Enforcing Passivity of Macromodels via Spectral Perturbation of Hamiltonian Matrices

S. Grivet-Talocia

Dept. of Electronics, Politecnico di Torino, C. Duca degli Abruzzi 24, 10129 Torino, Italy
Tel: +39-011-564-4104, Fax: +39-011-564-4099, E-mail: grivet@polito.it

Abstract

This paper presents a new technique for the enforcement of passivity of linear macromodels. The proposed algorithm is applicable to state-space realizations of the macromodels in case the input-output transfer function is in admittance, impedance, hybrid, or scattering form. The core of the algorithm is the application of first-order perturbation theory to the eigenvalues of an associated Hamiltonian matrix. This allows both a precise definition of the frequency bands where passivity violations occur, and the determination of a new set of state matrices leading to passivity compensation. The main algorithm is very efficient in the case of small passivity violations, so that first-order perturbation is feasible. An application to passive macromodeling of a package structure is presented.

Introduction

The research that motivates this work is focused on the generation of linear lumped macromodels for multiport interconnects. Such macromodels are of paramount importance for the analysis and design of any high-speed electronic system. In fact, the Signal Integrity (SI) of such systems can only be assessed by accurate system-level simulations including suitable models for all the parts of the system that have some influence on the signals. Each of these models must be passive, otherwise serious instabilities may occur during the simulations.

The standard procedure for the generation of lumped macromodels is to derive some rational approximation of the transfer matrix for the structure under investigation. Some techniques are available for the direct generation of passive macromodels (see, e.g., [4, 8]). These techniques require a circuit description (possibly including transmission lines) of the structure. However, such description is not always available. Therefore, alternative approaches have been proposed for the identification of lumped macromodels starting from input-output port responses, either in time or frequency domain (see, e.g., [5, 6]). Although very accurate approximations can be generated even for quite complex structures, the typical outcome of such methods is are stable but possibly non-passive models. This paper intends to propose a technique for the detection, characterization, and compensation of such passivity violations.

Passivity may be defined in a loose sense as the inability of a given structure to generate energy. The precise definition of passivity [1] requires that the transfer matrix under investigation be positive real (in case of hybrid representations of the multiport) or bounded real (in case of scattering representations). The direct application of these definitions for testing passivity, however, requires a frequency sweep since these conditions need to be checked at any frequency. The results of such tests, therefore, depend on an accurate sampling of the frequency axis, which is not a trivial task. Erroneous results may occur. For this reason, purely algebraic passivity tests are highly desirable.

Fortunately, the Positive Real Lemma and the Bounded Real Lemma provide an answer to this problem [3]. These results provide a connection between the passivity definitions and various equivalent algebraic conditions. These conditions can be expressed via feasibility of Linear Matrix Inequalities (LMI), or via existence of solutions to equivalent Algebraic Riccati Equations (ARE), or via the spectral properties of associated Hamiltonian matrices. For an excellent review and for a rich bibliography on the subject we refer the reader to [3]. In this work, we focus on the latter formulation using Hamiltonian matrices, since a study of their spectral properties leads not only to a precise criterion for passivity check, but also to a simple algorithm for the passivity compensation in case some passivity violations are detected. Both the passivity check and the compensation algorithms proposed in this paper are based on first-order spectral perturbations of the associated Hamitonian matrices.

Preliminary and Notations

In this work, we concentrate on the characterization and compensation of passivity of a given linear macromodel. Our starting point will be a state-space realization of the macromodel, ragardless of the particular macromodeling algorithm that was used to derive it. Therefore, we consider a linear time-invariant multiport system $\mathcal{M}$ in state space form

$$\mathcal{M}: \begin{cases} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\,, \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)\,. \end{cases} \tag{1}$$

where the dot denotes time differentiation. The number of ports and the dynamic order will be denoted by p and n, respectively, so that the state vector $\boldsymbol{x} \in \mathbb{R}^n$, and the input and output vectors $\boldsymbol{u}$, $\boldsymbol{y} \in \mathbb{R}^p$. Two main multiport representations will be discussed, namely the scattering and the hybrid representation. In the former case, port input and output vectors $\boldsymbol{u}$, $\boldsymbol{y}$ will be identified with either voltage, current, or power waves entering and exiting the system, respectively. In the latter case, they will denote port voltages and currents. Note that we include in the hybrid case both impedance (open-circuit) and admittance (short-circuit) representations.

The input-output transfer matrix associated to (1) is

$$\boldsymbol{H}(s) = \boldsymbol{D} + \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}\,, \tag{2}$$

where s is the Laplace variable. Throughout the following we will assume that the system (1) is strictly stable, so that all eigenvalues of $\boldsymbol{A}$, or equivalently the poles of $\boldsymbol{H}(s)$ have a strictly negative real part. In addition, we will postulate both controllability and observability [7] for the state-space realization (1), which is thus assumed to be minimal. If this is not the case, standard reduction techniques can be applied. Finally, no assumptions on reciprocity will be made.

The characterization of passivity for the macromodel in (1) depends on the type of representation being adopted. For the particular case of hybrid representations, the transfer matrix must be positive real [1]. Since we consider only the case of strictly stable systems with no poles in the imaginary axis, the passivity condition may be expressed by requiring that the Hermitian part of the transfer matrix must be nonnegative definite on the imaginary axis, i.e.,

$$\boldsymbol{G}(j\omega) = (\boldsymbol{H}(j\omega) + \boldsymbol{H}^H(j\omega))/2 \geq 0, \quad \forall\omega\,, \tag{3}$$

where the superscript ^{H} denotes the complex conjugate transpose. This condition can be checked by ensuring that

$$\lambda_i(j\omega) \geq 0, \quad \forall\lambda_i(j\omega) \in \lambda(\boldsymbol{G}(j\omega))\,, \quad \forall\omega\,, \tag{4}$$

where we denoted as $\lambda(\boldsymbol{G})$ the set of all eigenvalues of matrix $\boldsymbol{G}$. In case of scattering representations, the transfer matrix must be unitary bounded, i.e.,

$$\boldsymbol{I} - \boldsymbol{H}^H(j\omega)\boldsymbol{H}(j\omega) \geq 0, \quad \forall\omega\,. \tag{5}$$

This condition is equivalent to

$$\max_{i,\omega} \sigma_i(j\omega) \leq 1, \quad \forall\sigma_i(j\omega) \in \sigma(\boldsymbol{H}(j\omega))\,, \tag{6}$$

where we denoted as $\sigma(\boldsymbol{H})$ the set of all singular values of matrix $\boldsymbol{H}$. We will denote the system as *locally passive* for $\omega \in (\omega_a, \omega_b)$ if the above conditions are satisfied in this frequency band.

Throughout this paper, we will assume that the passivity conditions are strictly satisfied for $s \to \infty$, i.e., we will postulate *strict asymptotic passivity*. In the hybrid case such condition reads

$$\min\{\lambda((\boldsymbol{D} + \boldsymbol{D}^T)/2)\} = \lim_{\omega\to\infty} \min\{\lambda(\boldsymbol{G}(j\omega))\} > 0\,. \tag{7}$$

whereas in the scattering case we have

$$\max\{\sigma(\boldsymbol{D})\} = \lim_{\omega\to\infty} \max\{\sigma(\boldsymbol{H}(j\omega))\} < 1\,. \tag{8}$$

Characterization of passivity violations

The passivity of a multiport described by its transfer matrix (2) can be characterized at various levels of detail. One may be interested in a binary test answering the simple question whether the multiport is passive or not. However, more refined characterizations are possible. In particular, we will describe here an algebraic procedure that allows to pinpoint very accurately the frequency bands where passivity violations occur, i.e., where either (4) or (6), according to the specific representation being investigated, are not satisfied.

The following two theorems, which we report from [2] without proof, motivate the introduction of Hamiltonian matrices for passivity characterization.

Theorem 1 *(Scattering representation). Assume* $\boldsymbol{A}$ *has no imaginary eigenvalues,* $\gamma > 0$ *is not a singular value of* $\boldsymbol{D}$*, and* $\omega_0 \in I\!R$*. Then,* $\gamma \in \sigma(\boldsymbol{H}(j\omega_0))$ *if and only if* $j\omega_0 \in \lambda(\boldsymbol{M}_\gamma)$*, where*

$$\boldsymbol{M}_\gamma = \begin{pmatrix} \boldsymbol{A} - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{D}^T\boldsymbol{C} & -\gamma\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T \\ \gamma\boldsymbol{C}^T\boldsymbol{S}^{-1}\boldsymbol{C} & -\boldsymbol{A}^T + \boldsymbol{C}^T\boldsymbol{D}\boldsymbol{R}^{-1}\boldsymbol{B}^T \end{pmatrix}, \tag{9}$$

and $\boldsymbol{R} = (\boldsymbol{D}^T\boldsymbol{D} - \gamma^2\boldsymbol{I})$ *and* $\boldsymbol{S} = (\boldsymbol{D}\boldsymbol{D}^T - \gamma^2\boldsymbol{I})$.

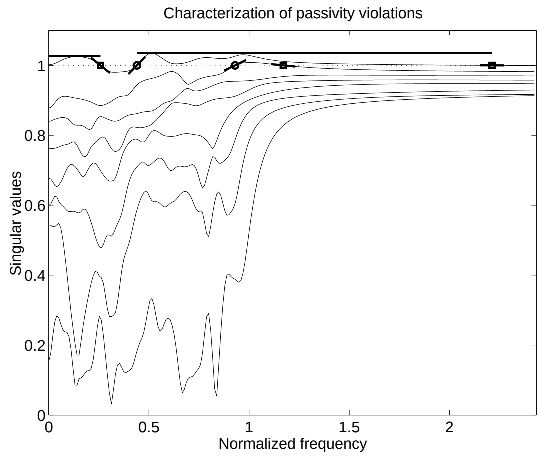

Figure 1: Collective information gathered from the proposed passivity characterization algorithms for a synthetic non-passive system. Breakpoints, slopes, and bounds for singular values are depicted with thick lines. The thin lines represent all singular values plotted versus frequency.

Theorem 2 *(Hybrid representations). Assume* $\boldsymbol{A}$ *has no imaginary eigenvalues,* $\delta > 0$ *is not an eigenvalue of* $(\boldsymbol{D} + \boldsymbol{D}^T)/2$*, and* $\omega_0 \in I\!R$*. Then,* $\delta \in \lambda(\boldsymbol{G}(j\omega_0))$ *if and only if* $j\omega_0 \in \lambda(\boldsymbol{N}_\delta)$*, where*

$$\boldsymbol{N}_\delta = \begin{pmatrix} \boldsymbol{A} + \boldsymbol{B}\boldsymbol{Q}^{-1}\boldsymbol{C} & \boldsymbol{B}\boldsymbol{Q}^{-1}\boldsymbol{B}^T \\ -\boldsymbol{C}^T\boldsymbol{Q}^{-1}\boldsymbol{C} & -\boldsymbol{A}^T - \boldsymbol{C}^T\boldsymbol{Q}^{-1}\boldsymbol{B}^T \end{pmatrix}, \tag{10}$$

and $\boldsymbol{Q} = (2\delta\boldsymbol{I} - \boldsymbol{D} - \boldsymbol{D}^T)$.

Matrices $\boldsymbol{M}_\gamma$ and $\boldsymbol{N}_\delta$ are said to be Hamiltonian due to their particular block structure. Note that these theorems allow to compute the exact frequencies (if any) at which the singular values of the eigenvalues cross or touch any given threshold. These frequencies are the imaginary eigenvalues of the Hamiltonian matrices. Therefore, setting the threshold to the critical level for passivity (i.e., $\gamma = 1$ for scattering and $\delta = 0$ for hybrid representations), allows to derive a simple algebraic procedure for passivity characterization. We illustrate this procedure using the scattering represenation, and we will only summarize the results for the hybrid case.

Let us consider a non-passive system and collect the (positive imaginary parts of the) imaginary eigenvalues at the critical level $\gamma = 1$ in the set

$$\Omega = \{\omega_i > 0 \;:\; j\omega_i \in \lambda(\boldsymbol{M}_1)\}\;. \tag{11}$$

We remark that only the particular case of simple eigenvalues will be herewith considered. The precise characterization of the frequency bands where passivity violations occur can be obtained via application of perturbation theory to each eigenvalue in the set Ω. More precisely, if we indicate with $\lambda_i^{(\gamma)} \in \lambda(\boldsymbol{M}_\gamma)$ the perturbation of some eigenvalue $j\omega_i$ for $\gamma \simeq 1$, we have the following convergent power series representation

$$\lambda_i^{(\gamma)} = j\omega_i + k_i'(\gamma - 1) + k_i''(\gamma - 1)^2 + \ldots \tag{12}$$

The first-order coefficient k_i' can be computed as [9]

$$k_i' = \frac{\boldsymbol{w}_i^T\boldsymbol{M'}_1\boldsymbol{v}_i}{\boldsymbol{w}_i^T\boldsymbol{v}_i}\,, \tag{13}$$

where $\boldsymbol{w}_i$, $\boldsymbol{v}_i$ are the left and right eigenvectors of $\boldsymbol{M}_1$ associated to $j\omega_i$ and

$$\boldsymbol{M}_\gamma = \boldsymbol{M}_1 + (\gamma - 1)\boldsymbol{M'}_1 + \dots \qquad (14)$$

It can be easily veryfied that $\Re\{k'_i\} = 0$. Consequently, if the singular values are plotted versus frequency (see Fig. 1 for an example), the quantities

$$\xi_i = \frac{1}{\Im\{k'_i\}} = \frac{j\,\boldsymbol{w}_i^T \boldsymbol{v}_i}{\boldsymbol{w}_i^T \boldsymbol{M'}_1 \boldsymbol{v}_i} \qquad (15)$$

represent the slopes of the singular value curves at the crossing points $\omega_i \in \Omega$. These observations can be summarized by the following theorem, which gives a condition for the characterization of local passivity in the frequency bands determined by the breakpoints in the set Ω.

Theorem 3 *(Scattering representation) Let Ω, defined as in (11), collect the (positive imaginary parts of the) imaginary eigenvalues of the Hamiltonian matrix $\boldsymbol{M}_\gamma$ in (9) at the critical level $\gamma = 1$, sorted in ascending order. Let all these imaginary eigenvalues be simple. Finally, let ξ_i be defined as in (15). Then, $\boldsymbol{H}(j\omega)$ is locally passive for $\omega \in (\omega_{i-1}, \omega_i)$ if and only if*

$$\Lambda_i = \sum_{k \geq i} \mathrm{sgn}(\xi_k) = 0\,, \qquad (16)$$

where $\mathrm{sgn}(\cdot)$ extracts the sign of its argument and $\omega_0 = 0$.

Proof. The proof follows immediately from the above considerations if the system is asymptotically passive. In fact, starting from the largest element in Ω and moving towards decreasing frequencies, each time a crossing (i.e., an element of Ω) is found with a negative slope, the number of singular values exceeding the critical level increases by one. Conversely, when a crossing is found with negative slope, this number decreases by one. Therefore, the sum in (16) indicates exactly the number of singular values exceeding the critical level in the frequency band (ω_{i-1}, ω_i). If this number equals zero, the system is locally passive in (ω_{i-1}, ω_i). □

We give now the corresponding result for the case of hybrid representations. The passivity characterization is summarized in the following theorem, that we state without proof.

Theorem 4 *(Hybrid representation) Let Ω, defined as in (11), collect the (positive imaginary parts of the) imaginary eigenvalues of the Hamiltonian matrix $\boldsymbol{N}_\delta$ in (10) at the critical level $\delta = 0$, sorted in ascending order. Let all these imaginary eigenvalues be simple. Finally, let ζ_i be defined as*

$$\zeta_i = \frac{j\,\boldsymbol{w}_i^T \boldsymbol{v}_i}{\boldsymbol{w}_i^T \boldsymbol{N'}_0 \boldsymbol{v}_i} \qquad (17)$$

where

$$\boldsymbol{N}_\delta = \boldsymbol{N}_0 + \delta\boldsymbol{N'}_0 + \dots \qquad (18)$$

Then, $\boldsymbol{H}(j\omega)$ is locally passive for $\omega \in (\omega_{i-1}, \omega_i)$ if and only if

$$\Lambda_i = \sum_{k \geq i} \mathrm{sgn}(\xi_k) = 0\,, \qquad (19)$$

where $\omega_0 = 0$.

The information provided by Theorems 3 and 4 can be complemented by a precise quantification of the passivity violation in each frequency band. In particular, if (ω_{i-1}, ω_i) has been detected to be a violation band, the bisection algorithm found in [2] can be used to determine the maximum singular value (or the minimum eiganvalue for hybrid representations) within this band to any prescribed accuracy. The passivity characterization information that can be extracted using the above tools are collectively depicted in Fig. 1 for a synthetic non-passive system.

Enformcement of Passivity

In this section, we address the problem of finding a passive approximation to a given stable but non-passive macromodel. In particular, we want to derive a new passive system $\mathcal{M}_p$ which is "close" in some sense to the original $\mathcal{M}$. More precisely, a new set of perturbed state matrices will be derived, with the aim of minimizing the induced perturbation on the input-output responses of the macromodel. As a particular case, we will perturb only the state matrix $\boldsymbol{C}$ by the amount $\boldsymbol{dC}$, leaving the other three matrices unchanged. In such case, if we denote with $dh_{ij}(t)$ the induced perturbation on the system impulse responses, it can be easily veryfied that

$$\sum_{i,j=1}^{p} \int_0^\infty |(dh)_{i,j}(t)|^2\,\mathrm{d}t \;=\; \mathrm{tr}\left(\boldsymbol{dC}\,\boldsymbol{W}\,\boldsymbol{dC}^T\right)\,, \qquad (20)$$

where $\boldsymbol{W}$ is the controllability Gramian [7]. The latter can be computed as the unique symmetric and positive definite solution of the Lyapunov equation

$$\boldsymbol{AW} + \boldsymbol{WA}^T = -\boldsymbol{BB}^T\,. \qquad (21)$$

The perturbation $\boldsymbol{dC}$ to be determined will be constrained to minimize the norm in (20).

We now derive suitable conditions on the perturbation $\boldsymbol{dC}$ leading to the enforcement of passivity. As noted in previous sections, this is equivalent to obtaining a new Hamiltonian matrix without imaginary eigenvalues. Therefore, the main algorithm is aimed at displacing these eigenvalues by some controlled amount in order to force them to move off the imaginary axis. Let us consider the perturbed Hamiltonian matrix (in the scattering case)

$$\boldsymbol{M}_1|_p \simeq \boldsymbol{M}_1 + \boldsymbol{dM}_1\,, \qquad (22)$$

obtained when $||\boldsymbol{dC}|| \ll ||\boldsymbol{C}||$, so that only first-order terms can be considered. Suppose that an imaginary eigenvalue $j\omega_i \in \lambda(\boldsymbol{M}_1)$ is perturbed by a small amount into a new location $j\omega_{i,p}$. We have the following result

$$j\omega_{i,p} - j\omega_i \simeq \frac{\boldsymbol{w}_i^T \boldsymbol{dM}_1 \boldsymbol{v}_i}{\boldsymbol{w}_i^T \boldsymbol{v}_i}\,. \qquad (23)$$

A simple check shows that the above expression is linear in the elements of the perturbation matrix $\boldsymbol{dC}$. Therefore, this condition can be restated as a standard underdetermined linear least squares problem

$$\boldsymbol{Z}\,\boldsymbol{c} \;=\; \boldsymbol{r}\,, \qquad (24)$$

where all the entries of $\boldsymbol{dC}$ have been collected in the column array $\boldsymbol{c}$, and where array $\boldsymbol{r}$ collects the desired perturbations on all imaginary eigenvalues.

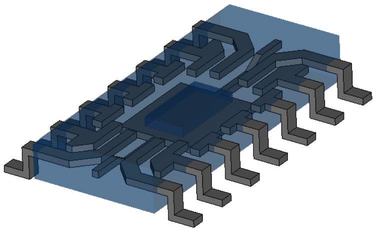

Figure 2: Surface mount package structure with 14 pins (28 ports). Bonding wires and printed circuit board on which package is mounted are not shown.

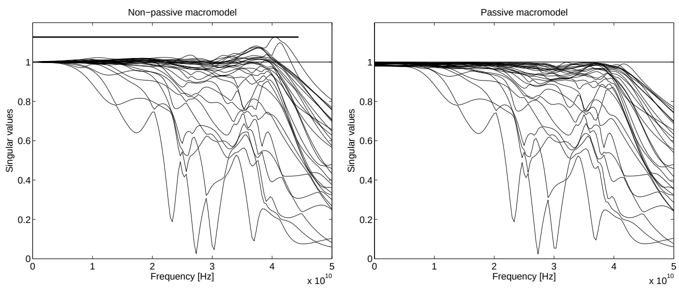

Figure 3: Passivity compensation for the package structure of Fig. 2. The singular values are plotted versus frequency for the non-passive macromodel (left panel) and for the passive macromodel (right panel).

We focus now on the determination of the new eigenvalue locations. We may refer to the example in Fig. 1 for illustration. We want to obtain the effect of lowering the singular values curves below the critical level $\gamma = 1$. It is clear that this effect can be achieved by moving each crossing point in the direction pointed by its slope, i.e., eigenvalues with positive slopes are moved towards higher frequencies, while eigenvalues with negative slopes are moved towards lower frequencies. The amount of displacement for each eigenvalue should be determined in order to satisfy the first-order assumptions. The solution of (24) with the constraint (20) is iterated until the passivity is met and no imaginary eigenvalues are found.

Example: package macromodeling

We illustrate the proposed passivity compensation algorithm through an application to macromodeling of a commercial 14-pin surface mount package, depicted in Fig. 2. The structure has $p = 28$ ports, half being defined between a corresponding pin and the printed circuit board on which the package is mounted, and half being defined between the bonding pad on the included die and the reference plane below the die itself. The structure has been meshed and analyzed with a full-wave electromagnetic solver based on the Finite-Difference Time-Domain (FDTD) method. The raw dataset obtained by FDTD is a set of 28×28 transient scattering responses due to Gaussian pulse excitation having a 30 GHz frequency bandwidth. This set of responses has been processed by a Vector Fitting algorithm [6] in order to derive a rational macromodel. The resulting state-space system is characterized by dynamic order $n = 180$ and results non-passive, with a singular value distribution depicted in the left panel of Fig. 3 (the crossing points and the slopes are not shown for clarity). A total number of 62 pairs of imaginary eigenvalues of the Hamiltonian matrix were detected. Passivity was achieved by the proposed algorithm in 50 iterations. The resulting singular values distribution is depicted in the right panel of Fig. 3. Some of the transient scattering responses of the passive macromodel are compared in Fig. 4 to the original responses obtained by the FDTD simulations, that were used for the identification of the macromodel. We can see that the passivity compensation did not degrade the accuracy in the approximation.

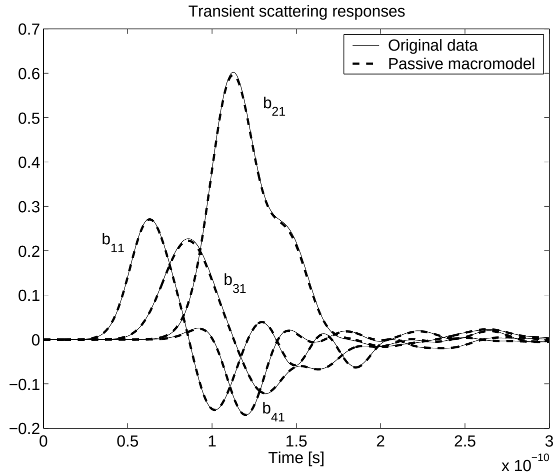

Figure 4: Selected transient responses for the package structure of Fig. 2.

Acknowledgements

This work is supported by the Italian Ministry of University (PRIN grant 2002093437), and by CERCOM, Politecnico di Torino. The author is grateful to Prof. Canavero, Prof. Maio, and Dr. Stievano for their valuable comments and suggestions.

References

[1] V. Belevitch, *Classical Network Theory*, Holden-Day, San Francisco, 1968.

[2] S. Boyd, V. Balakrishnan, P. Kabamba, "A bisection method for computing the H_∞ norm of a transfer matrix and related problems", *Math. Control Signals Systems*, Vol. 2, 1989, pp. 207–219.

[3] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan, *Linear matrix inequalities in system and control theory*, *SIAM studies in applied mathematics*, SIAM, Philadelphia, 1994.

[4] M. Celik, L. Pileggi, A. Obadasioglu, *IC Interconnect Analysis*, Kluwer, 2002.

[5] S. Grivet-Talocia, F. Canavero, I. Maio, and I. Stievano, "Reduced-order macromodeling of complex multiport interconnects," in *URSI General Assembly, Maastricht, Belgium*, August 19–23, 2002.

[6] B. Gustavsen, A. Semlyen, "Rational approximation of frequency responses by vector fitting", *IEEE Trans. Power Delivery*, Vol. 14, July 1999, pp. 1052–1061.

[7] T. Kailath, *Linear systems*, Prentice Hall, Englewood Cliffs, NJ, 1980.

[8] M. Nakhla and R. Achar, "Simulation of High-Speed Interconnects", *Proc. IEEE*, May 2001, Vol. 89, No. 5, pp. 693–728.

[9] J. H. Wilkinson, *The algebraic eigenvalue problem*, Oxford University Press, London, 1965.