(x_1, y, t_1) = A(x_1, t_1)\zeta_1(x_1, y)$, and that the complete solution (order 0 + order 1) is

$$\psi = (\varphi_0 + \varepsilon\varphi_1)e^{i\theta} = A((x_1, t_1)(\zeta_0 + \varepsilon\zeta_1)e^{i\theta} = (\zeta_0 + \varepsilon\zeta_1)e^{i\theta + i\theta_1} \ , \tag{9}$$

the complete phase will then be $\Theta = \theta + \theta_1$. In this way the initial condition can be given in terms of a distribution of phases. We considered the phase distribution corresponding to zero order spatial x sequence of saddle points, $\partial_{h_0}\sigma = 0$. Due to the multiscaling, the wave number is $h = \partial\Theta/\partial x = h_0 \partial\Theta/\partial\theta + \varepsilon\partial\Theta/\partial x_1 = h_0 + \varepsilon\partial\theta_1/\partial x_1$ and the pulsation is $-\sigma = -\partial\Theta/\partial t = -\sigma_0\partial\Theta/\partial\theta + \varepsilon\partial\theta_1/\partial t_1 = -\sigma_0 + \varepsilon\partial\theta_1/\partial t_1$. The first order correction of the comples instability characterisitcs are thus obtained as $h_1 = \partial\theta_1/\partial x_1$ and $\sigma_1 = -\partial\theta_1/\partial t_1$. In fig.1 the distribution of the module of the coefficients of the modulation equation is shown. Fig.2 shows the first order correction of the instability charactheristics, which increases with R and moving towards the body. From this, fixing a threshold for the maximum accettable corrections proportional to ε, it is possible to discuss both the limits of validity of the locally parallel theory, and the portion of the wake where the extension to the second order can be consider meaningful. It is observed, that even if the position ($x \approx 10$) where the absolute instability ($r > 0$) is reached would be accettable on the base of the actual laboratory knowledges, the perturbative theory there is no more reliable.

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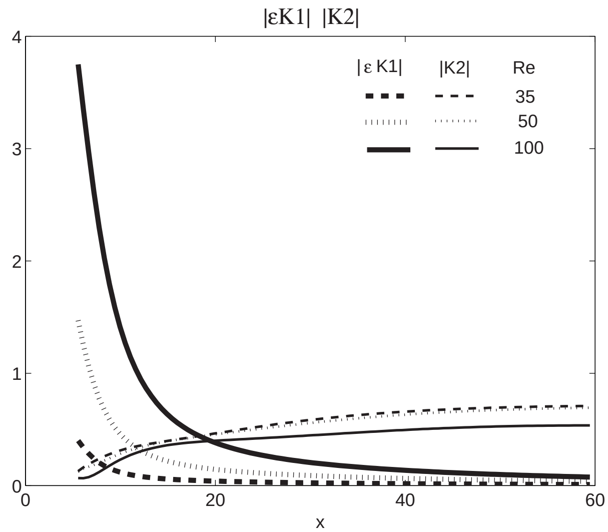

Figure 1: Coefficients of the modulation equation.

List of results

– The position where the wave number and spatial growth rate of the local saddle point coincide with those of the pertubation is called x_0: x_0 is thus the parameter of the perturbation, let us call it the center of the perturbation.

– R is the flow control parameter

∘ Moving downstream the center of the perturbation, the absolute instability condition is never reached in the nearby of the near wake, even rising R to the value 100.

• Placing x_0 close to the near wake $x_0 = 10$, the first order (supposing to accept the first order correction without restriction) yields absolute instability (AI, i.e. r=0) at $x = 15$ and R=100, nearly AI at $x \sim 6$ at R=50 (extrapolate data)

∘ k variation:

large in the left region of the domain (next to the near wake), $k(x)$, when R=const, decreases with x_0, $k(x)$, when x_0 =const, increases with R.

• ω, pulsation:

increases with R, decreases with x_0

the order one correction are larger close to the near wake, their intensity grows with R

∘ regions of minimal (less) stability (regions about the points $x = x_{ls}$ where $r = r_{max}$)

the maximum of $r(x)$ is always placed upstream with respect to the perturbation center (x_0)

when x_0 is low, in the points of less stability, r_{max} grows with R

when R=35 and x_0 rises, the position of less stability settle near $x \sim 15$

for higher R, the less stable points are placed in locations where $x < x_0$ (e.g., at R=50, $x_{ls} \sim 10$ for $x_0 \sim 21$, $x_{ls} \sim 18$ for $x_0 \sim 40$; at R=100, $x_{ls} \sim 14$ for $x_0 \sim 21$, $x_{ls} \sim 25$ for $x_0 \sim 40$)

the maximum values of r decreases with x_0 and, beyond a point absolute maximum (presumably near $R_{cr} = 40$,) decreases with R

•• analysis of the limits of the near-parallel stability theory.

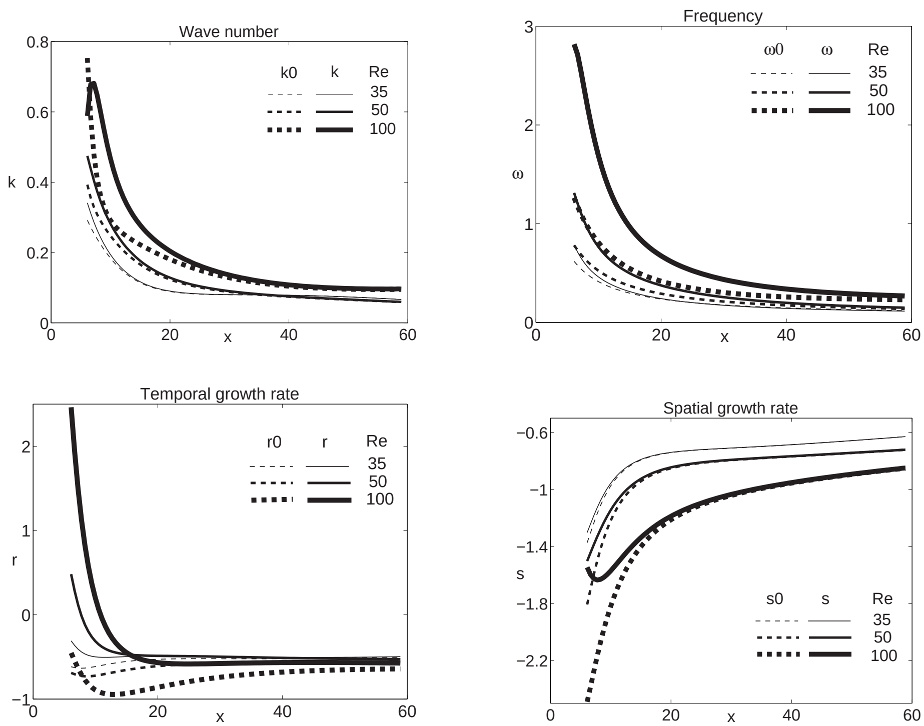

Figure 2: Instability characterisitcs downstream of the body, x is the distance from the body; - - - zero order, —— first order