

POLITECNICO DI TORINO  
Repository ISTITUZIONALE

Tutorial lecture on Characterization and Macromodeling of 3D Interconnects

*Original*

Tutorial lecture on Characterization and Macromodeling of 3D Interconnects / GRISET TALOCIA, Stefano. - ELETTRONICO. - (2004). (Intervento presentato al convegno 8th IEEE Workshop on Signal Propagation on Interconnects (SPI) tenutosi a Heidelberg (Germany) nel May 9-12, 2004).

*Availability:*

This version is available at: 11583/1412861 since: 2015-07-15T07:09:37Z

*Publisher:*

*Published*

DOI:

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)



# Characterization and Macromodeling of 3D Interconnects

**S. Grivet-Talocia**

Politecnico di Torino, Italy

[grievet@polito.it](mailto:grievet@polito.it)

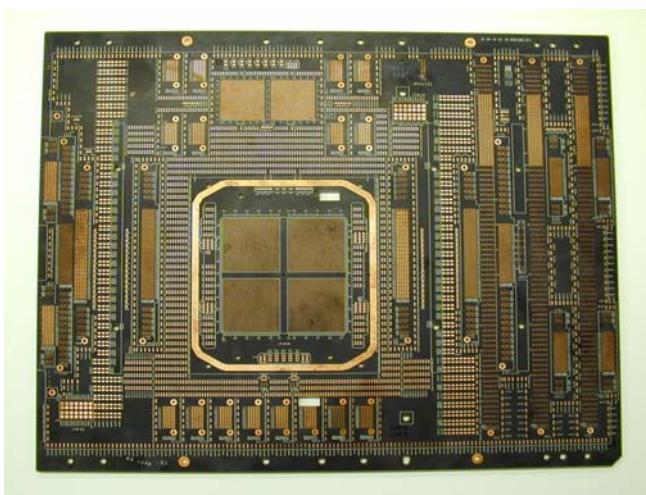
<http://www.eln.polito.it/research/emc>

**EMC**  
GROUP

S. Grivet-Talocia, SPI tutorial, 9 May 2004



## Introduction



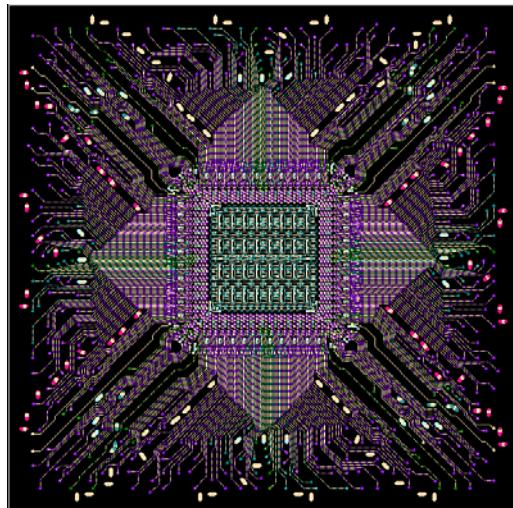
S. Grivet-Talocia, SPI tutorial, 9 May 2004

2

**EMC**  
GROUP



## Introduction



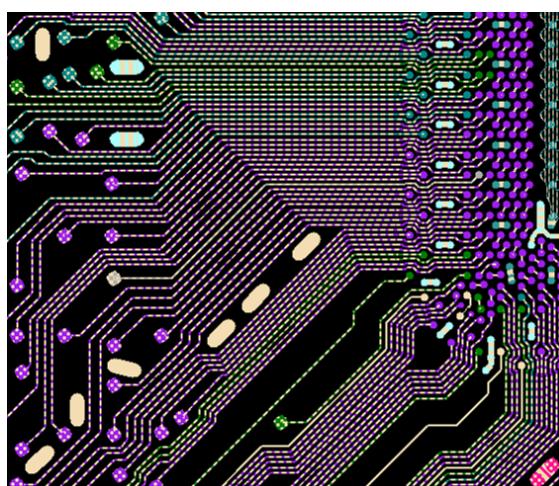
S. Grivet-Talocia, SPI tutorial, 9 May 2004

3

**E***M***C**  
GROUP



## Introduction



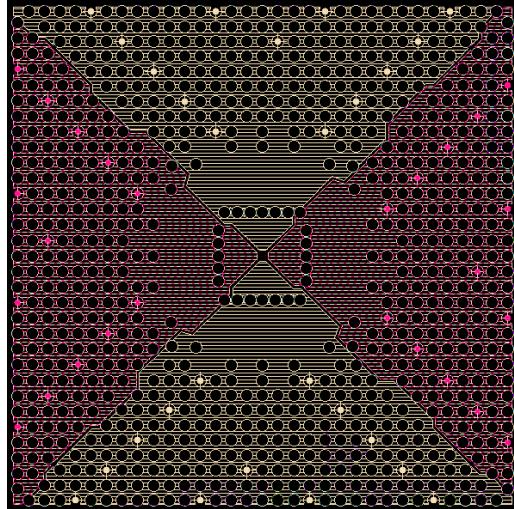
S. Grivet-Talocia, SPI tutorial, 9 May 2004

4

**E***M***C**  
GROUP



## Introduction



S. Grivet-Talocia, SPI tutorial, 9 May 2004

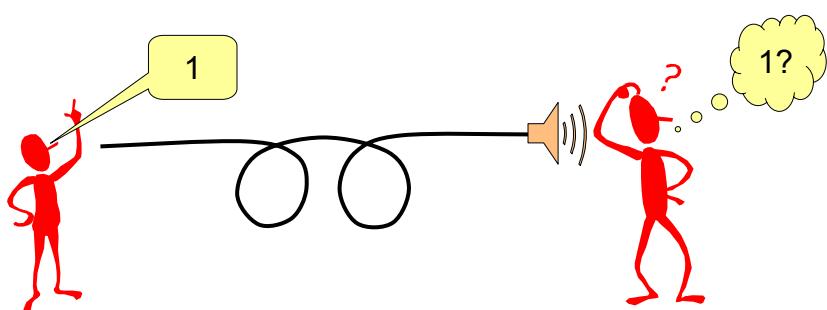
5

**E***M***C**  
GROUP



## Introduction

**High-speed Data transmission requires  
integrity of the signals  
thru lines, bends, vias, connectors, ...**



S. Grivet-Talocia, SPI tutorial, 9 May 2004

6

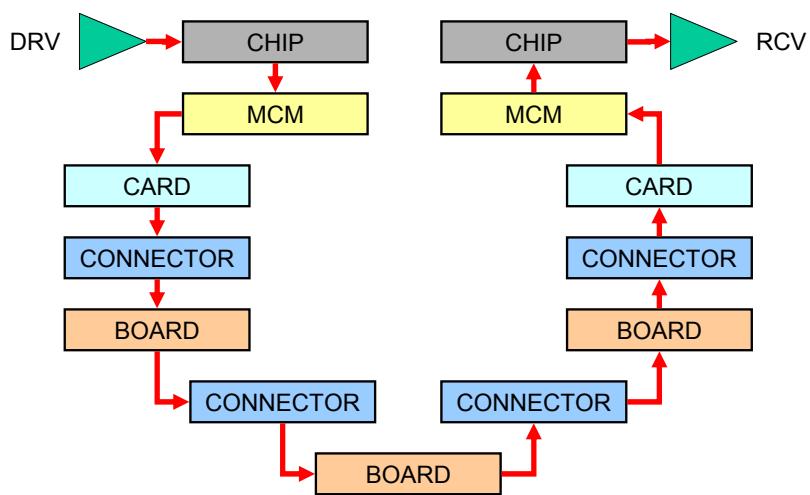
**E***M***C**  
GROUP



## An example

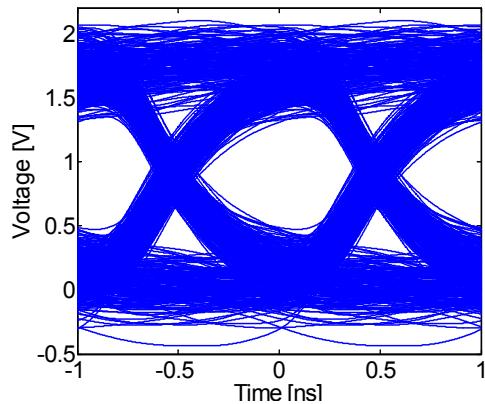


## An example





## An example



S. Grivet-Talocia, SPI tutorial, 9 May 2004

9

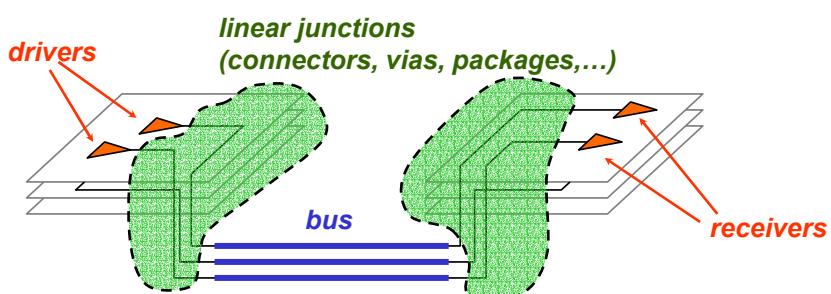
EMC  
GROUP



## Introduction

Signal Integrity issues in high-speed digital systems

Crosstalk, couplings, reflections, losses, dispersion, attenuation, resonances, ground noise, nonlinear effects, radiation, EMI, ...



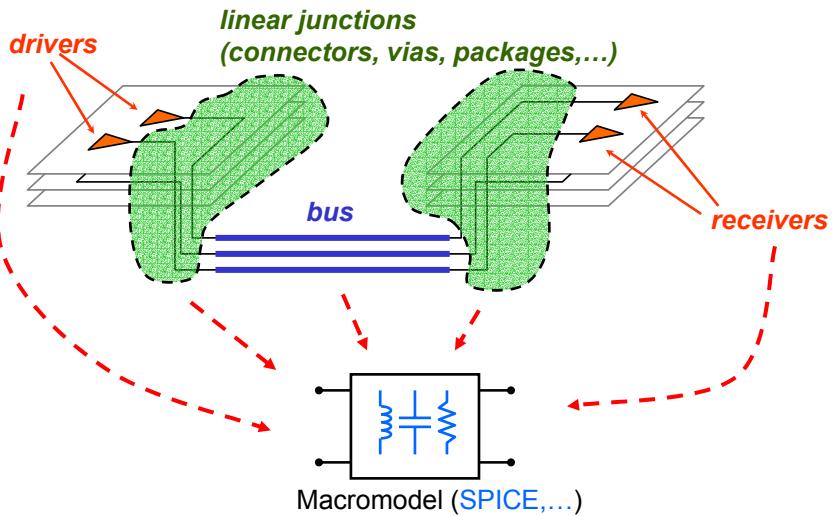
S. Grivet-Talocia, SPI tutorial, 9 May 2004

10

EMC  
GROUP



## Introduction



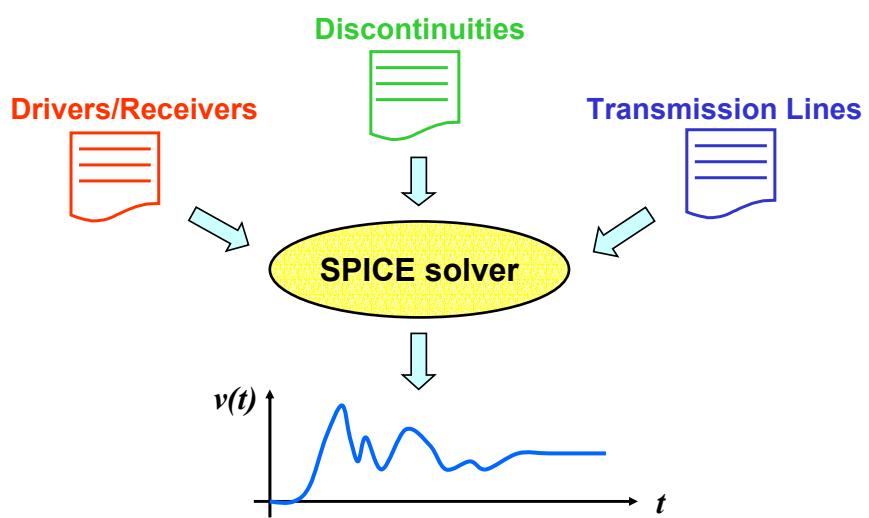
S. Grivet-Talocia, SPI tutorial, 9 May 2004

11

EMC  
GROUP



## “Lego” approach



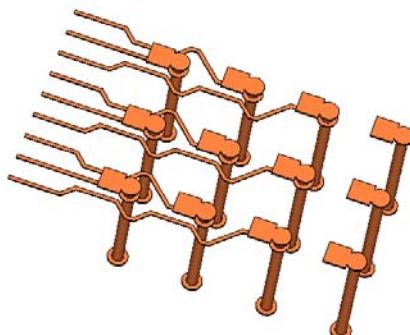
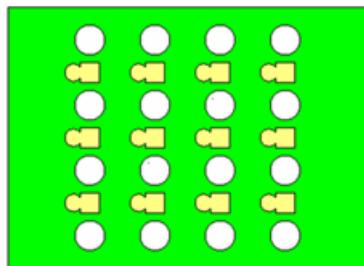
S. Grivet-Talocia, SPI tutorial, 9 May 2004

12

EMC  
GROUP



## 3D Interconnects



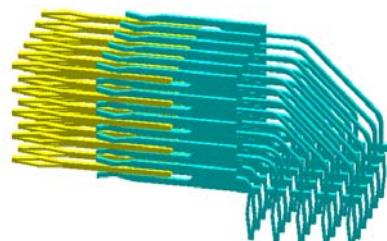
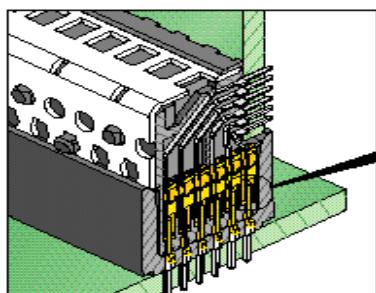
S. Grivet-Talocia, SPI tutorial, 9 May 2004

13

**E***M***C**  
GROUP



## 3D Interconnects



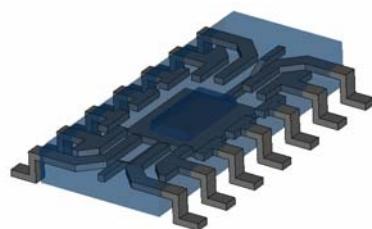
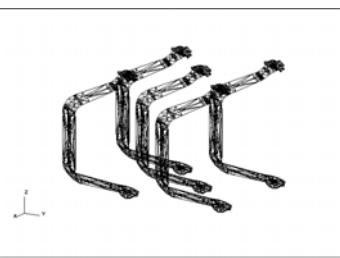
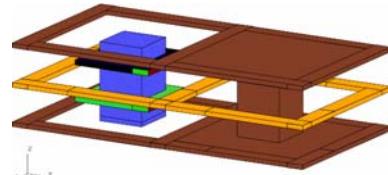
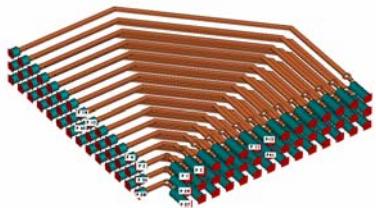
S. Grivet-Talocia, SPI tutorial, 9 May 2004

14

**E***M***C**  
GROUP



## 3D Interconnects



S. Grivet-Talocia, SPI tutorial, 9 May 2004

15

**E**<sup>MC</sup>  
GROUP



## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

S. Grivet-Talocia, SPI tutorial, 9 May 2004

16

**E**<sup>MC</sup>  
GROUP



## Macromodeling approaches

Macromodeling of 3D interconnects for Signal Integrity

1. Capture physical effects leading to signal degradation
  - Must take into account **3D electromagnetic fields**
  - **Simulation or measurement**
  - Many different characterizations are possible!
2. Use this information to build a macromodel
  - **Many macromodeling approaches available!**



## Macromodeling approaches

### Characterization via equations

Discretization of Maxwell full-wave equations

**Model Order Reduction** methods: build a simplified model from an existing (large) one

### Characterization via port responses (Black Box)

Time or frequency domain

Simulated or measured

**Reduced-Order Model Identification** methods: build a model from samples of the port responses



## Macromodeling approaches

Main goal of all (lumped) macromodeling methods:  
**produce a rational approximation**



$$\mathbf{H}_q(s) = \mathbf{H}_{\infty} + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

### Lumped circuits

- have rational transfer functions
- are governed by Ordinary Differential Equations

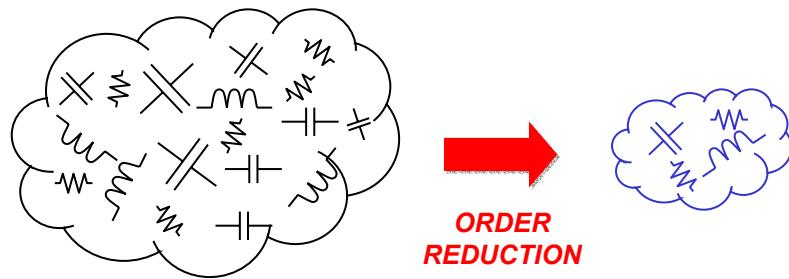


## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



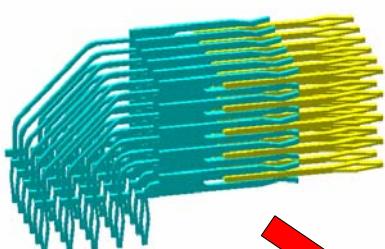
## Moder Order Reduction



$$\mathbf{H}(s) \approx \mathbf{H}_q(s)$$

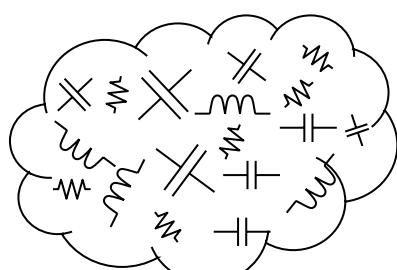


## Possible scenarios



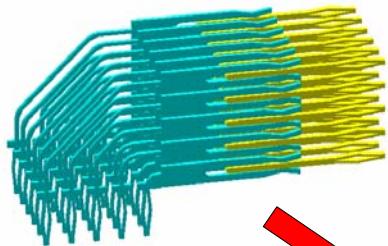
Large circuit

PEEC (Partial Element  
Equivalent Circuit)  
discretization





## Possible scenarios



Large system

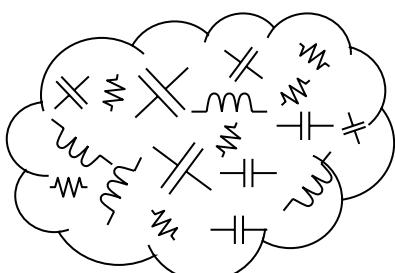
Spatial discretization of  
Maxwell equations  
(FDTD, FEM, MoM, ...)

Set of ODEs  
(Ordinary  
Differential  
Equations)

E<sup>M</sup>C  
GROUP



## Possible scenarios



Set of ODEs  
(Ordinary  
Differential  
Equations)

MNA

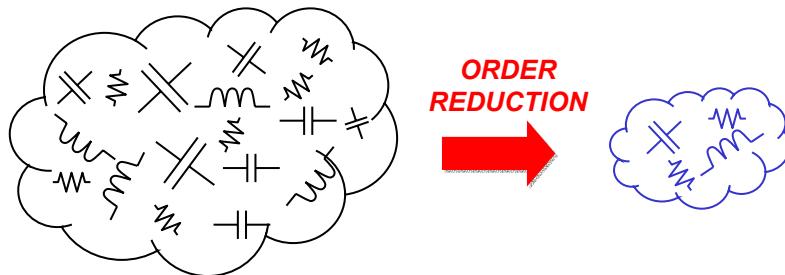
$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T \mathbf{x} \end{cases}$$

$$\mathbf{H}(s) = \mathbf{L}^T (\mathbf{G} + s\mathbf{C})^{-1} \mathbf{B}$$

E<sup>M</sup>C  
GROUP



## Approximation via moment matching



$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_N s^N + \cdots$$

$$\mathbf{H}_q(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \cdots + \mathbf{M}_q s^q + \cdots$$

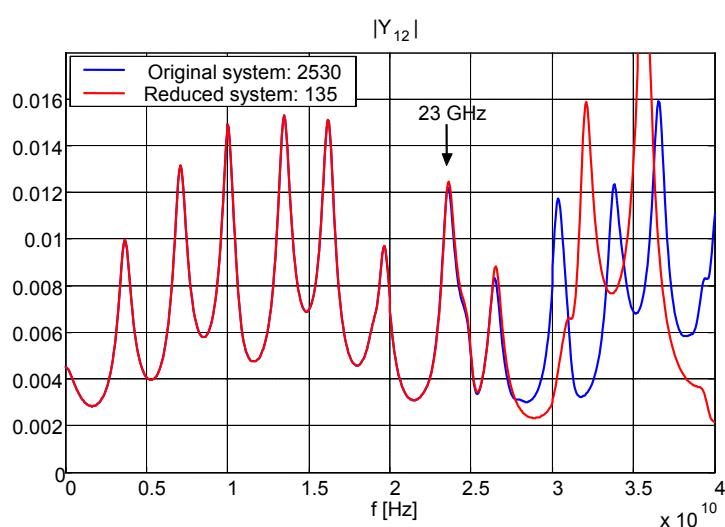
S. Grivet-Talocia, SPI tutorial, 9 May 2004

25

**E***M***C**  
GROUP



## Moment matching: an example



S. Grivet-Talocia, SPI tutorial, 9 May 2004

26

**E***M***C**  
GROUP



## Moments

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T \mathbf{x} \end{cases} \quad \begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T \mathbf{x} \end{cases} \quad \begin{matrix} \mathbf{A} = -\mathbf{G}^{-1}\mathbf{C} \\ \mathbf{R} = \mathbf{G}^{-1}\mathbf{B} \end{matrix}$$

$$\mathbf{H}(s) = \mathbf{L}^T (\mathbf{I} - s\mathbf{A})^{-1} \mathbf{B}\mathbf{u}$$

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \dots$$

↓              ↓              ↓  
**Moments**     $\mathbf{L}^T \mathbf{R}$          $\mathbf{L}^T \mathbf{A} \mathbf{R}$          $\mathbf{L}^T \mathbf{A}^2 \mathbf{R}$



## Moment matching techniques



**Explicit**       $\mathbf{M}_i = \mathbf{L}^T \mathbf{A}^i \mathbf{R} \rightarrow \mathbf{H}_q(s)$

**Asymptotic Waveform Evaluation (AWE)**

**Pade` Approximations**

**Complex Frequency Hopping (CFH)**

- Good theoretical properties, convergence
- Bad numerical properties, intrinsic ill-conditioning due to
  - Moment generation
  - Moment matching



## Moment matching techniques



### Implicit

#### Krylov subspace projection methods

- Same information stored in moments
- Much better numerical performance, robustness
- Several versions

Arnoldi, PRIMA, Lanczos, ...

- Possibility of preserving stability and passivity by construction!



## Krylov subspaces

$$\mathbf{H}(s) = \mathbf{M}_0 + \mathbf{M}_1 s + \mathbf{M}_2 s^2 + \dots$$

↓      ↓      ↓  
Moments     $\mathbf{L}^T \mathbf{R}$      $\mathbf{L}^T \mathbf{A} \mathbf{R}$      $\mathbf{L}^T \mathbf{A}^2 \mathbf{R}$

$$Kr(\mathbf{A}, \mathbf{R}, q) = \text{span}\{\mathbf{R}, \mathbf{A}\mathbf{R}, \mathbf{A}^2\mathbf{R}, \dots, \mathbf{A}^{q-1}\mathbf{R}\}$$

$\mathbf{V}_q$  = basis of  $Kr(\mathbf{A}, \mathbf{R}, q)$

Constructed via iterative (stable) algorithms



## Arnoldi (basic) algorithm

$$\begin{cases} \mathbf{x} = \mathbf{A}\dot{\mathbf{x}} + \mathbf{R}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

$$\boxed{\mathbf{V}_q^T} \quad \boxed{\mathbf{A}} \quad \boxed{\mathbf{V}_q} = \boxed{\mathbf{A}_q}$$

$$\boxed{\mathbf{x}} \approx \boxed{\mathbf{V}_q} \boxed{\mathbf{x}_q}$$

$$\begin{cases} \mathbf{x}_q = \mathbf{A}_q \dot{\mathbf{x}}_q + \mathbf{R}_q \mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T \mathbf{x}_q \end{cases}$$

**E***M***C**  
GROUP



## PRIMA algorithm

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

$$\boxed{\mathbf{V}_q^T} \quad \boxed{\mathbf{C}} \quad \boxed{\mathbf{V}_q} = \boxed{\mathbf{C}_q}$$
  

$$\boxed{\mathbf{V}_q^T} \quad \boxed{\mathbf{G}} \quad \boxed{\mathbf{V}_q} = \boxed{\mathbf{G}_q}$$

$$\boxed{\mathbf{x}} \approx \boxed{\mathbf{V}_q} \boxed{\mathbf{x}_q}$$

$$\begin{cases} \mathbf{G}_q \mathbf{x}_q + \mathbf{C}_q \dot{\mathbf{x}}_q = \mathbf{B}_q \mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T \mathbf{x}_q \end{cases}$$

**E***M***C**  
GROUP



## Passivity conditions (PRIMA)

$$\begin{cases} \mathbf{G}\mathbf{x} + \mathbf{C}\dot{\mathbf{x}} = \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{L}^T\mathbf{x} \end{cases}$$

Can often be enforced  
by construction building  
the original system



$$\begin{cases} \mathbf{G}_q \mathbf{x}_q + \mathbf{C}_q \dot{\mathbf{x}}_q = \mathbf{B}_q \mathbf{u} \\ \mathbf{y} = \mathbf{L}_q^T \mathbf{x}_q \end{cases}$$

$$\mathbf{G} \geq 0 \quad \mathbf{C} \geq 0$$

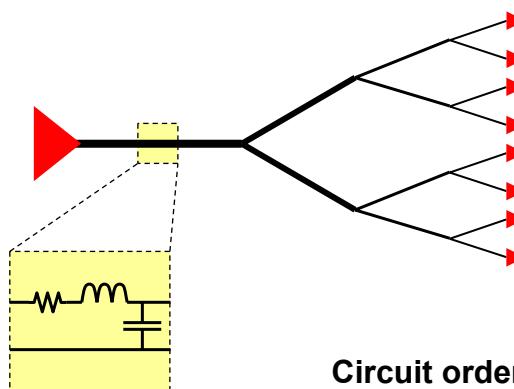
$\mathbf{C}$  symmetric

$$\mathbf{L} = \pm \mathbf{B}$$

$\mathbf{V}_q$  must be full rank



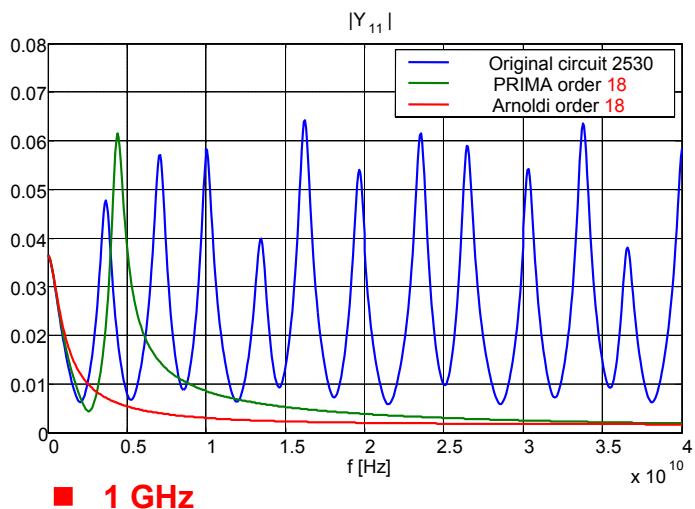
## An example: RLC tree circuit



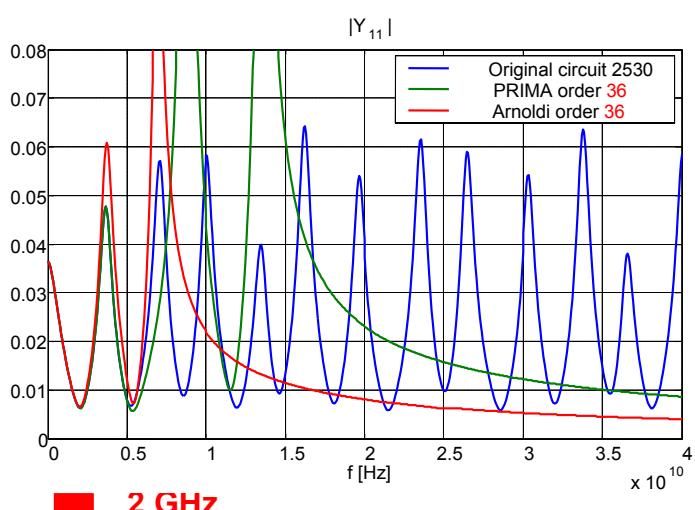
Circuit order: 2530  
Ports: 9



## RLC tree circuit: order reduction

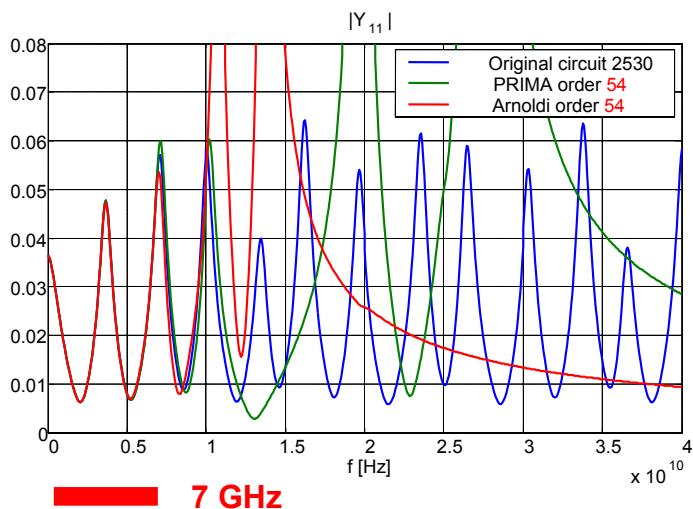


## RLC tree circuit: order reduction

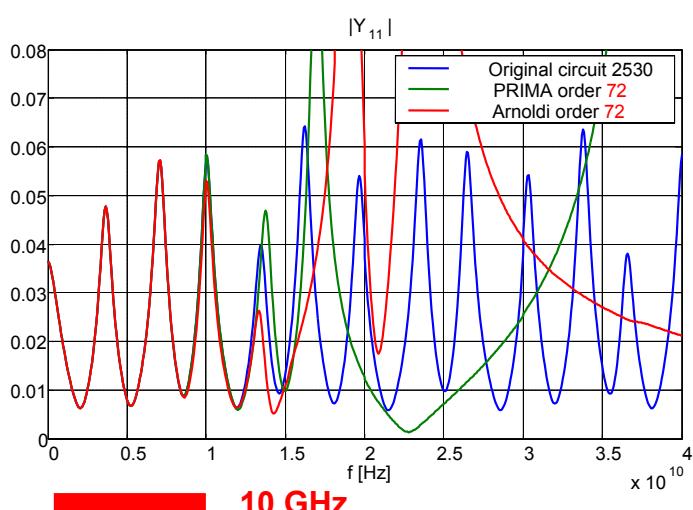




## RLC tree circuit: order reduction

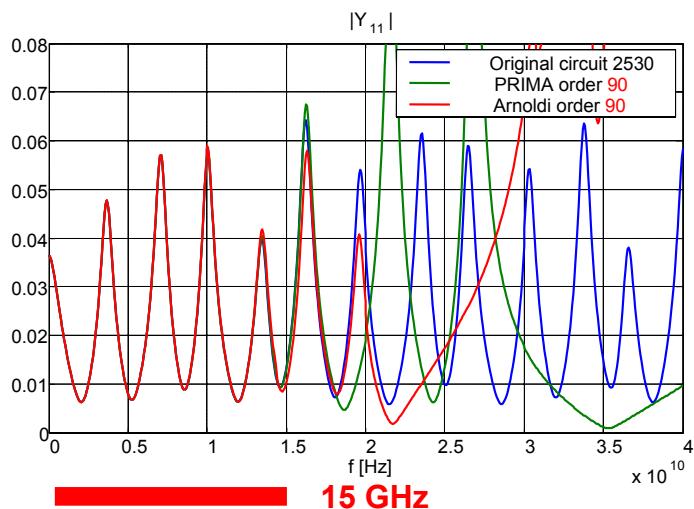


## RLC tree circuit: order reduction





## RLC tree circuit: order reduction



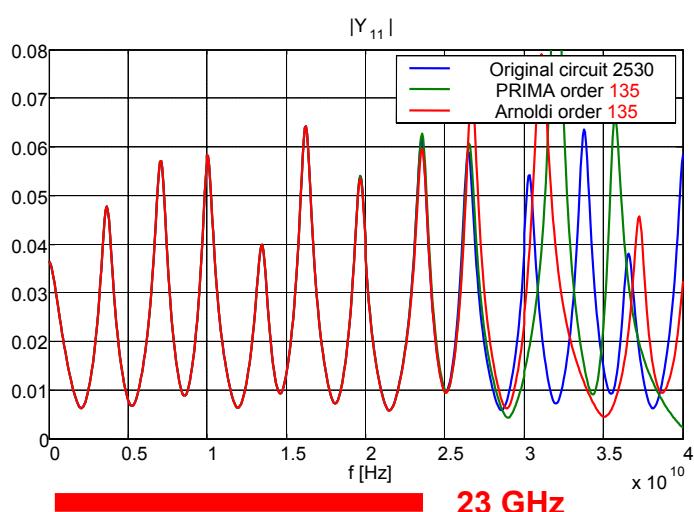
S. Grivet-Talocia, SPI tutorial, 9 May 2004

39

**E**  
**M**  
**C**  
GROUP



## RLC tree circuit: order reduction



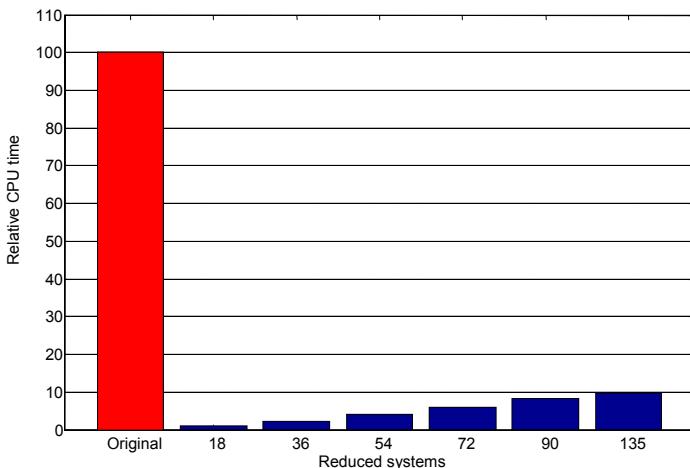
S. Grivet-Talocia, SPI tutorial, 9 May 2004

40

**E**  
**M**  
**C**  
GROUP



## RLC tree circuit: efficiency



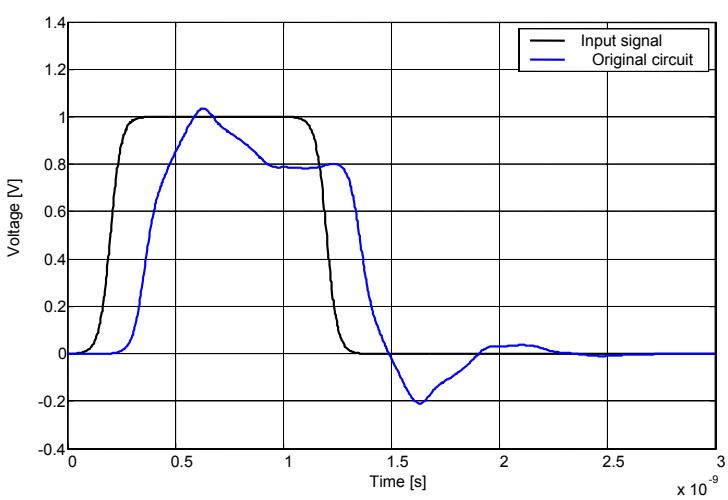
S. Grivet-Talocia, SPI tutorial, 9 May 2004

41

**E***M***C**  
GROUP



## RLC tree circuit: transient analysis



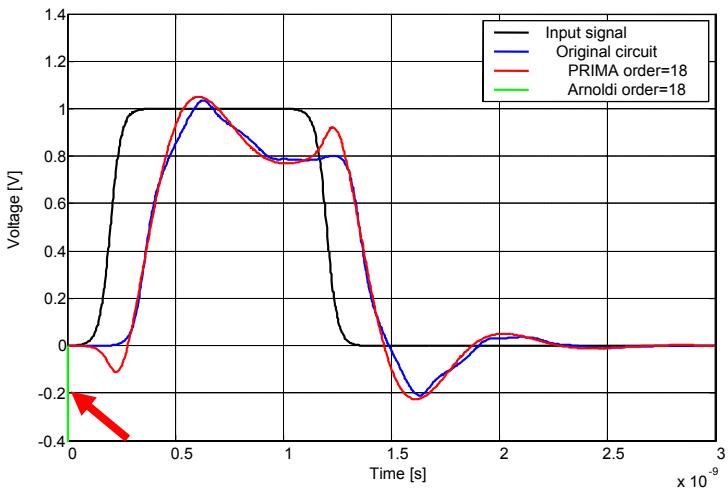
S. Grivet-Talocia, SPI tutorial, 9 May 2004

42

**E***M***C**  
GROUP



## RLC tree circuit: transient analysis



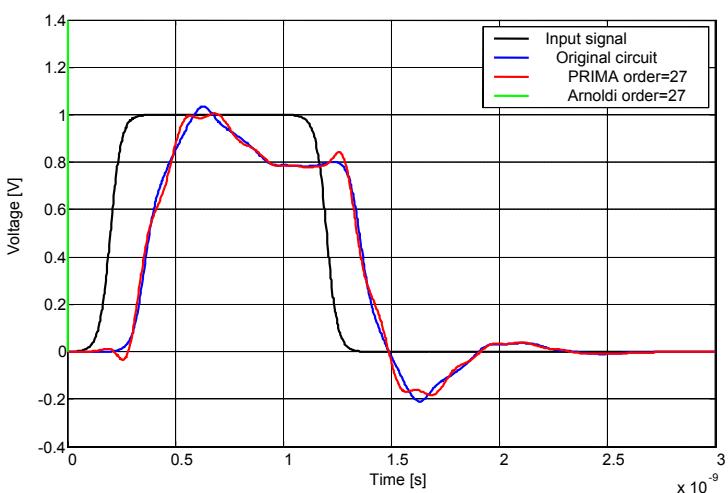
S. Grivet-Talocia, SPI tutorial, 9 May 2004

43

EMC  
GROUP



## RLC tree circuit: transient analysis



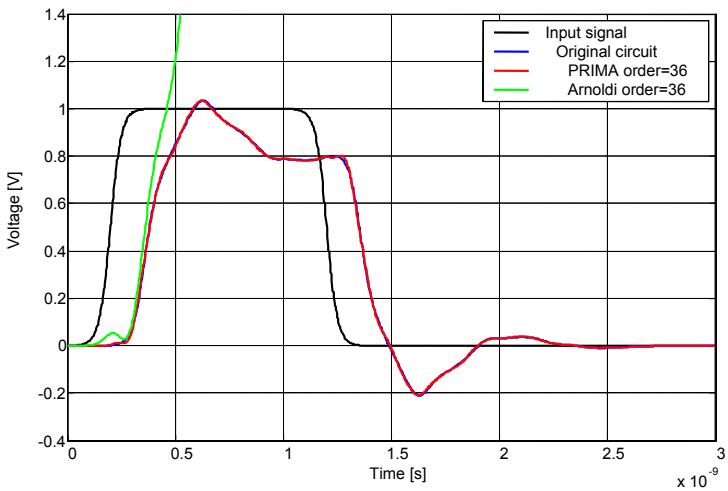
S. Grivet-Talocia, SPI tutorial, 9 May 2004

44

EMC  
GROUP



## RLC tree circuit: transient analysis



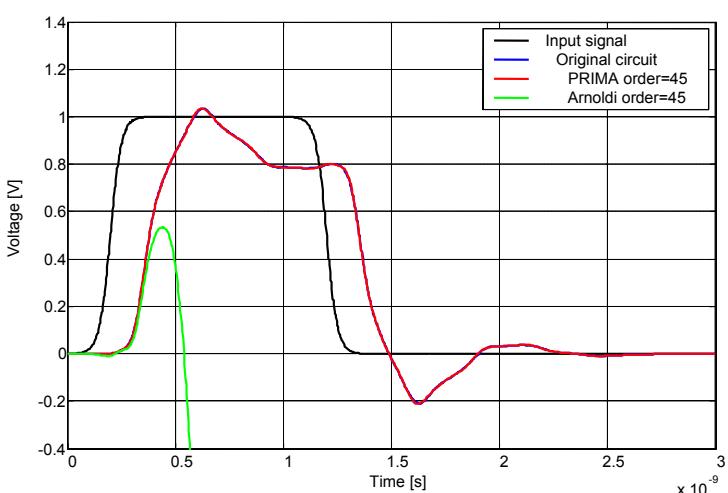
S. Grivet-Talocia, SPI tutorial, 9 May 2004

45

EMC  
GROUP



## RLC tree circuit: transient analysis



S. Grivet-Talocia, SPI tutorial, 9 May 2004

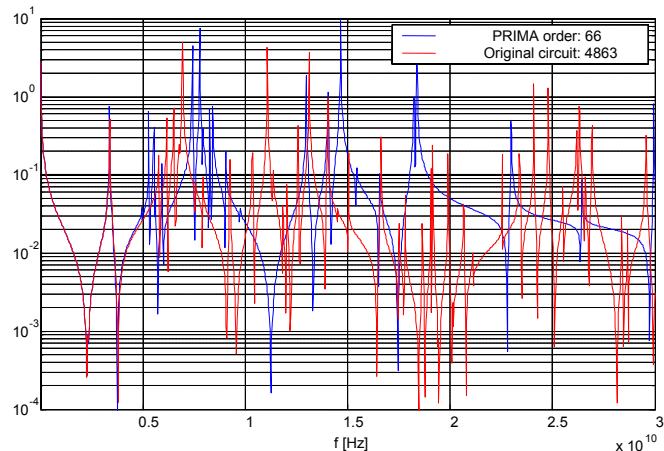
46

EMC  
GROUP



## Example: MNA, 22 ports, order 4863

([www.win.tue.nl/niconet/NIC2/benchmodred.html](http://www.win.tue.nl/niconet/NIC2/benchmodred.html))



3 GHz

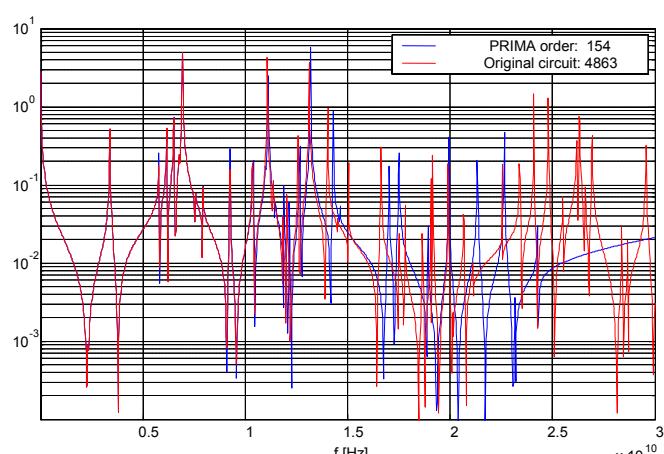
EMC  
GROUP

S. Grivet-Talocia, SPI tutorial, 9 May 2004

47



## Example: MNA, 22 ports, order 4863



6 GHz

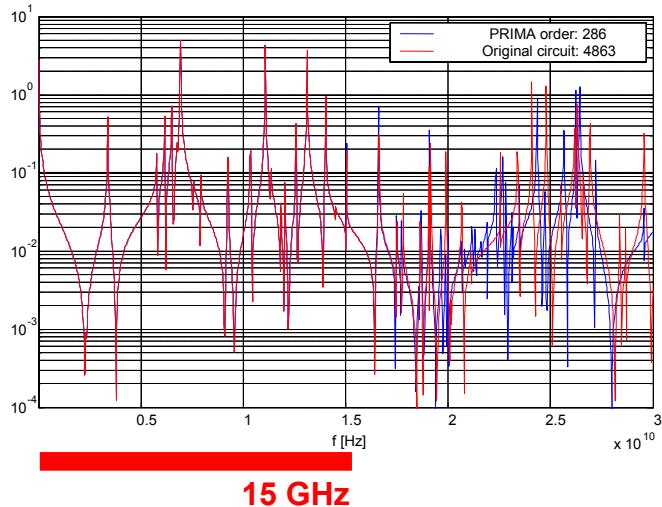
EMC  
GROUP

S. Grivet-Talocia, SPI tutorial, 9 May 2004

48



## Example: MNA, 22 ports, order 4863



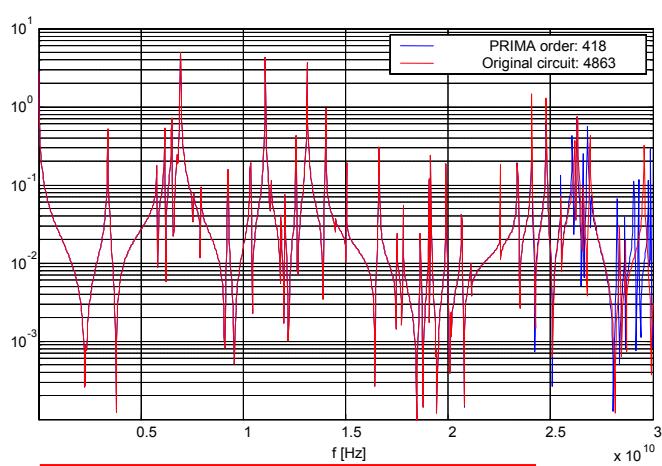
S. Grivet-Talocia, SPI tutorial, 9 May 2004

49

**E***M***C**  
GROUP



## Example: MNA, 22 ports, order 4863



S. Grivet-Talocia, SPI tutorial, 9 May 2004

50

**E***M***C**  
GROUP



## Key references

M.Celik, L.Pileggi, A.Odabasioglu, IC Interconnects Analysis, Kluwer, 2002

*...and references therein*

R.Achar, M.S.Nakhla, Proceedings of the IEEE, Vol.89, 2001, 693-728

*... and references therein*

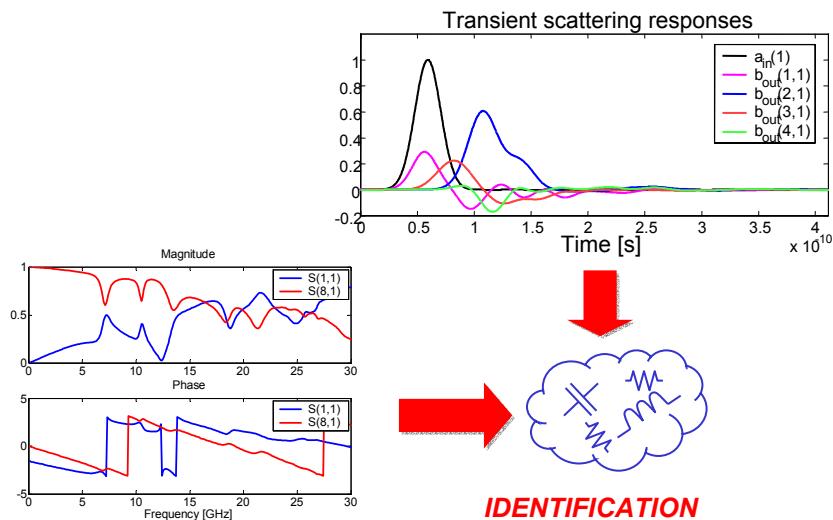


## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Model Identification



S. Grivet-Talocia, SPI tutorial, 9 May 2004

53

**E**  
**M**  
**C**  
GROUP



## Model identification

From samples to model: **identification** process

Reduced-order identification: **approximation** process

Several identification methods exist

Characterized by use of different:

- Input data
- Modeling criteria
- Model parameter estimation

S. Grivet-Talocia, SPI tutorial, 9 May 2004

54

**E**  
**M**  
**C**  
GROUP



## Identification methods

### Block Complex Frequency Hopping (BCFH)

[R.Achar, M.S.Nakhla, IEEE Proceedings, Vol.89, 2001]

Rational Padé approximation of network functions

Convergence property in a neighborhood of the expansion point

Hopping along frequency axis to cover the modeling bandwidth

May lead to ill-conditioned numerical systems when used for identification from sampled responses



## Identification methods

### Global Rational Approximation

[M.Elzinga, K.L.Virga, J.L.Prince, IEEE Trans. MTT, 9/2000]

[W.Beyene, J.Schutt-Aine, IEEE Trans. CPMT, Vol.21, 3/1998]

[K.L.Chi, M.Swaminathan, IEEE Trans. CAS II, vol.47, 4/2000]

[J.Morse, A.C.Cangellaris, Proc. EPEP, 2001]

[... many, many, many others... ]

A matrix of rational functions is fitted to the samples of a network function matrix (e.g. the Y matrix)





## Identification methods

### Pencil of Functions

[Y.Hua, T.Sarkar, *IEEE Trans. AP*, vol.37, 2/1989]

Time-domain data

Estimates model poles by **fitting** a sum of exponential functions to the samples of transient port responses

**Poles** obtained as eigenvalues of a generalized eigenvalue problem

**Automatic order estimation**



## Identification methods

### Subspace-based State-Space System Identification methods (4SID)

[M. Viberg, *Automatica*, 12/1995]

[T.McKelvey, H.Akcay, L.Ljung, *IEEE Trans. AC*, vol.41, 7/1996]

Based on **projections** of data onto orthogonal subspaces, leading to direct state-space estimation

Built-in **automatic order estimation** (based on SVD)

Available in both **time and frequency** domain

Equivalent to Pencil of Functions methods



## Identification methods

### Nevanlinna-Pick Interpolation

[C.P. Coelho, J.R. Phillips, L.M. Silveira, Proc. DATE 2002]

Interpolation of samples of the scattering matrix with a  
**(unitary bounded)** matrix rational function

Nice theoretical properties

Very complex

Leads to models with large dynamical order



## Identification methods

### Vector Fitting

[B. Gustavsen, A. Semlyen, IEEE Trans. PD, vol. 14, 7/1999]

Performs data **fitting** with rational functions avoiding nonlinear optimization

**Iterative** process converging to the dominant poles

Available for both time and frequency domain



## Identification methods

**Identification methods are not expected to work for every possible problem**

**Any method performs well for a certain class of identification problems**

**Vector Fitting is selected here as one of the most promising methods for a wide range of applications**



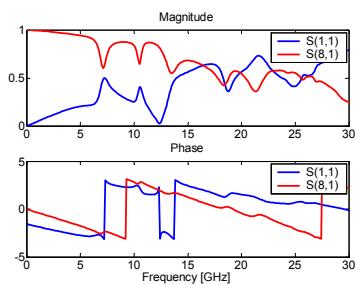
## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis



## Frequency-domain macromodeling

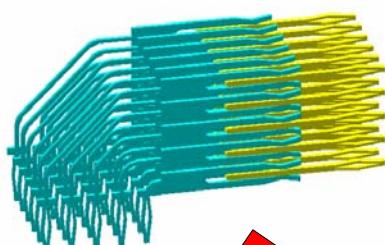
Model identification from frequency-domain responses



**IDENTIFICATION**



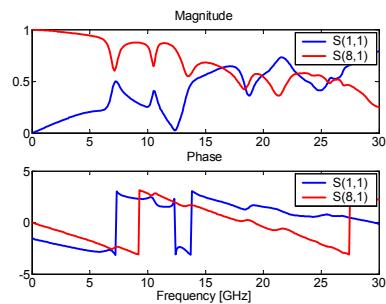
## Possible scenarios



Frequency-Domain  
full-wave simulation  
(MoM, FEM, ...)

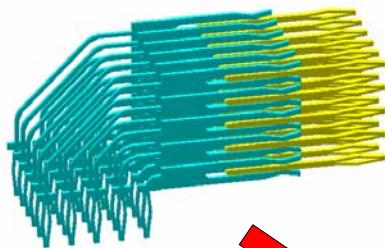


Frequency tables  
of transfer matrix  
( $S$ ,  $Y$ ,  $Z$ , ...)



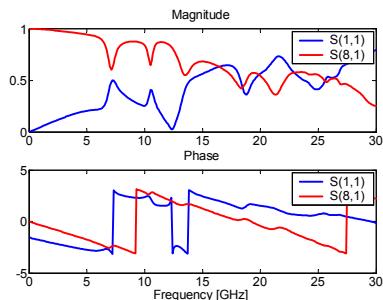


## Possible scenarios



Frequency tables  
of transfer matrix  
(S, Y, Z, ...)

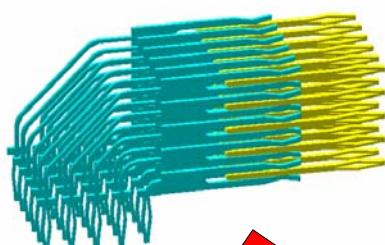
Time-Domain full-wave  
simulation (FIT, FDTD)  
FFT postprocessing



EMC  
GROUP

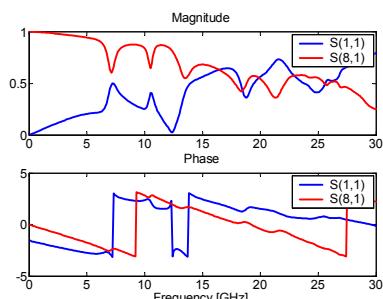


## Possible scenarios



Frequency tables of  
transfer matrix (S)

Direct VNA  
measurement



EMC  
GROUP



## Frequency-Domain Macromodeling

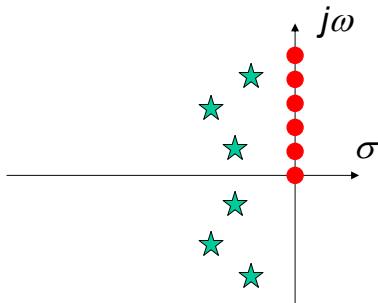
Input data  
 $\{\hat{H}(j\omega_k), k = 1, \dots, K\}$

Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$



Unknowns:

- Poles  $p_n$
- Residues  $R_n$
- Constant  $H_\infty$



## Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^N \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

- Nonlinear dependence on poles
- Requires nonlinear optimization  
(e.g. nonlinear least squares)
- Convergence problems (local minima, etc...)



## Frequency-Domain Macromodeling

Direct fitting condition: nonlinear!

$$\sum_{n=1}^N \frac{R_n}{j\omega_k - p_n} + H_\infty \approx \hat{H}(j\omega_k), \quad \forall k$$

### Vector Fitting avoids nonlinear optimization

B. Gustavsen, A. Semlyen, ``Rational approximation of frequency responses by **vector fitting**'', *IEEE Trans. Power Delivery*, Vol.14, July 1999, pp.1052-1061



## Frequency-Domain Vector Fitting

### Input data

$$\{\hat{H}(j\omega_k), \quad k = 1, \dots, K\}$$

### Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

### Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$

- $w(s)$  is unitary for  $s \rightarrow \infty$
- poles  $q_n$  are fixed a priori
- residues  $c_n$  are unknown

### Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

### The poles of

$w(s) H(s)$   
are  $\{q_n\}$  only!



## Frequency-Domain Vector Fitting

Input data

$$\{\hat{H}(j\omega_k), \quad k=1,\dots,K\}$$

Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$

$$= \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

There are  $N$  zeros  $\{z_n\}$

Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$

$$\{p_n\} \cong \{z_n\}$$



## Frequency-Domain Vector Fitting

Input data

$$\{\hat{H}(j\omega_k), \quad k=1,\dots,K\}$$

Weight function

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1$$

$$\left\{ \sum_{n=1}^N \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

Vector Fitting condition

$$w(s) H(s) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{s - q_n} + \tilde{H}_\infty$$



## Frequency-Domain Vector Fitting

Unknowns

$$\{c_n, \tilde{c}_n, \tilde{H}_\infty\}$$

$$\left\{ \sum_{n=1}^N \frac{c_n}{j\omega_k - q_n} + 1 \right\} \hat{H}(j\omega_k) \cong \sum_{n=1}^N \frac{\tilde{c}_n}{j\omega_k - q_n} + \tilde{H}_\infty$$

**Linear least squares problem: easy to solve!**

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

Poles of  $H(s)$

**E***M***C**  
GROUP



## Frequency-Domain Vector Fitting

$$w(s) = \sum_{n=1}^N \frac{c_n}{s - q_n} + 1 = \prod_{n=1}^N \frac{(s - z_n)}{(s - q_n)}$$

Poles of  $H(s)$

**Theorem:** the zeros  $\{z_n\}$  are the eigenvalues of

$$\mathbf{Q} = \mathbf{A} - \mathbf{b} \mathbf{c}^T$$

where

$$\mathbf{A} = \text{diag} \{ q_n \}$$

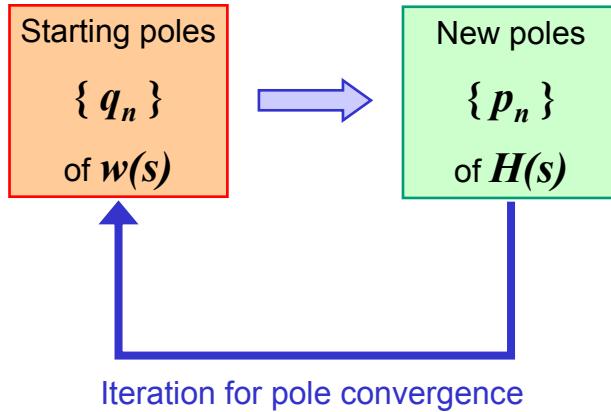
$$\mathbf{b} = (1 \quad 1 \quad \cdots \quad 1)^T$$

$$\mathbf{c} = (c_1 \quad c_2 \quad \cdots \quad c_N)^T$$

**E***M***C**  
GROUP



## Pole relocation

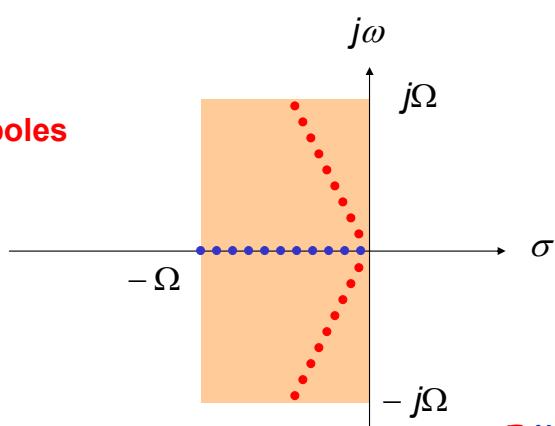


## Choice of starting poles

Uniform coverage of approximation bandwidth

Real poles

Complex poles





## Vector Fitting: residues

Input data

$$\{\hat{H}(j\omega_k), \quad k = 1, \dots, K\}$$

Approximation

$$H(s) = \sum_{n=1}^N \frac{R_n}{s - p_n} + H_\infty$$

Fitting condition

$$H(j\omega_k) \approx \hat{H}(j\omega_k), \forall k$$

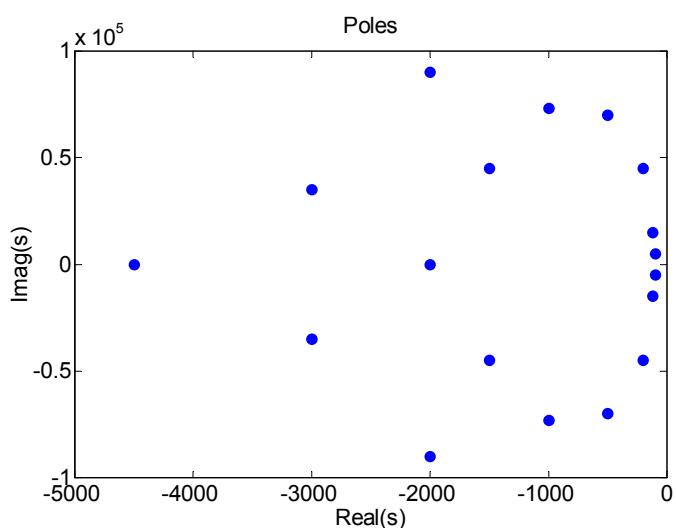
Another linear

least squares problem!



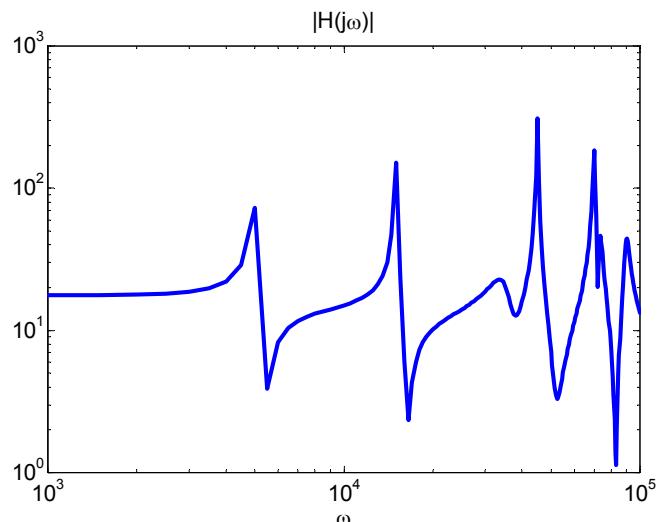
Residues  $R_n$  and constant  $H_\infty$

## Example 1: 18 random poles/residues





## Example 1



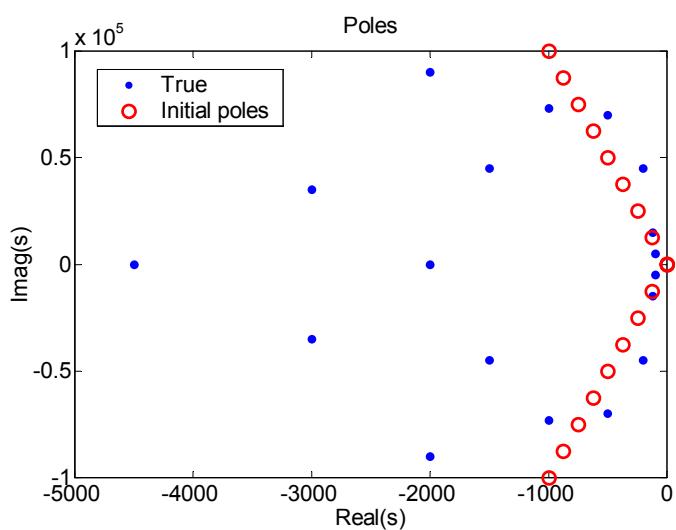
S. Grivet-Talocia, SPI tutorial, 9 May 2004

79

EMC  
GROUP



## Example 1



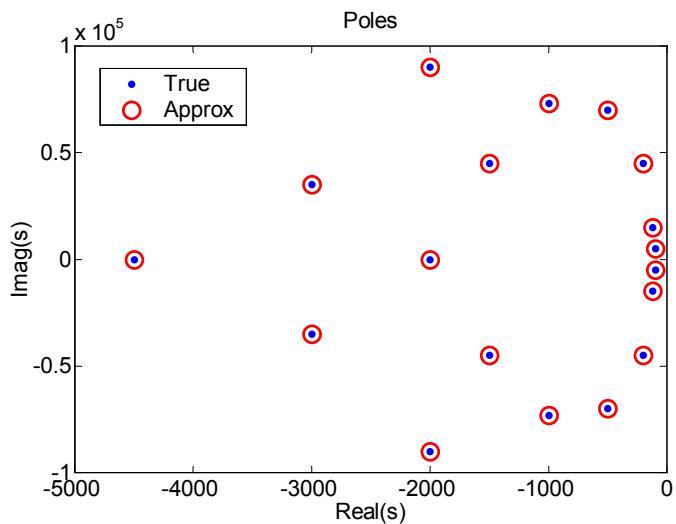
S. Grivet-Talocia, SPI tutorial, 9 May 2004

80

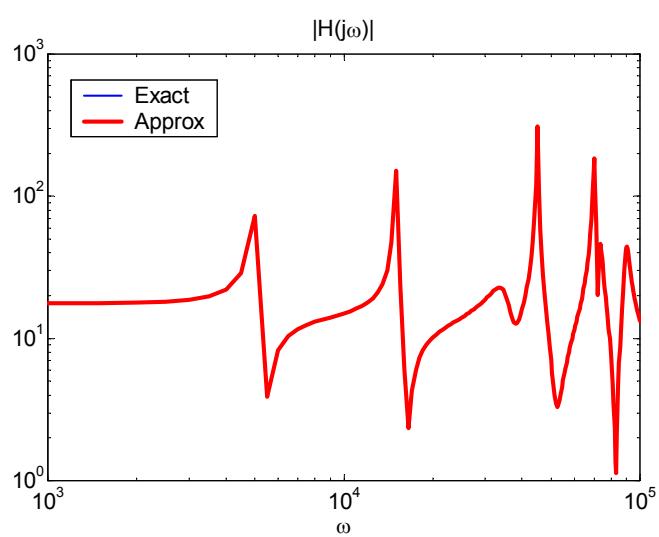
EMC  
GROUP



## Example 1

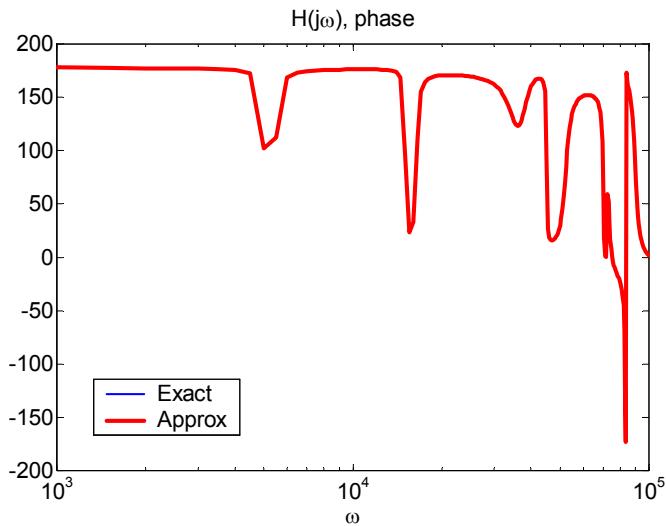


## Example 1





## Example 1



S. Grivet-Talocia, SPI tutorial, 9 May 2004

83

EMC  
GROUP



## Example 2

Same 18-pole rational function

Reduced-order fitting (12th order)

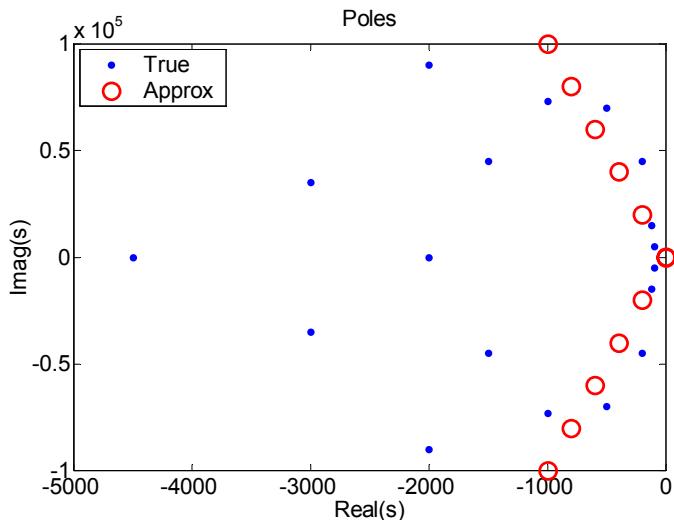
S. Grivet-Talocia, SPI tutorial, 9 May 2004

84

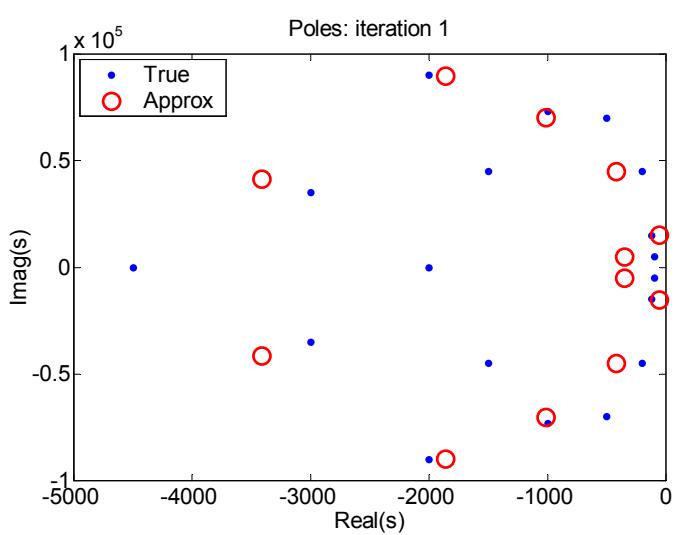
EMC  
GROUP



## Example 2

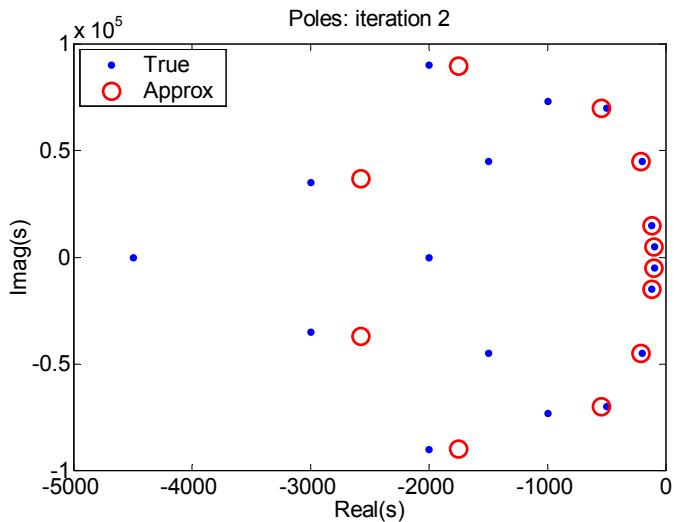


## Example 2

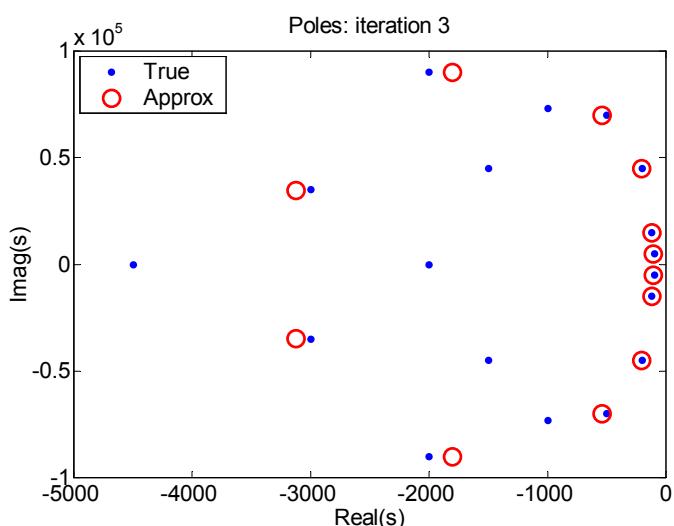




## Example 2

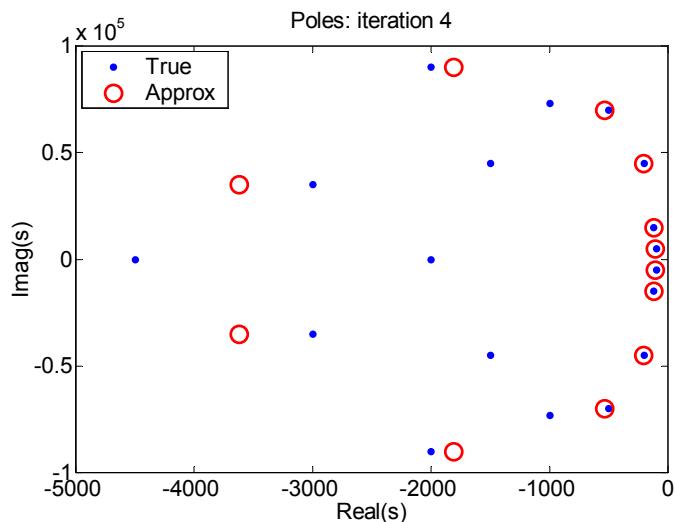


## Example 2

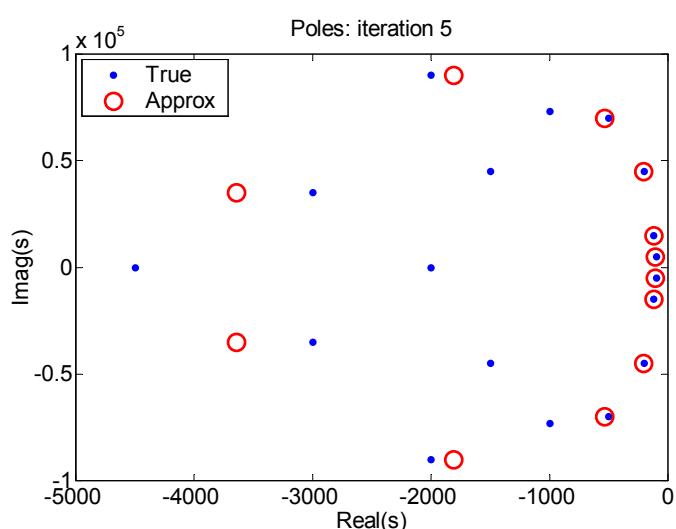




## Example 2

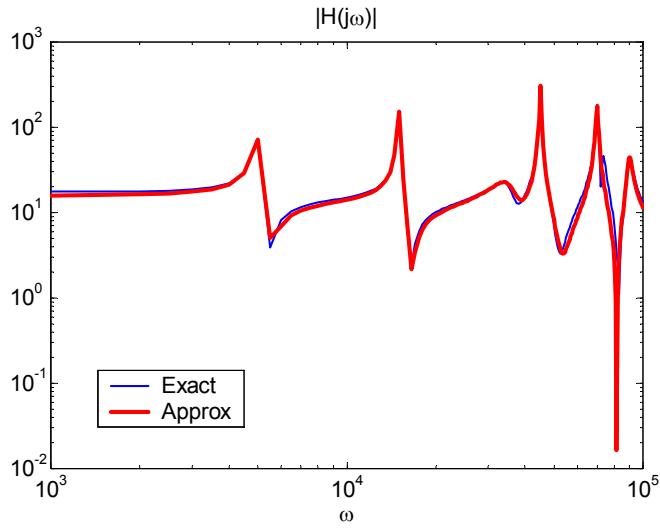


## Example 2





## Example 2



S. Grivet-Talocia, SPI tutorial, 9 May 2004

91

**E***M***C**  
GROUP



## Example 3

**Same 18-pole rational function**

**Reduced-order fitting (12th order)**

**Different starting poles (real poles)**

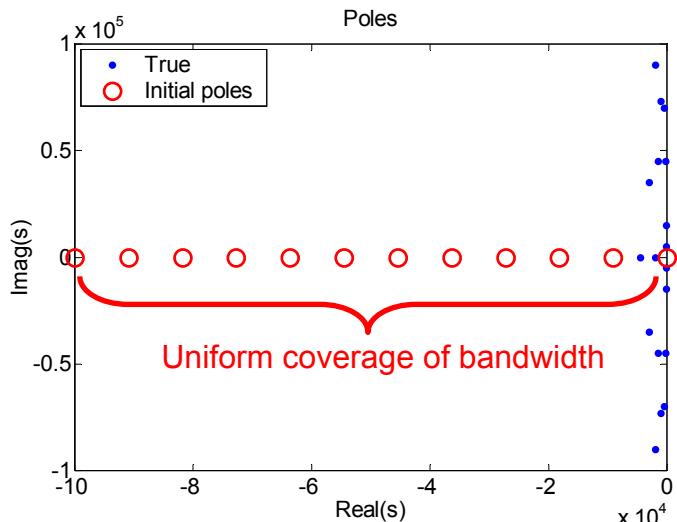
S. Grivet-Talocia, SPI tutorial, 9 May 2004

92

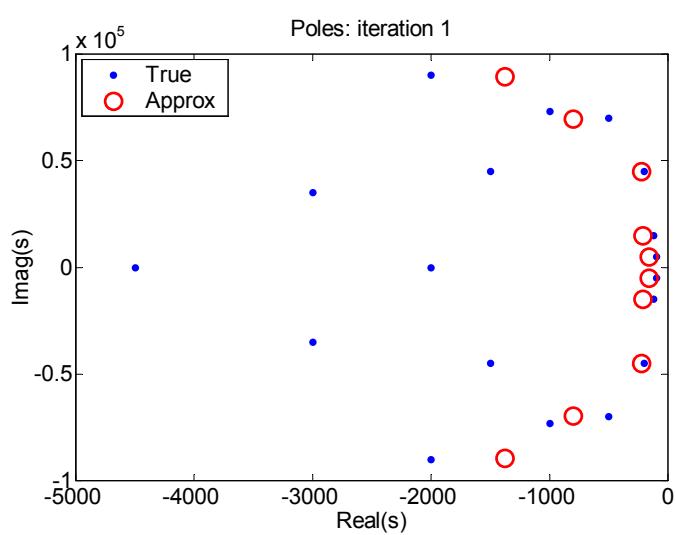
**E***M***C**  
GROUP



### Example 3

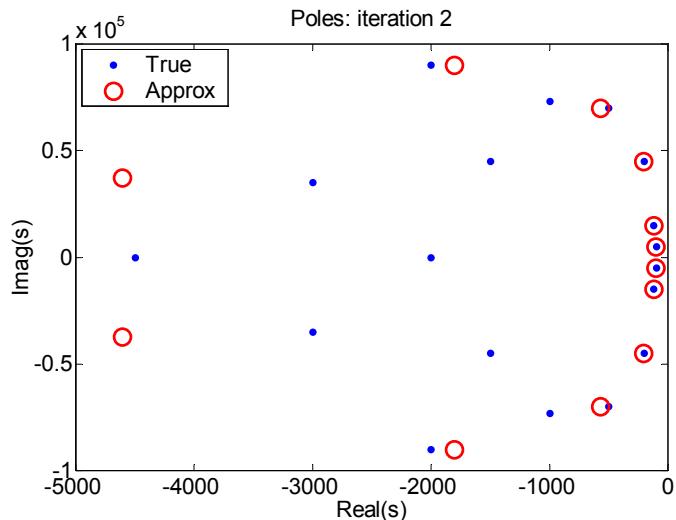


### Example 3

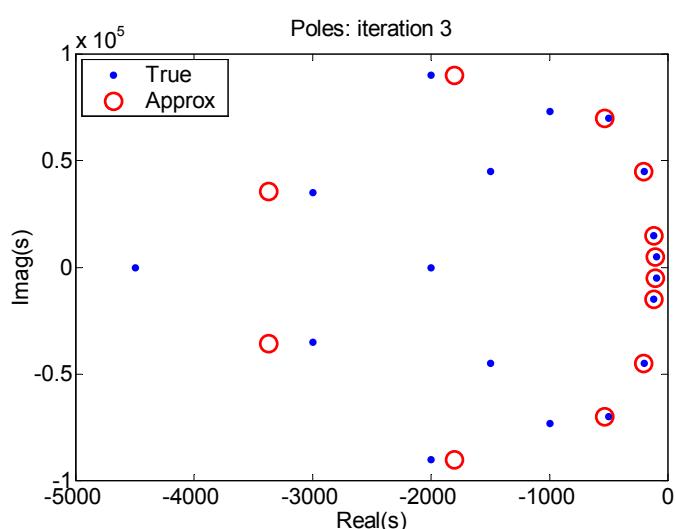




### Example 3

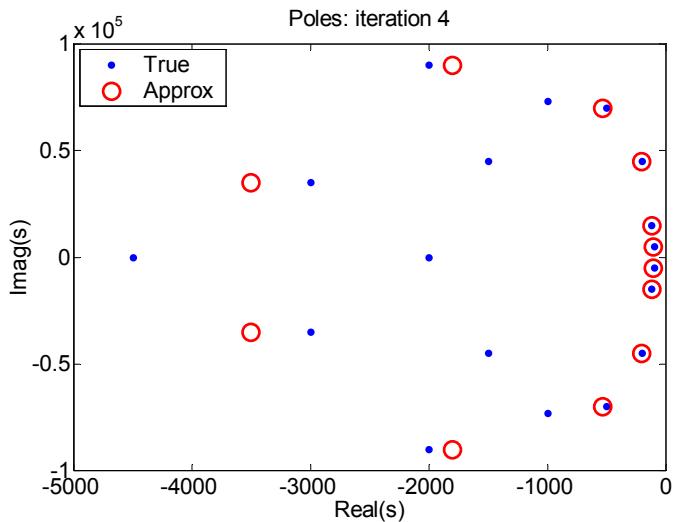


### Example 3

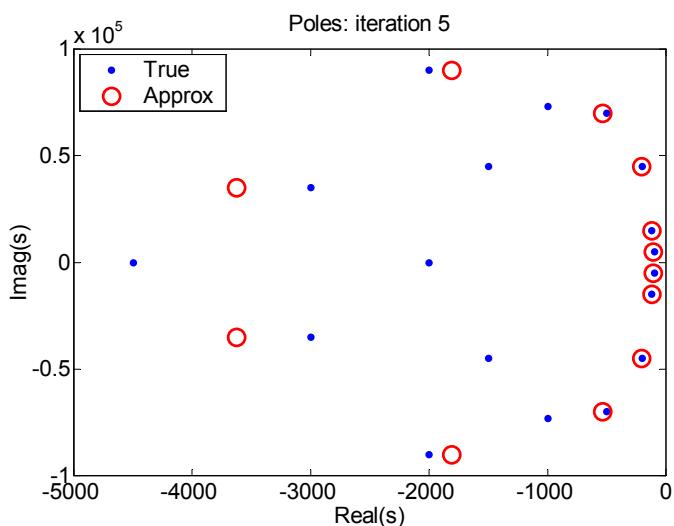




### Example 3

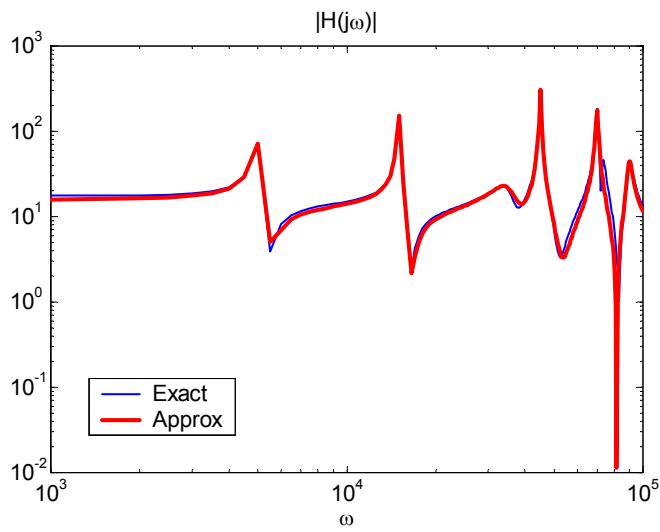


### Example 3





### Example 3



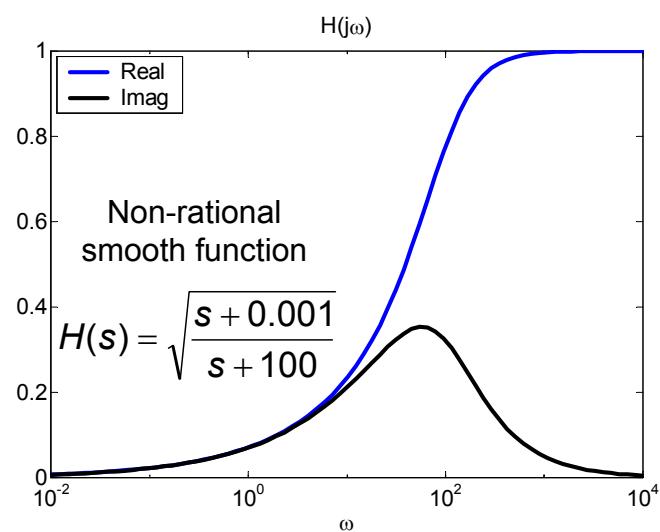
S. Grivet-Talocia, SPI tutorial, 9 May 2004

99

EMC  
GROUP



### Example 4



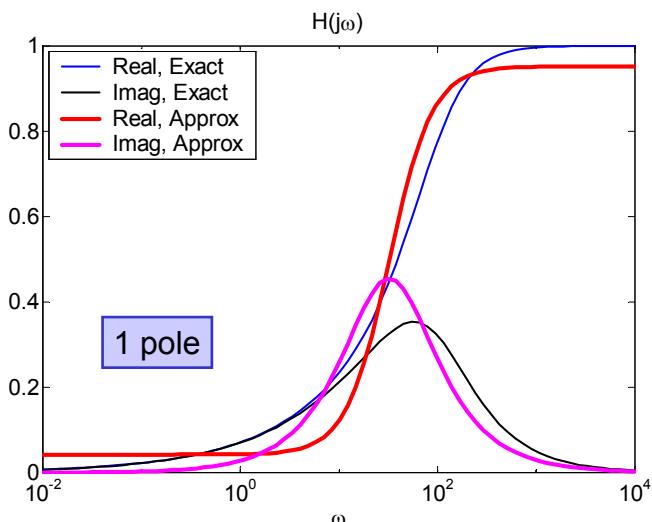
S. Grivet-Talocia, SPI tutorial, 9 May 2004

100

EMC  
GROUP



## Example 4



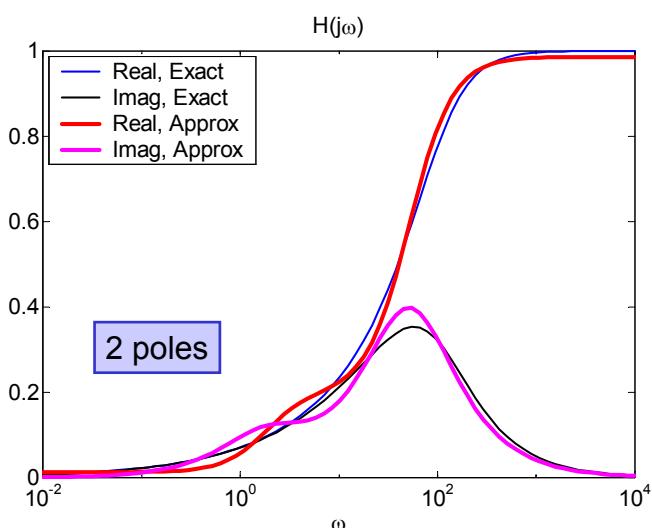
S. Grivet-Talocia, SPI tutorial, 9 May 2004

101

**E***M***C**  
GROUP



## Example 4



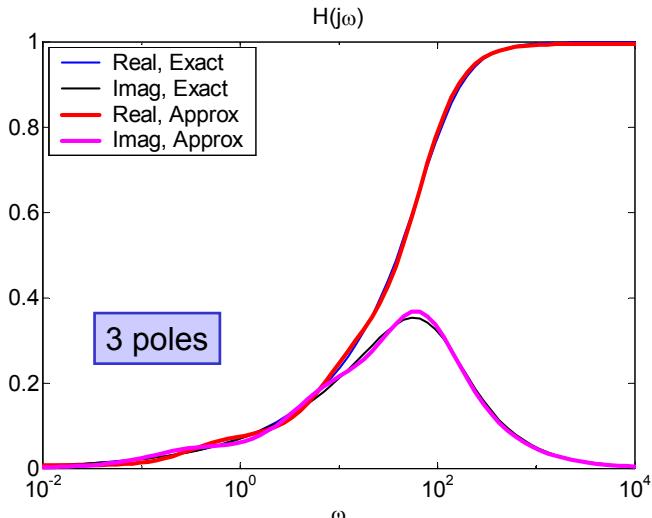
S. Grivet-Talocia, SPI tutorial, 9 May 2004

102

**E***M***C**  
GROUP



## Example 4



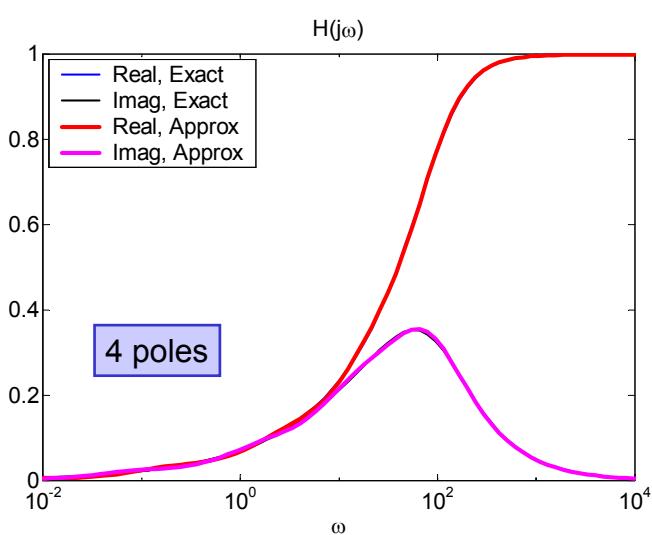
S. Grivet-Talocia, SPI tutorial, 9 May 2004

103

EMC  
GROUP



## Example 4



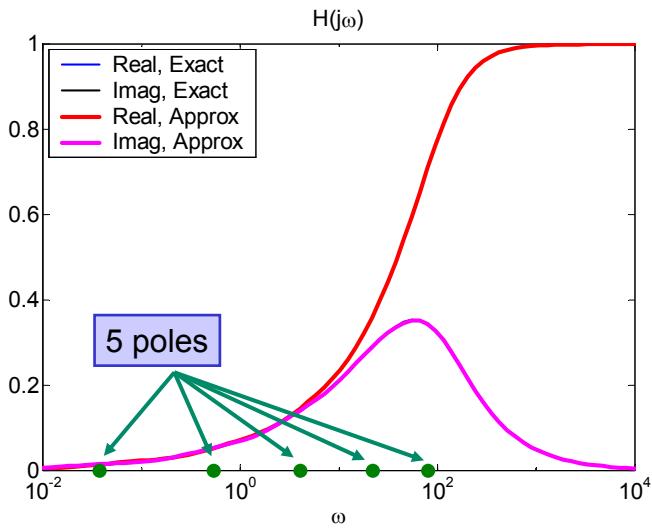
S. Grivet-Talocia, SPI tutorial, 9 May 2004

104

EMC  
GROUP



## Example 4



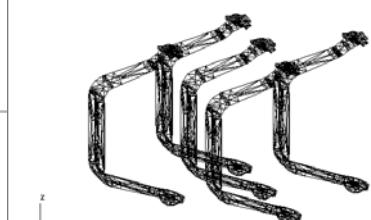
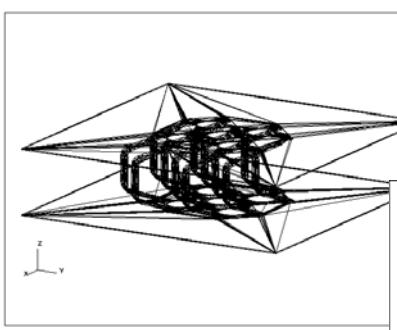
S. Grivet-Talocia, SPI tutorial, 9 May 2004

105

**E**  
**M**  
**C**  
GROUP



## Example 5: MCM-board connector



S. Grivet-Talocia, SPI tutorial, 9 May 2004

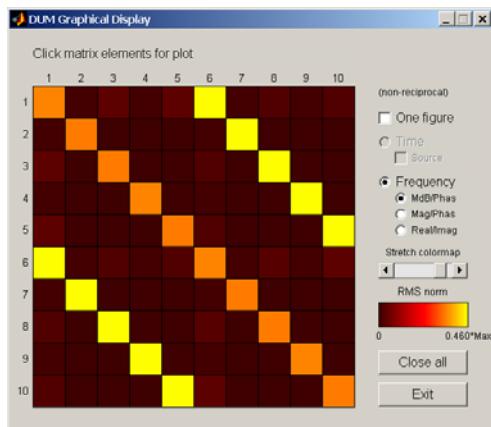
106

**E**  
**M**  
**C**  
GROUP

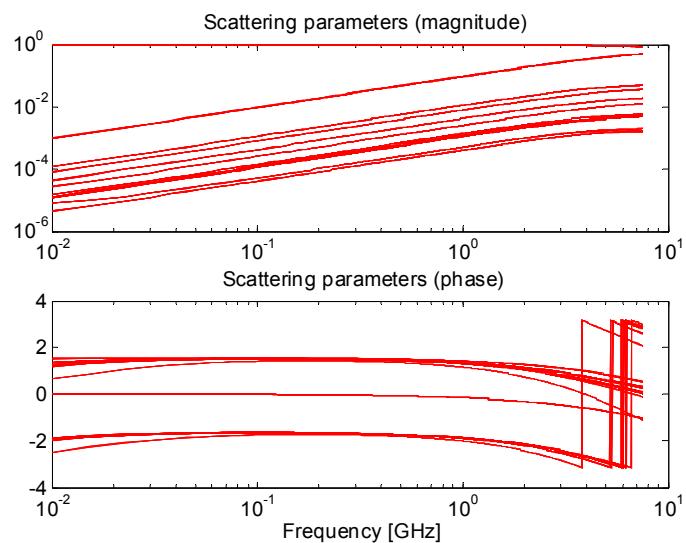


## Example 5: MCM-board connector

Data: 10-port structure, frequency-domain S-matrix



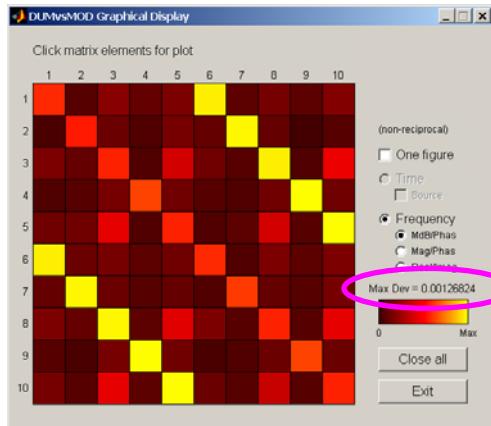
## Example 5: MCM-board connector





## Example 5: MCM-board connector

Macromodel: 4-poles



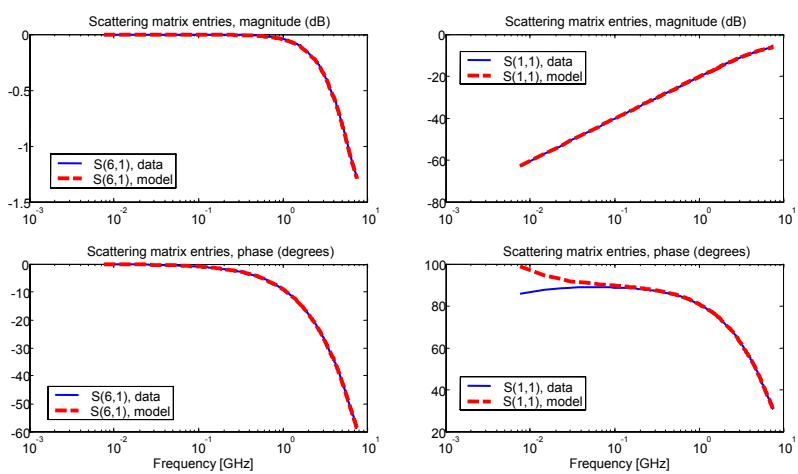
S. Grivet-Talocia, SPI tutorial, 9 May 2004

109

EMC  
GROUP



## Example 5: MCM-board connector



S. Grivet-Talocia, SPI tutorial, 9 May 2004

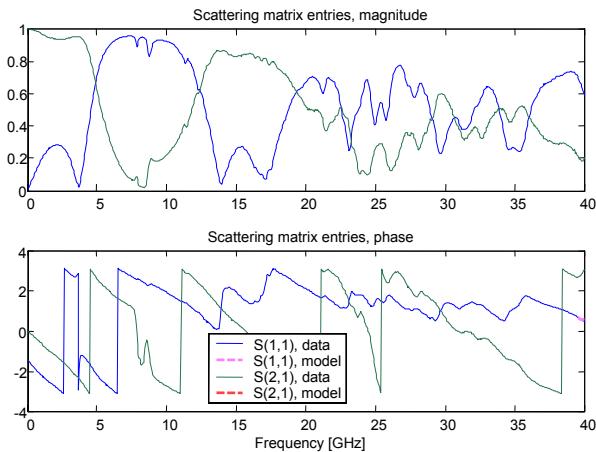
110

EMC  
GROUP



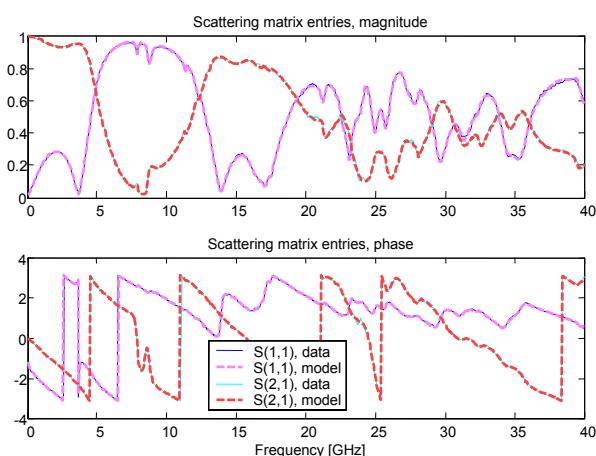
## Example 6: stripline+lauches

Data: measured S-parameters



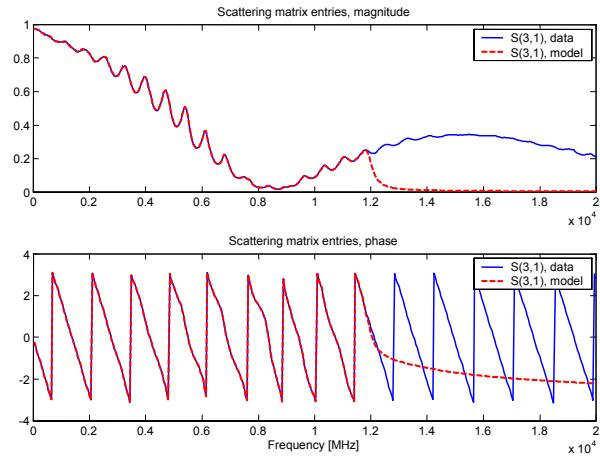
## Example 6: stripline+lauches

Macromodel: 60 poles





## Example 7: PCB path, measured VF with frequency-selective weighting



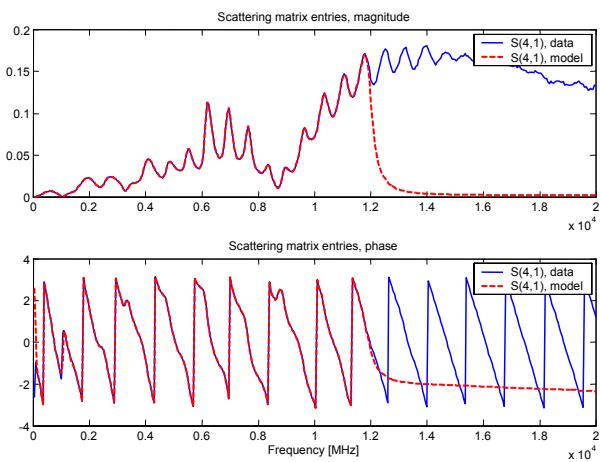
S. Grivet-Talocia, SPI tutorial, 9 May 2004

113

EMC  
GROUP



## Example 7: PCB path, measured VF with frequency-selective weighting



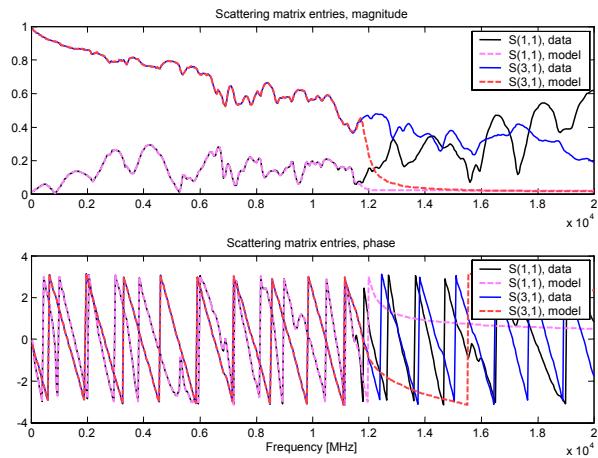
S. Grivet-Talocia, SPI tutorial, 9 May 2004

114

EMC  
GROUP



## Example 8: Connector, measured VF with frequency-selective weighting



## Vector Fitting: summary

- Tool for frequency-domain rational approximation
  - rational transfer functions (system identification)
  - rational transfer functions (reduced-order modeling)
  - non-rational transfer functions
- Data from full-wave simulations
- Direct frequency-domain measurements



## Vector Fitting: summary

[www.energy.sintef.no/produkt/VECTFIT/home.asp](http://www.energy.sintef.no/produkt/VECTFIT/home.asp)

- Very accurate and robust
- Only linear least squares + eigenvalues required
- Stability is not guaranteed
  - can be fixed by flipping real part during relocation
- Passivity is not guaranteed
  - can be fixed a posteriori (see later)



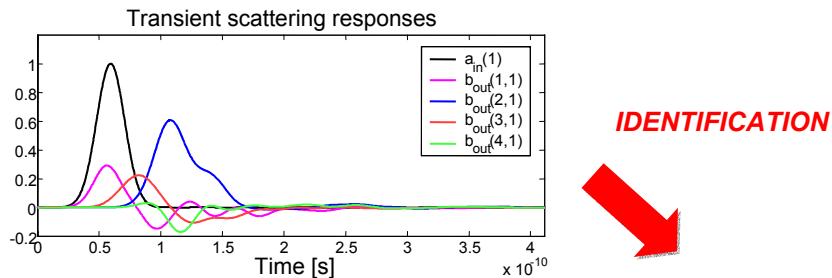
## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

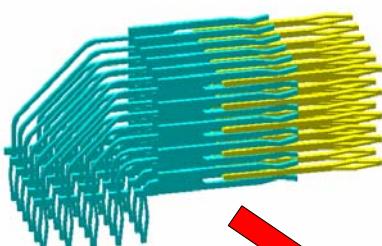


## Time-domain macromodeling

Model identification from time-domain responses

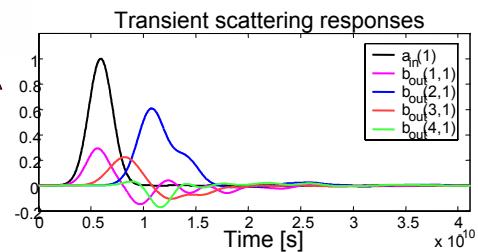


## Possible scenarios



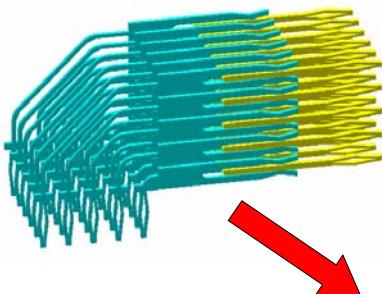
Time-Domain full-wave simulation (FIT, FDTD)

Port responses to transient excitations (usually gaussian)





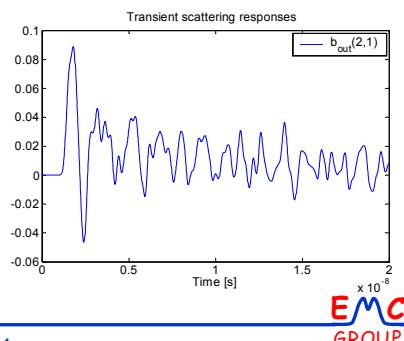
## Possible scenarios



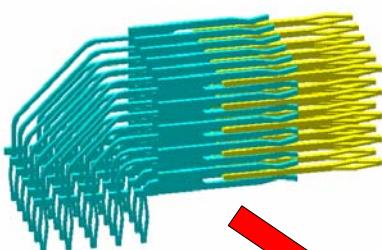
Port responses to transient excitations (usually gaussian)

Time-Domain full-wave simulation (FIT, FDTD)

Truncated waveforms from short FDTD runs



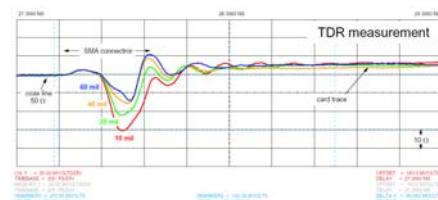
## Possible scenarios



Port responses to transient excitations (usually gaussian)

Time-domain measurements

(work in progress)





## Time-Domain Macromodeling

Input pulse

$x(t)$  t – domain

Output responses

$y(t)$  t – domain

Transfer function

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s)$$

Rational approximation

$$\mathbf{H}(s) \approx \mathbf{H}_\infty + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

Unknowns:

- Poles  $p_n$
- Residues  $\mathbf{R}_n$
- Constant  $\mathbf{H}_\infty$



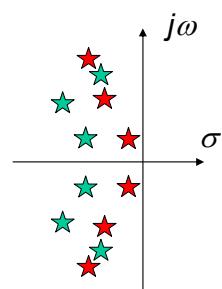
## Time-Domain Vector Fitting

Step 1. Find the dominant poles via “relocation”

Guess poles  
 $\{q_n\}$

New poles  
 $\{p_n\}$

Iterative refinement



How to do it using time-domain data?

How to insure convergence to the right poles?

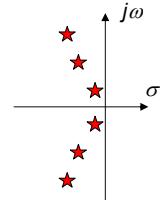


## Time-Domain Vector Fitting

1a. Start with initial poles:  $\{q_n\}$

1b. Define weight function: unknown  $\{k_n\}$

$$w(s) = 1 + \sum_n \frac{k_n}{s - q_n}$$



Starting poles

1c. Assume the following condition

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Poles of  $\mathbf{H}(s)$  = Zeros of  $w(s)$



## Time-Domain Vector Fitting

$$w(s)\mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n} \quad \text{Apply the input pulse } \mathbf{X}(s)$$

$$w(s)\mathbf{Y}(s) = \left( a + \sum_n \frac{b_n}{s - q_n} \right) \mathbf{X}(s) \quad \text{Compute inverse Laplace transform}$$

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$

$$\mathbf{x}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{x}(\tau) d\tau$$

$$\mathbf{y}_n(t) = \int_0^t e^{q_n(t-\tau)} \mathbf{y}(\tau) d\tau$$

Low-pass filtered input and output signals



## Time-Domain Vector Fitting

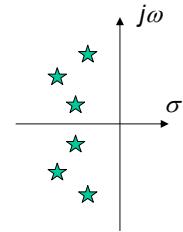
1d. Solve a linear least squares system for  $k_n, a, b_n$

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$

1e. Compute the zeros  $\{p_n\}$  of the weight function

$$w(s) = 1 + \sum_n \frac{k_n}{s - q_n} = \frac{\prod_n (s - p_n)}{\prod_n (s - q_n)}$$

These are the dominant poles!



S. Grivet-Talocia, "Package Macromodeling via Time-Domain Vector Fitting", *IEEE Microwave Wireless Comp. Lett.*, Nov. 2003

S. Grivet-Talocia, SPI tutorial, 9 May 2004

127

**E**  
**M**  
**C**  
GROUP



## Time-Domain Vector Fitting

Step 2. Compute the residues

2a. Low-pass filter input signals with new poles

$$\tilde{\mathbf{x}}_n(t) = \int_0^t e^{p_n(t-\tau)} \mathbf{x}(\tau) d\tau$$

2b. Solve a linear least squares system for  $\mathbf{R}_n$  and  $\mathbf{H}_\infty$

$$\mathbf{y}(t) = \mathbf{H}_\infty \mathbf{x}(t) + \sum_n \mathbf{R}_n \tilde{\mathbf{x}}_n(t)$$

S. Grivet-Talocia, SPI tutorial, 9 May 2004

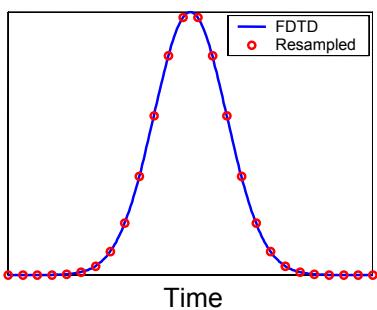
128

**E**  
**M**  
**C**  
GROUP

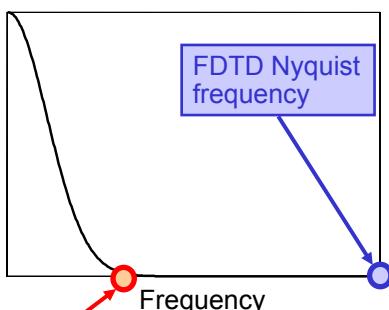


## Subsampling

Transient waveform



Spectrum



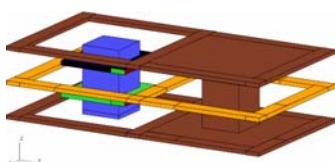
Nyquist frequency of  
resampled waveform

=  
Effective  
bandwidth

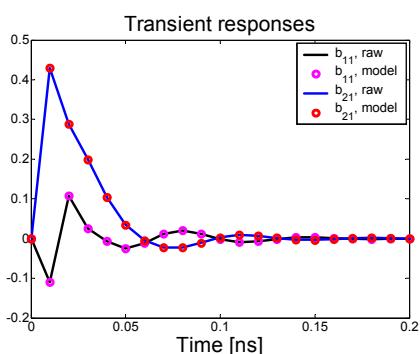
E  
M  
C  
GROUP



## Example 1: single via



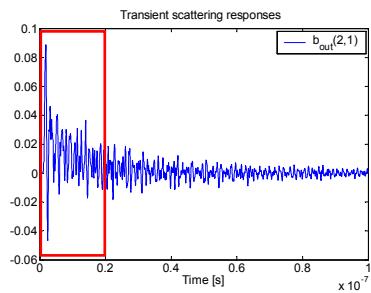
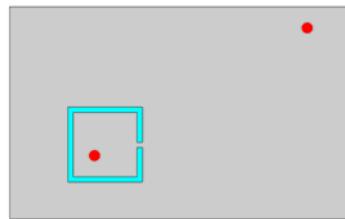
Raw data:  
Triangle Impulse Responses  
obtained by a transient PEEC  
solver (by Dr. Ruehli, IBM)



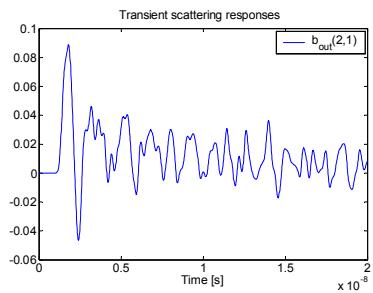
E  
M  
C  
GROUP



## Example 2: segmented power bus



- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- $50 \Omega$  port terminations



S. Grivet-Talocia, SPI tutorial, 9 May 2004

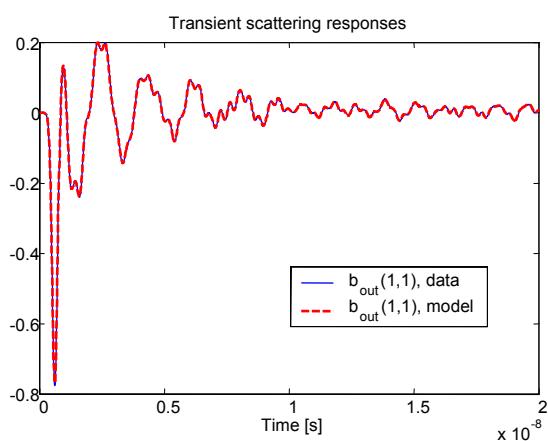
131

EMC  
GROUP



## Example 2: segmented power bus

80-poles model (Time-Domain Vector Fitting)



S. Grivet-Talocia, SPI tutorial, 9 May 2004

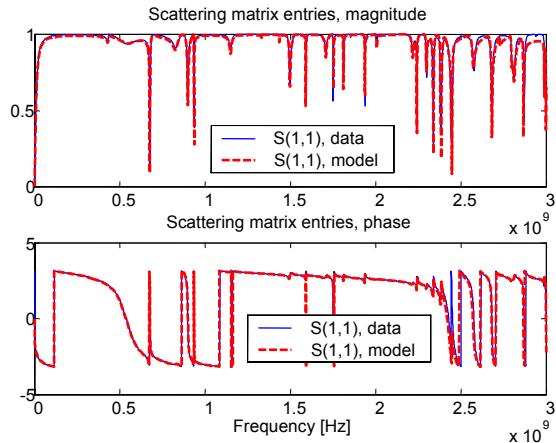
132

EMC  
GROUP



## Example 2: segmented power bus

Comparison vs. frequency-domain scattering data



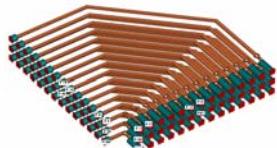
## Example 2: segmented power bus

Full-wave simulation time (CST) to compute...

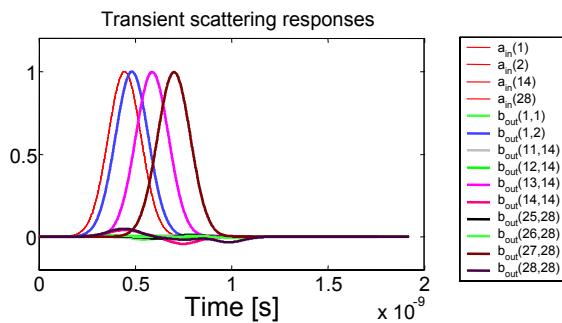
- ... frequency scattering data: **60 hours**  
(wait until transients are finished for reliable FFT)
- ... macromodel: **6 hours**  
(can use truncated waveforms for TD-VF)



## Example 3: 42-pin connector



3x14 pins, 84 ports  
Characterized via FIT  
(CST Microwave Studio 4)  
(Courtesy: Erni - AdMOS)



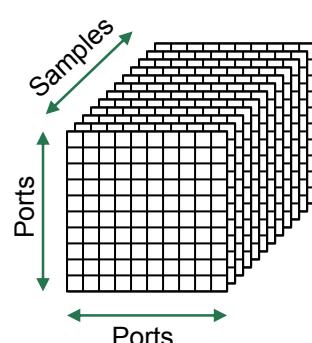
## Handling many ports

Frequency-Domain Vector Fitting

$$\left(1 + \sum_n \frac{k_n}{s - q_n}\right) \mathbf{H}(s) = a + \sum_n \frac{b_n}{s - q_n}$$

Time-Domain Vector Fitting

$$\mathbf{y}(t) + \sum_n k_n \mathbf{y}_n(t) = a \mathbf{x}(t) + \sum_n b_n \mathbf{x}_n(t)$$



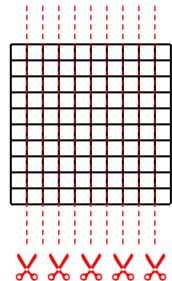
Processing **all** responses may lead to a **large** system!



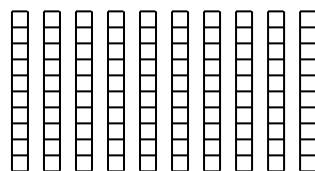
## Handling many ports

1. Split port responses into subsets

Transfer matrix  $\mathbf{H}(s)$



Subsets  $\{\mathbf{h}_k(s)\}$



## Handling many ports

2. Macromodel each subset via FD-VF or TD-VF



$$\mathbf{h}_k(s) \approx \mathbf{h}_{k,\infty} + \sum_n \frac{\mathbf{r}_{k,n}}{s - p_{k,n}}$$



Partial state-space representation

$$\begin{cases} \dot{\mathbf{w}}_k = \mathbf{A}_k \mathbf{w}_k + \mathbf{B}_k \mathbf{x}_k \\ \mathbf{y}_k = \mathbf{C}_k \mathbf{w}_k + \mathbf{D}_k \mathbf{x}_k \end{cases}$$

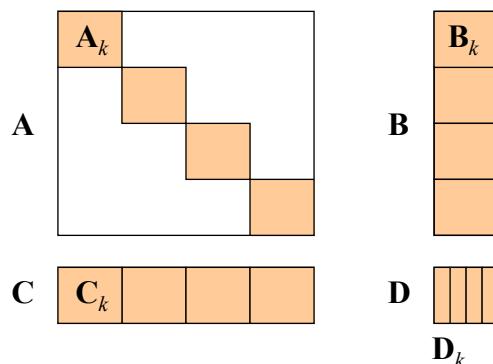


## Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$

All matrices  
can be  
constructed  
as sparse!



EMC  
GROUP

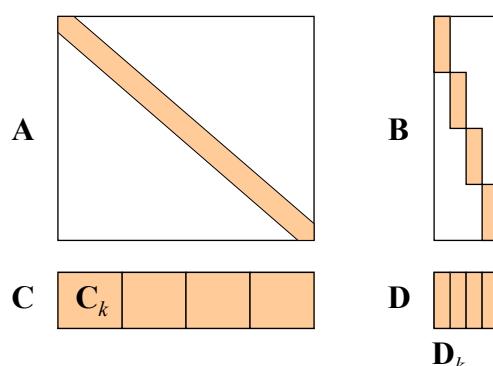


## Handling many ports

4. Assemble all partial models into a global model

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$

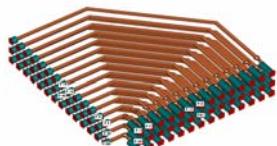
All matrices  
can be  
constructed  
as sparse!



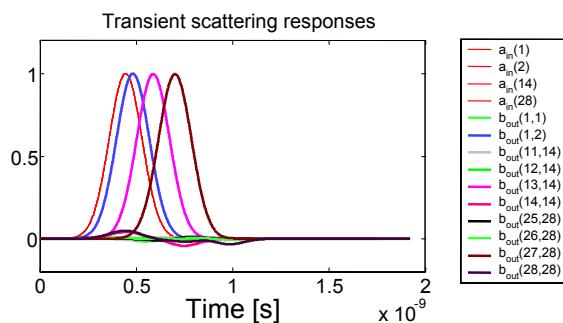
EMC  
GROUP



## Example 3: 42-pin connector

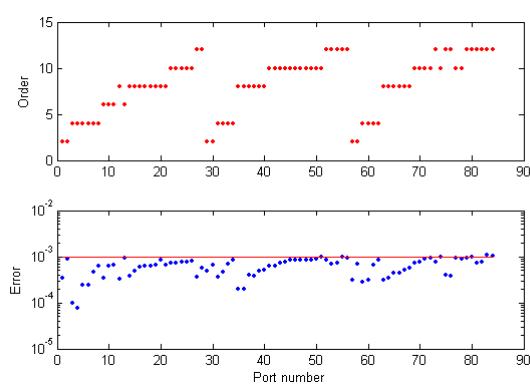


3x14 pins, 84 ports  
Characterized via FIT  
(CST Microwave Studio 4)  
(Courtesy: Erni - AdMOS)



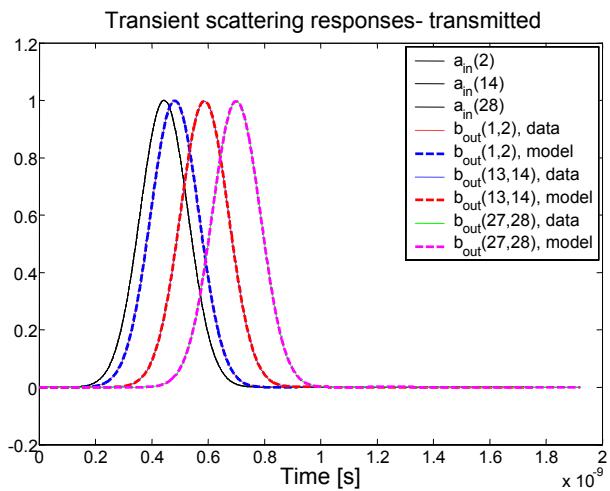
## Example 3: model order selection

Automatic (iterative) order selection on each of the 84 subsets of port responses (reduced model complexity)

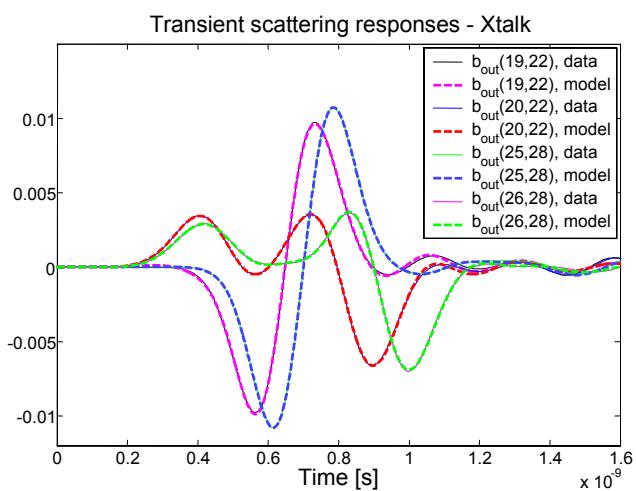




## Example 3: macromodel accuracy

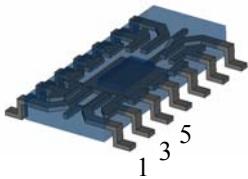


## Example 3: macromodel accuracy

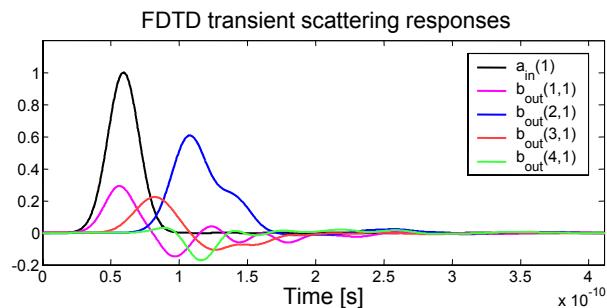




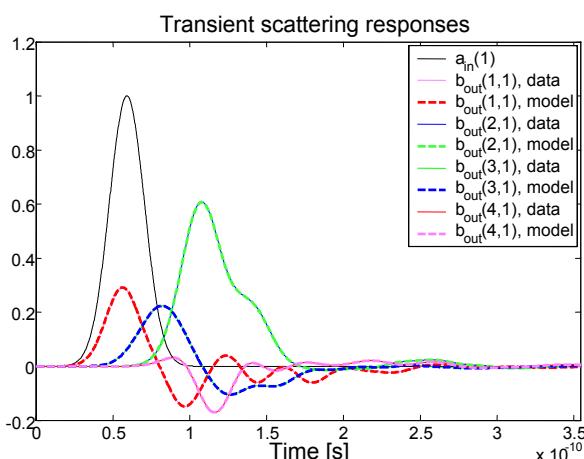
## Example 4: 14-pin package



14-pin SOIC package  
Simplified CAD for FDTD  
Bandwidth: 40 GHz  
50 Ω port terminations



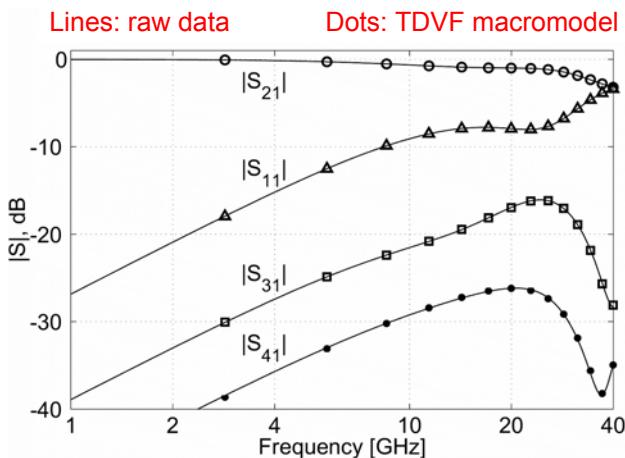
## Example 4: macromodel responses



No visible difference between data and model



## Example 4: macromodel responses

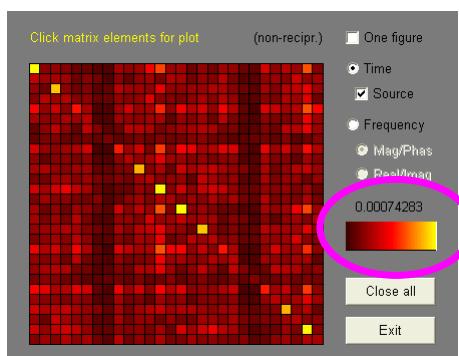


No visible difference between data and model



## Example 4: macromodel accuracy

Maximum deviation between model and data for all 28x28 responses



Largest:  
0.00074

TD-VF produces highly accurate macromodels



## Macromodel properties

### ☺ Accuracy

Good initial data  $\Rightarrow$  small approximation errors

### ☺ Stability

All poles with negative real part

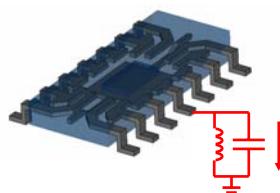
### ☹ Passivity

The macromodel may not be passive

EMC  
GROUP



## Example 4: change terminations

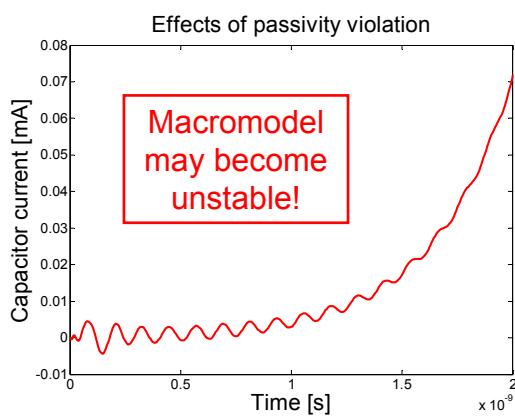


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \Omega$$

$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$



EMC  
GROUP



## Outline

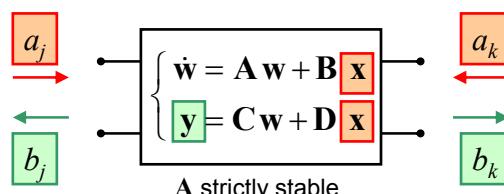
- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

EMC  
GROUP



## Passivity conditions

### Scattering representation



Scattering matrix: must be bounded real

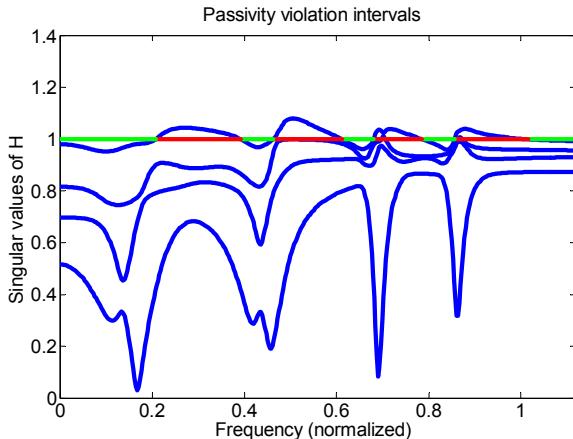
$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

EMC  
GROUP



## Passivity conditions Scattering representation

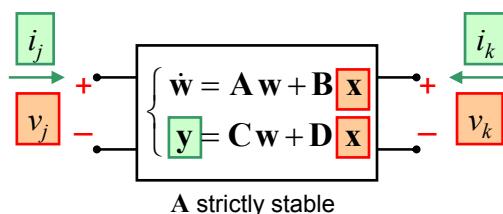


S. Grivet-Talocia, SPI tutorial, 9 May 2004

153

EMC  
GROUP

## Passivity conditions Admittance representation



Admittance matrix: must be positive real

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

$$\left\{ \text{eigenvalues of } (\mathbf{H}(j\omega) + \mathbf{H}^H(j\omega)) \right\} \geq \mathbf{0}, \quad \forall \omega$$

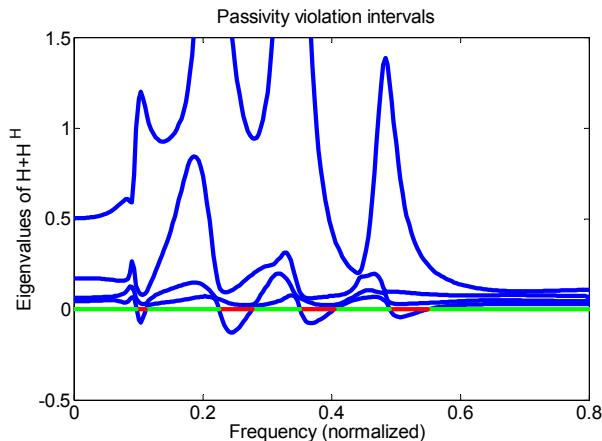
S. Grivet-Talocia, SPI tutorial, 9 May 2004

154

EMC  
GROUP



## Passivity conditions Admittance representation



## Checking passivity Scattering representation

$$\{ \text{singular values of } \mathbf{H}(j\omega) \} \leq 1, \quad \forall \omega$$

Several techniques can be used

**Frequency sweep test:** most straightforward

- Choose a set of frequency samples
- Compute  $\mathbf{H}$  and its singular values, and check
- **Time-consuming** for large models
- **May give wrong answers** due to poor sampling



## Checking passivity

### Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

**Equivalent purely algebraic conditions:**

- Linear Matrix Inequalities (**LMI**)
- Algebraic Riccati Equations (**ARE**)
- Eigenvalues of **Hamiltonian matrices**

S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, "Linear Matrix Inequalities in System and Control Theory, SIAM, Philadelphia, 1994



## Checking passivity

### Scattering representation

$$\{\text{singular values of } \mathbf{H}(j\omega)\} \leq 1, \quad \forall \omega$$

**Linear Matrix Inequality (LMI)**

$$\begin{pmatrix} \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{C}^T \mathbf{C} & \mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D} \\ \mathbf{B}^T \mathbf{P} + \mathbf{D}^T \mathbf{C} & \mathbf{D}^T \mathbf{D} - \mathbf{I} \end{pmatrix} \leq \mathbf{0} \quad \mathbf{P} = \mathbf{P}^T, \mathbf{P} > 0$$

Real matrix **P** is the variable



## Checking passivity Scattering representation

$$\{ \text{singular values of } \mathbf{H}(j\omega) \} \leq 1, \quad \forall \omega$$

### Algebraic Riccati Equation (ARE)

$$\begin{aligned} \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \mathbf{C}^T \mathbf{C} + (\mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D})(\mathbf{I} - \mathbf{D}^T \mathbf{D})^{-1}(\mathbf{P} \mathbf{B} + \mathbf{C}^T \mathbf{D})^T &= \mathbf{0} \\ \mathbf{P} &= \mathbf{P}^T \end{aligned}$$

Real matrix  $\mathbf{P}$  is the variable



## Checking passivity Scattering representation

$$\{ \text{singular values of } \mathbf{H}(j\omega) \} \leq 1, \quad \forall \omega$$

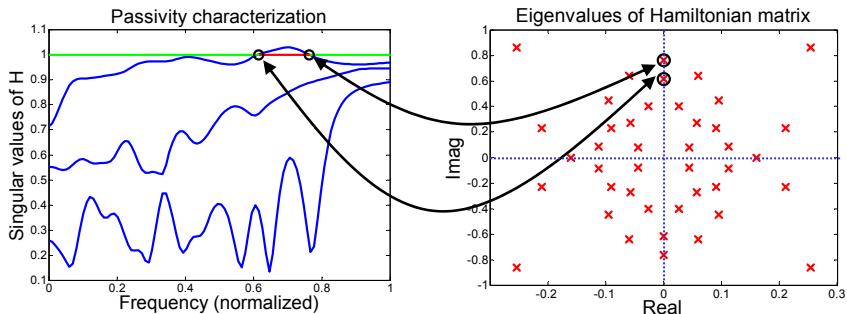
### Eigenvalues of Hamiltonian matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} - \mathbf{B}(\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{D}^T \mathbf{C} & -\mathbf{B}(\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{B}^T \\ \mathbf{C}^T (\mathbf{D} \mathbf{D}^T - \mathbf{I})^{-1} \mathbf{C} & -\mathbf{A}^T + \mathbf{C}^T \mathbf{D}(\mathbf{D}^T \mathbf{D} - \mathbf{I})^{-1} \mathbf{B}^T \end{pmatrix}$$

Real matrix  $\mathbf{M}$  must have no imaginary eigenvalues



## Checking passivity Scattering representation

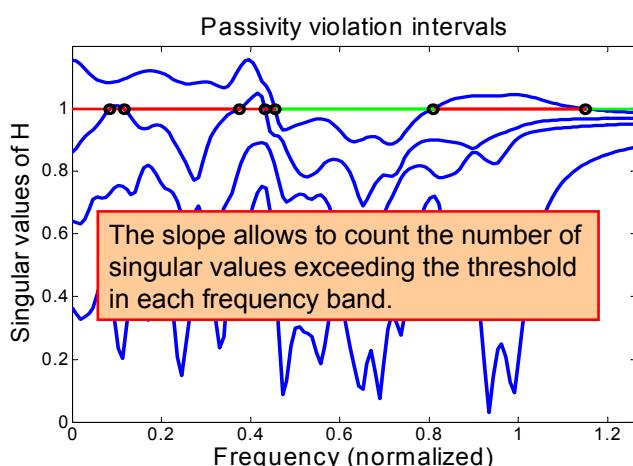


**Theorem** [Boyd, Balakrishnan, Kabamba, 1989]

$j\omega_0$  is an eigenvalue of  $\mathbf{M} \Leftrightarrow \sigma = 1$  is a singular value of  $\mathbf{H}(j\omega_0)$



## Checking passivity Scattering representation

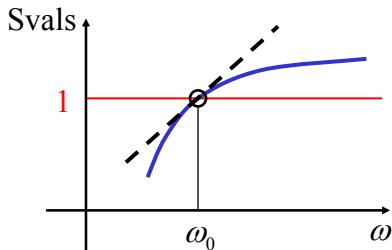




## Checking passivity Scattering representation

First-order perturbation of Hamiltonian eigenvalues

$$\boxed{\text{Slope} = \text{Im} \left\{ \frac{\mathbf{w}^T \mathbf{v}}{\mathbf{w}^T \mathbf{M}' \mathbf{v}} \right\}}$$



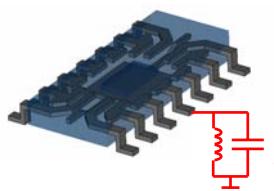
$\mathbf{w}, \mathbf{v}$ : Left and right eigenvectors of  $\mathbf{M}$  associated to  $\omega_0$

$\mathbf{M}'$ : Another Hamiltonian matrix (computed via  $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ )

**E**  
**M**  
**C**  
GROUP



## Example 4: change terminations

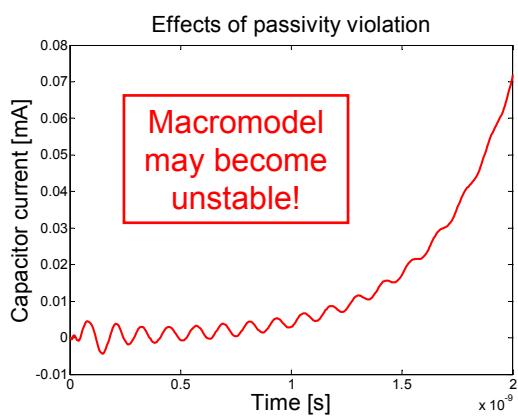


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \Omega$$

$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$

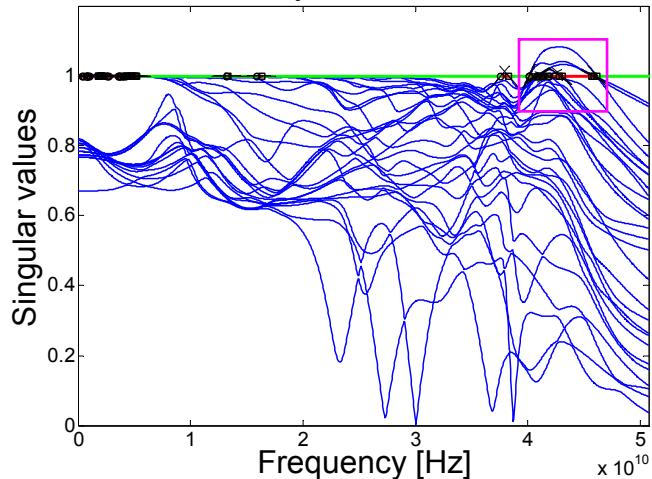


**E**  
**M**  
**C**  
GROUP



## Example 4: passivity characterization

Passivity characterization



S. Grivet-Talocia, SPI tutorial, 9 May 2004

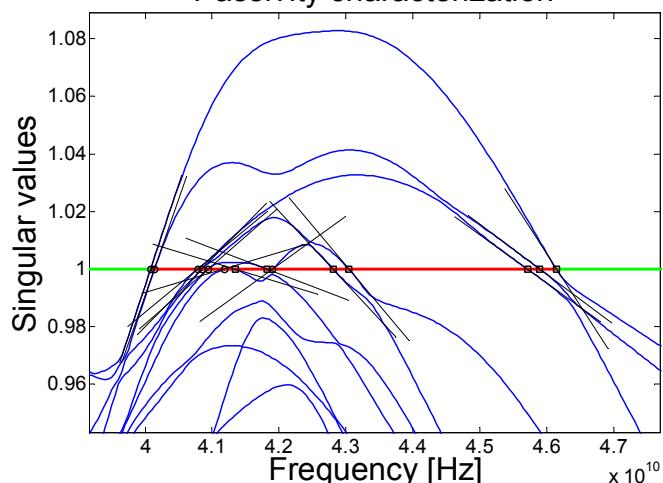
165

**E**  
**M**  
**C**  
GROUP



## Example 4: passivity characterization

Passivity characterization



S. Grivet-Talocia, SPI tutorial, 9 May 2004

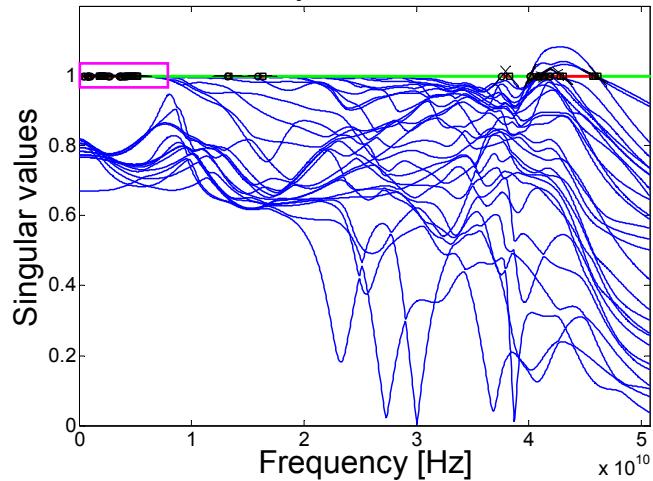
166

**E**  
**M**  
**C**  
GROUP



## Example 4: passivity characterization

Passivity characterization



S. Grivet-Talocia, SPI tutorial, 9 May 2004

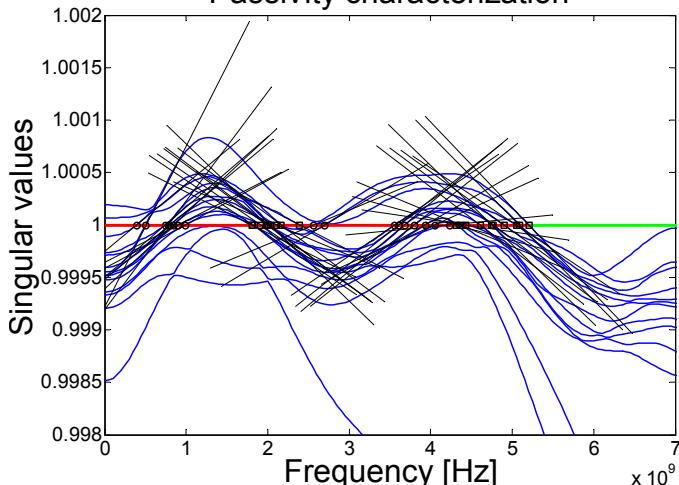
167

EMC  
GROUP



## Example 4: passivity characterization

Passivity characterization



S. Grivet-Talocia, SPI tutorial, 9 May 2004

168

EMC  
GROUP



## Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

$$\begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases} \xrightarrow{\hspace{1cm}} \begin{cases} \dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{x} \\ \mathbf{y} = (\mathbf{C} + \mathbf{dC}) \mathbf{w} + \mathbf{D} \mathbf{x} \end{cases}$$



## Passivity enforcement

Several different approaches are possible. Examples are

### Quadratic/convex optimization

[B.Gustavsen, A.Semlyen: IEEE Trans. Power Systems, vol.16, 2001]  
[C.P.Coelho, J.Phillips, L.M.Silveira, IEEE Trans. CADICAS, vol.23, 2004]

### Trace parameterization/Semi-Definite Programming

[H.Chen, J.Fang: Proc. EPEP, 2003]

### Perturbation of residues

[D.Saraswat, R.Achar, M.Nakhla: Proc. EPEP, 2003]

### Perturbation of Hamiltonian eigenvalues

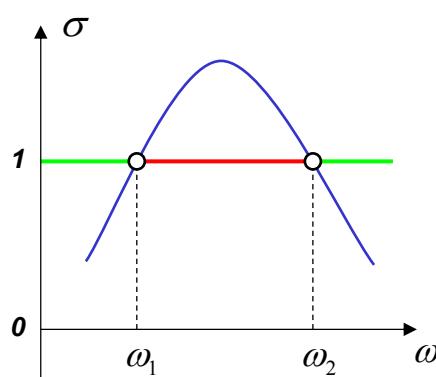
[S.Grivet-Talocia: Proc. SPI, 2003 and IEEE Trans. CAS (in press)]

### Many others... Hot research topic!

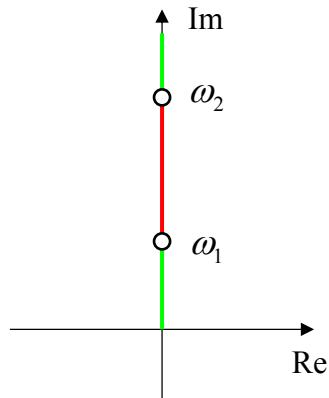


## Perturbation of Hamiltonian Eigs

Singular values of H

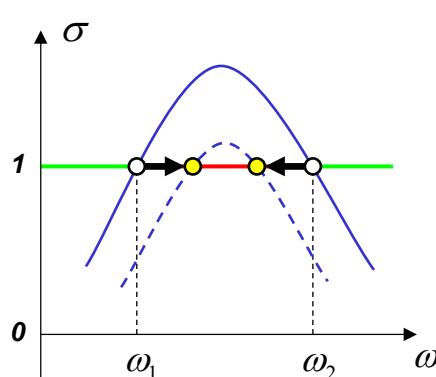


Eigenvalues of M

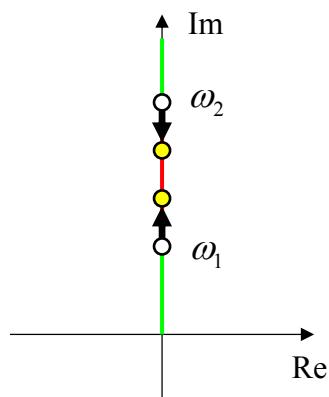


## Perturbation of Hamiltonian Eigs

Singular values of H



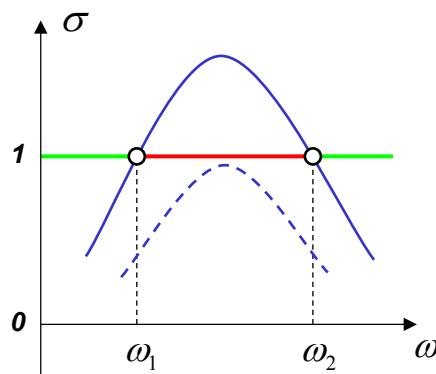
Eigenvalues of M



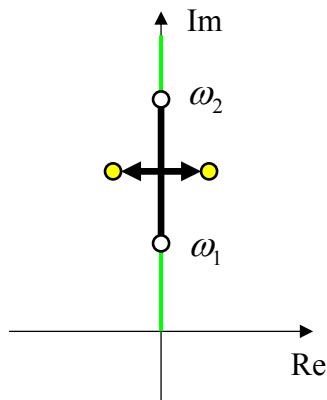


## Perturbation of Hamiltonian Eigs

Singular values of H



Eigenvalues of M



## Perturbation of Hamiltonian Eigs

First-order perturbation of eigenvalues (again)

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \end{cases}$$

Perturb state matrix  $\mathbf{C}$   
 $\tilde{\mathbf{C}} = \mathbf{C} + \mathbf{dC}$

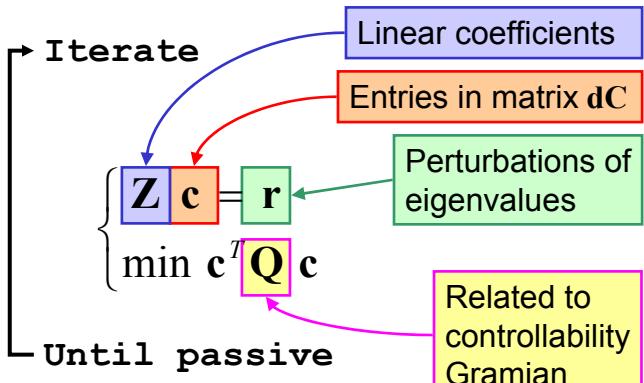
$$\tilde{\mathbf{M}} \approx \mathbf{M} + \mathbf{dM} \quad (\text{first-order: } \mathbf{dM} \text{ is linear in } \mathbf{dC})$$

$$\mathbf{w}_m^T \mathbf{dM} \mathbf{v}_m \approx j(\tilde{\omega}_m - \omega_m) \mathbf{w}_m^T \mathbf{v}_m$$

Linear constraint on the correction matrix  $\mathbf{dC}$



## Perturbation of Hamiltonian Eigs



## Preserve accuracy of macromodel

Minimize this norm !

$$\sum_{i,j} \int_0^\infty (\tilde{h}_{i,j}(t) - h_{i,j}(t))^2 dt = \| dC W dC^T \|_F^2$$

Induced perturbation in  
the impulse responses

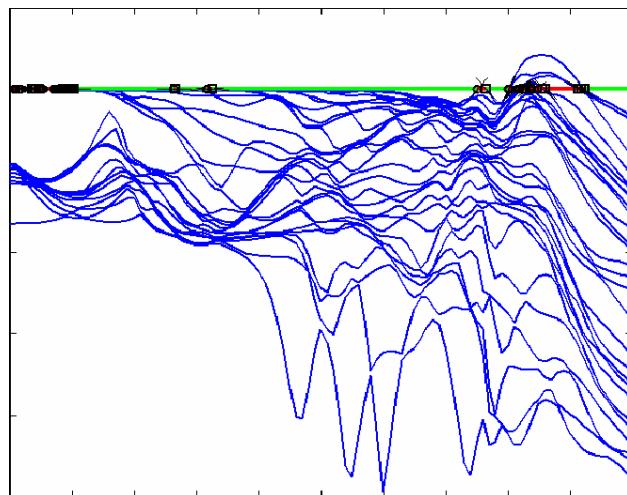
Weighted norm of state  
matrix perturbation

$W$ : controllability Gramian

$$AW + WA^T = -BB^T$$



## Example 4: passivity compensation



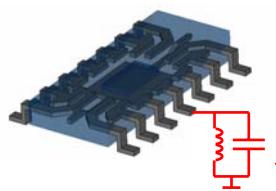
S. Grivet-Talocia, SPI tutorial, 9 May 2004

177

EMC  
GROUP



## Example 4 : passivity compensation

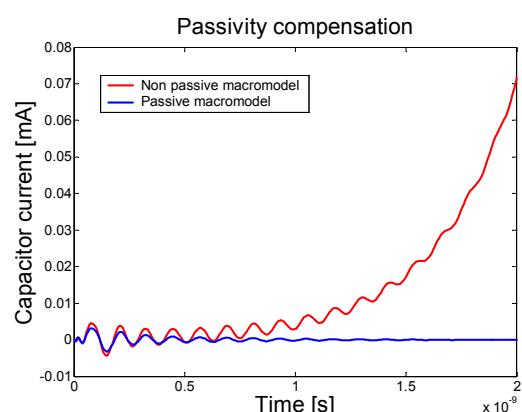


Port terminations:

$$R = 50 \text{ m}\Omega \div 50 \Omega$$

$$L = 1 \text{ nH}$$

$$C = 1 \text{ pF}$$



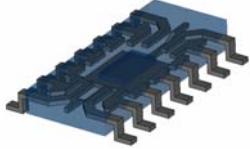
S. Grivet-Talocia, SPI tutorial, 9 May 2004

178

EMC  
GROUP



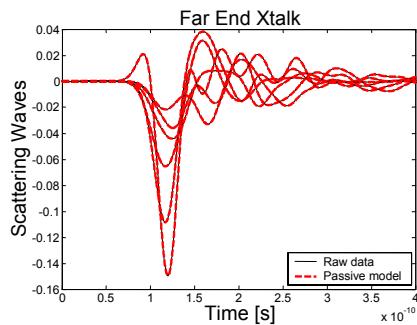
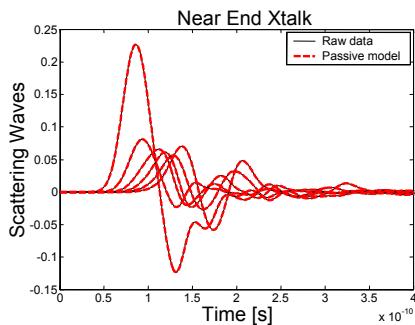
## Example 4: passive macromodel



Raw data

Macromodel

Passive macromodel



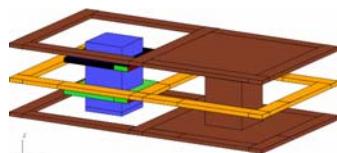
S. Grivet-Talocia, SPI tutorial, 9 May 2004

179

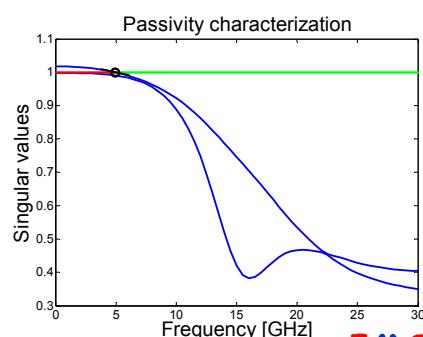
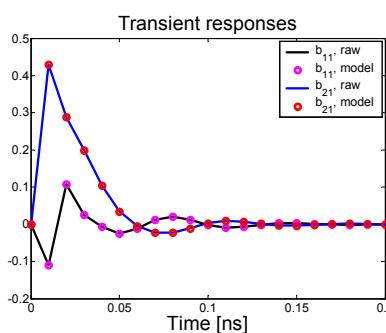
EMC  
GROUP



## Example 1: single via, nonpassive



Raw data:  
Triangle Impulse Responses  
obtained by a transient PEEC  
solver (by Dr. Ruehli, IBM)



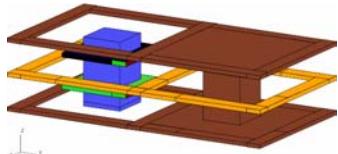
S. Grivet-Talocia, SPI tutorial, 9 May 2004

180

EMC  
GROUP

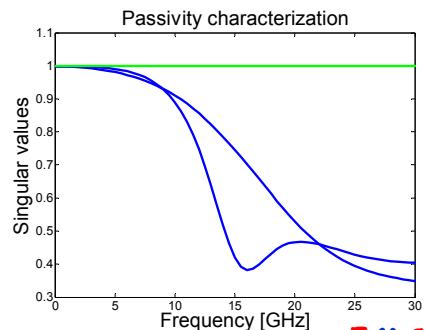
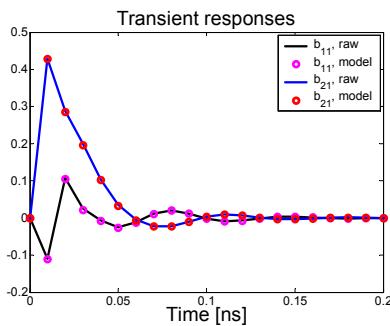


## Example 1: single via, passive



Raw data:

Triangle Impulse Responses obtained by a transient PEEC solver (by Dr. Ruehli, IBM)



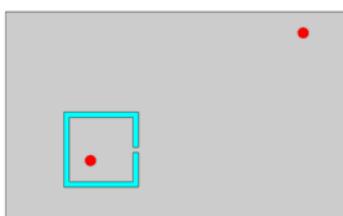
S. Grivet-Talocia, SPI tutorial, 9 May 2004

181

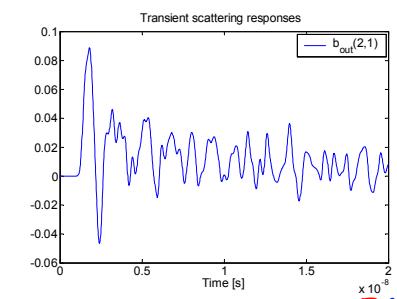
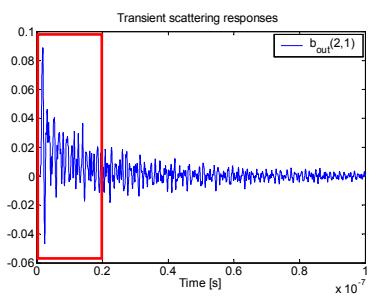
EMC  
GROUP



## Example 2: segmented power bus



- 2-port structure
- Time-Domain solution
- CST Microwave Studio
- Bandwidth: 3 GHz
- $50\ \Omega$  port terminations



S. Grivet-Talocia, SPI tutorial, 9 May 2004

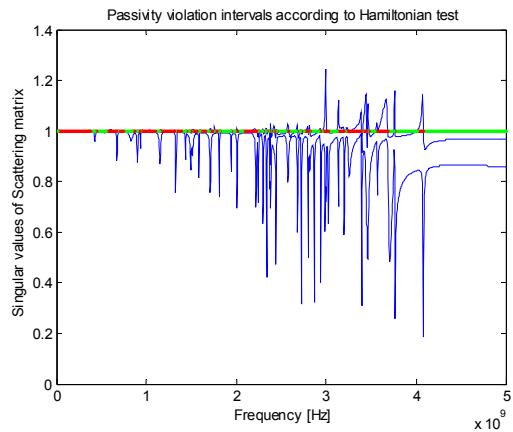
182

EMC  
GROUP



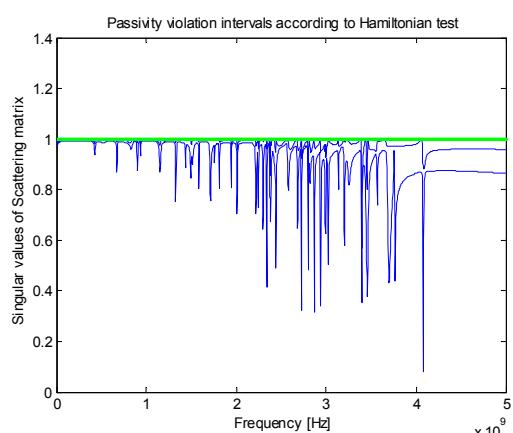
## Example 2: segmented power bus

### Passivity characterization



## Example 2: segmented power bus

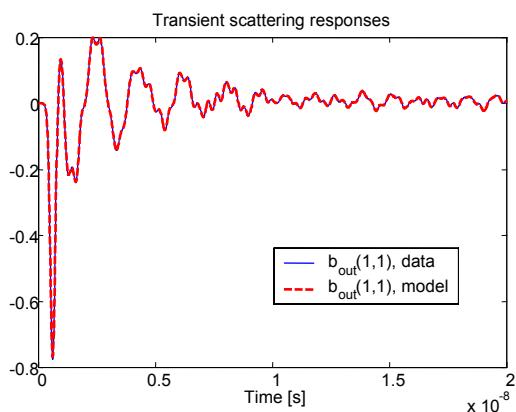
### Passivity compensation





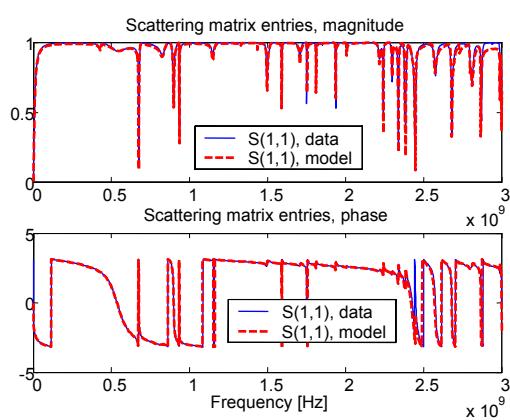
## Example 2: segmented power bus

80-poles **passive** model



## Example 2: segmented power bus

Comparison vs. frequency-domain scattering data





## More examples...

41 poles, 2 ports

Compensation

Accuracy

110 poles, 5 ports

Compensation

Accuracy

308 poles, 11 ports

Compensation

Accuracy

EMC  
GROUP



## Outline

- Introduction
- Macromodeling approaches for 3D Interconnects
- Model Order Reduction methods
  - PRIMA
- Model Identification methods
  - Frequency-Domain Vector Fitting
  - Time-Domain Vector Fitting
  - Passivity characterization and enforcement
- SPICE synthesis

EMC  
GROUP



## Macromodel implementation

### Main approaches

1. Synthesize an **equivalent circuit** in **SPICE** format
  - No access to SPICE kernel
  - Must use **standard circuit elements**
2. Direct **SPICE** implementation via **recursive convolution**
  - Laplace element**, most efficient
3. Other languages for **mixed-signal** analyses
  - Verilog-AMS**, **VHDL-AMS**, ...
  - Equation-based

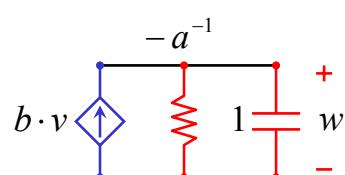
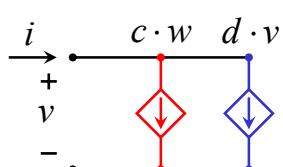


## SPICE synthesis

Admittance representation

One-port, one-pole

$$\begin{cases} \dot{w} = a w + b v \\ i = c w + d v \end{cases}$$





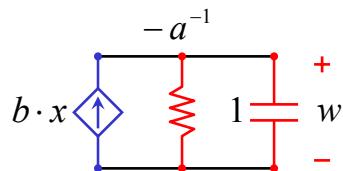
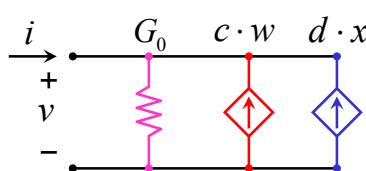
## SPICE synthesis

Scattering representation

One-port, one-pole

$$x = G_0 v + i, y = G_0 v - i$$

$$\begin{cases} \dot{w} = a w + b x \\ y = c w + d x \end{cases}$$

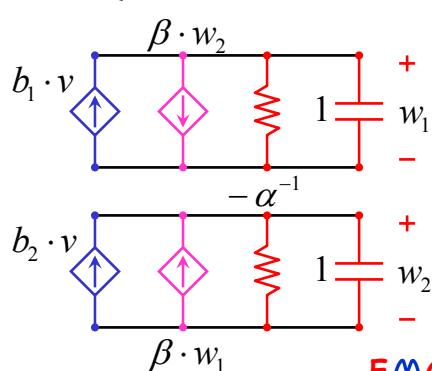
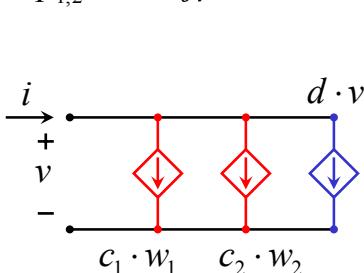


## SPICE synthesis

Admittance representation

One-port, two-poles (complex)

$$p_{1,2} = \alpha \pm j\beta$$





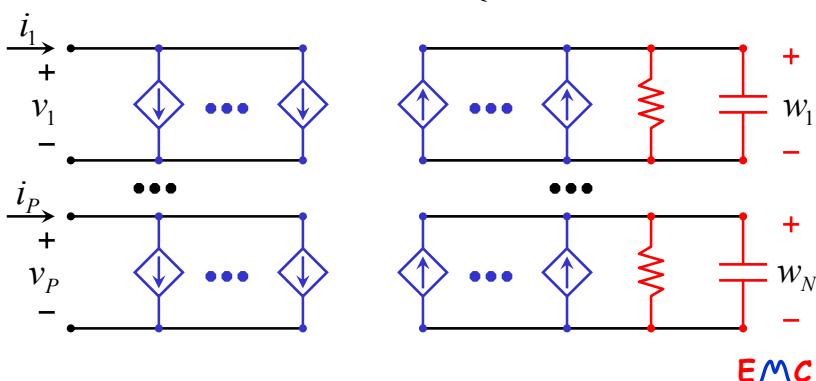
## SPICE synthesis

Admittance representation

General state-space synthesis

$$\dot{\mathbf{w}} = \mathbf{A} \mathbf{w} + \mathbf{B} \mathbf{v}$$

$$\mathbf{i} = \mathbf{C} \mathbf{w} + \mathbf{D} \mathbf{v}$$



## Recursive convolutions

$$\mathbf{H}(s) = \mathbf{D} + \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} = \mathbf{D} + \sum_n \frac{\mathbf{R}_n}{s - p_n}$$

$$\mathbf{h}(t) = \mathbf{D}\delta(t) + \sum_n \mathbf{R}_n e^{p_n t} u(t)$$

$$\mathbf{y}(t) = \mathbf{D}\mathbf{x}(t) + \sum_n \mathbf{R}_n \int_0^t e^{p_n(t-\tau)} \mathbf{x}(\tau) d\tau$$



## Recursive convolutions

$$\begin{aligned}\tilde{\mathbf{y}}(t_k) &= \int_0^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau && \text{Discrete time} \\ &= \int_0^{t_{k-1}} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau \\ &= e^{p\Delta t_k} \int_0^{t_{k-1}} e^{p(t_{k-1}-\tau)} \mathbf{x}(\tau) d\tau + \int_{t_{k-1}}^{t_k} e^{p(t_k-\tau)} \mathbf{x}(\tau) d\tau \\ &\approx e^{p\Delta t_k} \tilde{\mathbf{y}}(t_{k-1}) + \frac{1 - e^{p\Delta t_k}}{p} \mathbf{x}(t_k) && \text{Approximation!}\end{aligned}$$



## The macromodeling dream...

### Arbitrary characterization of the structure

- Equation-based or Black-Box
- Time or frequency, simulation or measurement

### Generation of a broadband macromodel

- Any order, any number of ports
- Any prescribed accuracy
- Stable and passive by construction
- Efficient (reduced-order and low-complexity)
- Fully automatic