New Higher Order Two-dimensional Singular Elements for FEM and MOM Applications

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Roberto D. Graglia, and Guido Lombardi *

Abstract — **A procedure to obtain in a unified and consistent manner singular vector bases complete to arbitrarily high order has been obtained for curved triangular and quadrilateral elements. These vector basis functions are fully compatible with the standard, high-order regular vector functions used in adjacent elements. The curl (divergence) conforming singular functions guarantee tangential (normal) continuity along the edges of the elements allowing for the discontinuity of normal (tangential) components, adequate modelling of the curl (divergence), and removal of spurious modes (solutions).**

1 Introduction.

Numberless structures of practical engineering interest contain conducting or penetrable edges and, in the vicinity of these edges, the surface charge [1]-[3] and the field behavior can be singular [3]-[6].

The best approach to numerically model this complex local behavior is to introduce and use singular functions able to precisely model the singular edge behavior of fields and currents. As far as the FEM treatment of edge singularities is concerned, important contributions to the development of scalar and vector expansion functions incorporating the singular behavior are provided in [7]-[14], whereas the continuous interest in incorporating edge conditions in MoM solutions dates back to the mid seventies of last century [15], with more recent contributions available in [16]-[18].

New curl- and divergence-conforming singular, high-order vector bases on curved two-dimensional domains will be discussed at the Conference. The bases are directly defined in the parent domain without introducing any intermediate reference frame, differently to what has been done by other authors [7, 12, 13]. Our bases incorporate the edge conditions and are able to approximate the unknown fields in the neighborhood of the edge of a wedge for any order of the singularity coefficient ν, that is supposed given and known *a priori*. The wedge can be penetrable in the curl-conforming case, while it is supposed impenetrable (metallic) in the divergence conforming case. Our curl (divergence) conforming singular bases are compatible with standard high-order interpolatory vector functions [19] in adjacent elements and guarantee tangential (normal) continuity along the edges of the elements allowing for the discontinuity of normal (tangential) components, adequate modelling of the curl (divergence), and removal of spurious modes (solutions).

A thorough investigation of the previous literature has shown that the fundamental question to be raised before deriving singular vector bases regards the number of basis functions that define the *lowest-order* singular bases. For example, the six triangular basis functions given in [12] are compatible with regular first-order curl-conforming elements adjacent to the edge opposite to the sharp-edge vertex. In this case, however, zeroth-order regular elements cannot be made adjacent to singular elements. On the contrary, [13] introduces eight basis functions to define a singular triangular element compatible to adjacent first order elements. Once again, zeroth-order regular elements cannot be made adjacent to this singular element. For triangular elements, the lowest number of curl-conforming functions required to achieve completeness and singular conformity to adjacent first-order elements could be proved to be equal to eleven, whereas six vector functions are at least necessary for completeness and singular conformity to adjacent curl-conforming zeroth-order elements.

2 Curl conforming functions.

We investigated several ways to derive *singular* and *complete* lowest-order vector bases. We define singular curl-conforming bases to be of the *lowest-order* when the following properties are fulfilled:

1. the basis set is complete just to the regular zeroth order, and the curl of the bases is also complete to regular zeroth order;
2. the element is fully compatible to adjacent zeroth-order regular elements attached to its non-singular edges, and to adjacent singular elements of the same order attached to the other edges;
3. the basis functions can model the static, $\rho^{\nu-1}$ singular behavior of the transverse field *in the neighborhood* of the sharp-edge (first term of Meixner's series [4]);

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4. the basis functions are able to model a non-singular field with curl that vanishes at the edge of the wedge as ρ^ν ($\nu \neq 1$ and not integer).

3 Divergence conforming functions.

Singular divergence conforming functions are useful to model the surface current distribution on impenetrable wedges. The wedge faces in the neighborhood of the edge profile should be meshed by using edge singularity quadrilaterals and/or two types of singularity triangles: the edge (e) and the vertex (v) singularity triangle, with local edge numbering schemes shown in Figure 1. One has no interest in considering vertex singularity quadrilaterals since the only element-filler required to mesh in the neighborhood of the edge profile is the vertex singularity triangle.

The local edge-numbering scheme sketched in Figure 1 has been chosen so to associate the $i \pm 1$ labels to the element edges departing from the edge profile.

Singular divergence conforming bases are defined to be of the *lowest-order* when the following properties are fulfilled:

1. the basis set is complete just to the regular zeroth order and the divergence of the bases is also complete to regular zeroth order;
2. the element is fully compatible to adjacent zeroth-order regular elements attached to its non-singular edges, and to adjacent singular elements of the same order attached to the other edges;
3. the basis functions can model the $\rho^{\nu-1}$ singular behavior of the current and charge density *in the neighborhood* of the sharp-edge, where $\rho \propto \chi$ is the distance from the sharp-edge profile;
4. the basis functions can model the normal component of the sharp-edge current density that vanishes at the sharp-edge as ρ^ν ($\nu \neq 1$ and not integer).

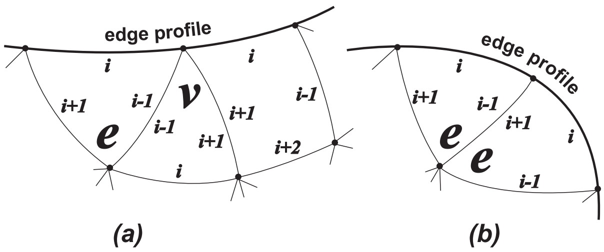

Figure 1: a) Local edge-numbering scheme used for edge singularity quadrilaterals and edge ($\boldsymbol{e}$) and vertex ($\boldsymbol{v}$) singularity triangles. Notice that the element edges $i \pm 1$ always depart from the edge profile. b) Although two edge singularity triangles can have an edge in common, the basis functions cannot model a corner singularity.

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