Accurate and efficient transient simulation of interconnect networks with nonlinear terminations

Accurate and Efficient Transient Simulation of Interconnect Networks with Nonlinear Terminations

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The Finite-Difference Time-Domain (FDTD) method is one of the most popular schemes for the transient simulation of interconnect structures. It provides quite accurate results as far as the number of discretization points per wavelength is high. Otherwise, it is well known that numerical dispersion strongly affects the accuracy of the simulation. This can be a relevant problem when nonlinear terminations are to be handled, since interaction of incident pulses with nonlinear circuits may lead to spikes and sharp variations that may not be well represented on the FDTD mesh.

To overcome these limitations, we propose to use high-order difference schemes for the transient simulation of interconnects with nonlinear loads. In particular, we focus our attention on centered fourth-order schemes [3, 2], which offer significant improvement with respect to the standard second-order FDTD scheme. As an example, we report a numerical dispersion test obtained by launching a gaussian pulse on an infinite length transmission line (modeled through periodic boundaries). Both the FDTD results and the fourth-order scheme results are shown in Fig. 1. Time stepping is provided by the standard fourth-order Runge-Kutta (RK4) integrator, which has been selected for its very large stability region and because its approximation order is matched to the order of the spatial discretization.

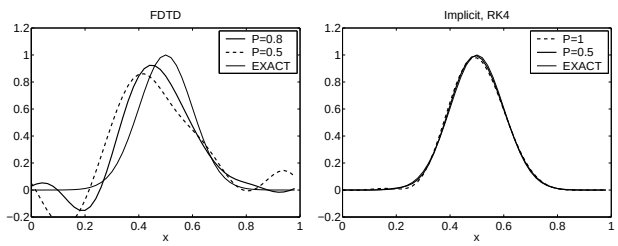

Figure 1: Accuracy of FDTD (left), and fourth-order implicit scheme with RK4 time integrator (right). The degrees of freedom are the same in the two cases, and p denotes the Courant number.

The numerical treatment of boundary conditions with high-order schemes presents some difficulties. It is well known that a standard treatment through elimination of the boundary unknowns and direct insertion of the termination equations into the difference scheme may lead to late time instability [2]. In this paper we show how this problem can be overcome by following a completely different strategy, based on a weak implementationon of the boundary conditions. The basic idea is to allow an approximation error in the imposition of the boundary conditions, provided that this error is of the same order of the difference scheme performing the spatial discretization of the interconnect equations. Since any numerical simulation leads to approximate results, it does not matter where exactly in the scheme the error is generated. It is crucial to keep this error under control, in terms of both local truncation error and its growth in time (i.e., stability).

The weak implementation of the boundary conditions is performed as follows. First, the transmission line equations are discretized in space throguh a fourth-order difference operator, including the boundary nodes. The basic principle is to treat the voltages and currents at the interconnect edges as dinstinct unknowns with respect to the voltages and currents of the termination networks. Their difference is regarded as an approximation error in the imposition of the boundary equations. Special penalty terms proportional to these errors are simply added to the discretized interconnect equations. They act as a "numerical glue", so that the dynamics of the resulting evolution equations lead to a stable control of the overall approximation error with the same order of accuracy both at inner and at boundary nodes. The formal proof, together with the precise formulation of the method for lossy multiconductor lines with nonlinear static loads, can be found in [4]. The extension to arbitrary transmission-line networks with either linear or nonlinear junctions is straightforward.

We show now a comparison between the standard second-order accuate FDTD method and two (explicit and implicit) fourth-order schemes. A normalized scalar transmission line ($Z_C = 1, T_D = 1$) is loaded with highly unmatched resistances, $R_S = 10^{-3}$, $R_L = 10^3$, with gaussian excitation. Table 1 reports the maximum er-

N	FDTD	Explicit	Implicit
40	1.6×10^{-1}	1.8×10^{-2}	1.2×10^{-1}
60	7.0×10^{-2}	4.0×10^{-3}	3.0×10^{-2}
90	3.0×10^{-2}	9.3×10^{-4}	6.0×10^{-3}
135	1.3×10^{-2}	2.1×10^{-4}	1.2×10^{-3}
200	6.1×10^{-3}	5.1×10^{-5}	2.3×10^{-4}

Table 1: Accuracy of FDTD and fourth-order schemes applied to a terminated line. Each entry reports the maximum voltage error at the right termination.

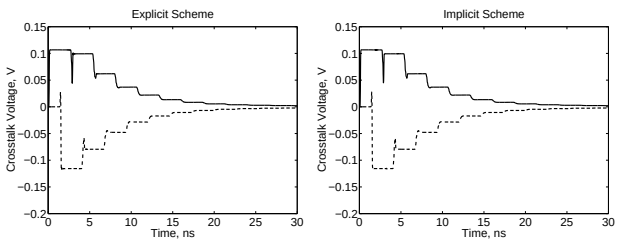

Figure 2: Near-End (continuous line) and Far-End (dashed line) crosstalk voltages for a low-loss three-conductor PCB line simulated with two fourth-order schemes.

ror at the right boundary between the exact solution and the numerical solution obtained with FDTD and with the fourth-order schemes with RK4 time advancement. The Courant number was set in all cases to $p = 0.8$, i.e., close to the stability limit of the FDTD scheme. It is evident that the fourth order schemes perform much better than FDTD in terms of both error values and decay rate under grid refinement. The achievement of a maximum error below a given threshold, say $\epsilon = 10^{-3}$, would require a very fine grid or, equivalently, a very large number of unknowns for the FDTD method (as the decay rate is N^{-2}, a simple extrapolation gives approximately $N \sim 1000$). The same error can be obtained with the fourth-order schemes with much less grid points, about $N = 140$ for the explicit one and $N = 90$ for the implicit one. In addition, the FDTD error is extremely sensitive to small variations of the Courant number p, while the high-order schemes, due to the choice of RK4 time advancement algorithm, are very stable. This means that the application of high-order schemes to the multiconductor case, where all propagation speeds for the various modes are different, allows to treat all modes approximately with the same approximation. This is obviously not possible with FDTD, which treats with high accuracy only the faster modes and deteriorates its performance for the slow modes.

Figure 2 illustrates a numerical test consisting of the crosstalk analysis of a PCB structure (see [1], pp. 317 and 351). The line is made on three PCB lands placed on one side of a glass epoxy ($\varepsilon_r = 4.7$) substrate 47 mils thick. The lands have width 15 mils, thickness 1.38 mils, and their separation is 45 mils. The line is 10 inches long and is terminated with diagonal 50 Ω loads. The excitation is a 50 ps rise-time step function applied between the two edge conductors, one of which is taken as the reference.

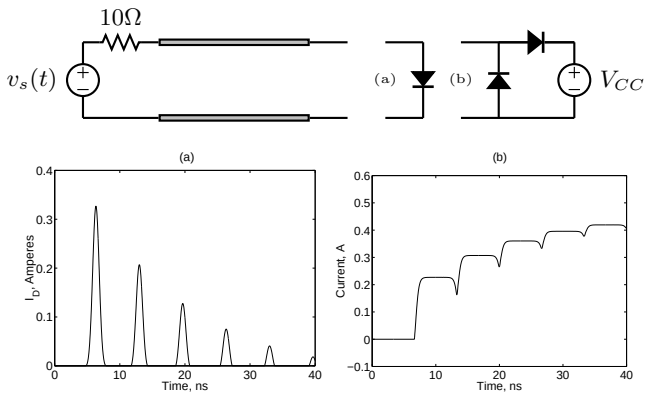

Figure 3: Currents at the right termination obtained with the fourth order scheme. Scalar line ($Z_c = 50$ Ω, $T_D = 3.33$ ns) loaded with a diode (a) and with a voltage protection circuit (b).

A further example illustrates the inclusion of nonlinear terminations. The results (see Fig. 3) obtained by loading a scalar line with a shunt diode (with a 10 V Gaussian source) and with a simple voltage protection circuit (with a 10 V step source) show no difference with the SPICE simulation, also reported in the plots.

The proposed schemes allow very accurate simulation of quite general types of interconnects with nonlinear loads. The interconnects can be lossy. In addition, also interconnect networks can be treated. These schemes can also be applied without modification to the analysis of nonuniform lines, characterized by per-unit-length parameters with longitudinal variations. These structures cannot be analyzed with SPICE. The presented results show that the transient simulation is performed at higher accuracy and reduced numerical dispersion with respect to the standard FDTD scheme. Therefore, the proposed discretizations seem to be very promising for the simulation of highly interconnected systems typically found in fast applications.

References

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