Modeling of Interconnect Junctions from measured scattering responses

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F. G. Canavero,* S. Grivet-Talocia* I. A. Maio* and I. S. Stievano*

Abstract — **This paper addresses the development of lumped equivalent circuits of interconnect junctions from their measured or simulated scattering responses. Two methods for the identification of linear dynamic systems are applied and their performances are compared for accuracy and efficiency.**

1 Introduction

Today's tendency towards an increasing use of higher frequencies both for RF and digital applications results in unwanted parasitic electromagnetic effects on the interconnection structure of digital and telecommunication systems. Signal distortions are introduced due to the mutual coupling among different conductors. Therefore, the modeling of these structures has become an unavoidable step in the analysis and design of high speed electronic systems.

In most cases, a complete electromagnetic simulation of the entire structure could be unaffordable (both in terms of computer memory needed and CPU time required). Therefore, it becomes crucial to derive equivalent models of subparts of the system. Such models must retain, in a given bandwidth, the major features of the details they reproduce, and must be simple enough to be handled in subsequent simulations. The possibility to generate circuit equivalents that are SPICE compatible is usually sought for.

This contribution addresses the modeling of linear dynamic multiport elements from measured or simulated responses, with the aim to generate lumped wideband equivalents of generic junctions. From a formal point of view, the modeling of multiport elements from their responses by means of lumped equivalents is a parametric identification problem and many different identification methods could be tried [1]. In this paper, we assess the performances of two identification approaches that seem particularly suited to the application at hand: the Block Complex Frequency Hopping (BCFH) method [2, 3] and the subspace identification methods. The BCFH algorithm allows accurate approximations of input data over wide frequency bands with simple models having a meaningful physical foundation. On the other hand, in the control system area, subspace methods are well established as the most effective approach for the modeling of multi-input multi-output linear systems from their responses.

We test both methods on realistic structures by using quasi-matched scattering responses. Such network function are preferred because, for typical interconnect structures, they have the simplest behavior and often allow the factorization of possible ideal delay terms.

2 BCFH Method

The BCFH algorithm is a moment matching order reduction method that can be exploited also for system identification, because its inputs are the moments of the response data [2, 3]. It works by estimating the true set of poles of the modeled circuit element within the bandwidth of interest (*i.e.,* in a region of the complex angular frequency plane where $\sigma < 0$ and $|\omega| < \omega_{MAX}$). This is achieved by computing the moments of some input scattering functions at several complex expansion points. The set of poles is then used to represent each scattering function with a partial fraction expansion, whose residues are computed by least square fitting the function samples, as in [2].

The estimation problem defined by the BCFH method is well conditioned and the obtained models have a solid physical foundation, because their poles are approximations of the actual poles of the modeled multiport element. This guarantees the stability of estimated models and helps the control of their passivity. In fact, as far as the accuracy of the identified models is good, the passivity condition

$$\sum_j |S_{ij}(j\omega)|^2 \leq 1, \quad \forall i, j, \text{ and } \omega \qquad (1)$$

where $S_{ij}(\omega)$ is the scattering function describing wave transmission from port i to port j, is guaranteed within the approximation bandwidth. Active spurious behavior of estimated models can arise only beyond ω_{MAX} and can be compensated a posteriori by extending ω_{MAX} to the region where the amplitudes of the scattering functions of the structure become negligible.

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The applied approach can be extended to distributed structure whose scattering characteristics allow the factorization of delay terms. For these cases, mixed models composed of lumped parts and of delay blocks can be used, thereby effectively including propagation effects without losing the advantages of lumped equivalents.

Finally, for the implementation of models as equivalent circuits, we replace wave variables with voltages and currents in the scattering equations and synthesize the resulting relations (*e.g.,* see [5]).

2.1 BCFH results

The BCFH approach has been tested on simple ideal multiport elements, obtaining good results. Even if the poles of quasi-matched scattering responses are far from the imaginary axis, their estimations via BCFH turn out to be sufficiently accurate to yield good models.

In order to check the performances of the approach on a realistic test case we apply it to the integrated circuit package of Fig. 1. This structure is composed of an ideal conductor reference plane, a dielectric layer and metallic lands between the chip and the board. It is characterized by the wave variables measured at the ends of each land with respect to the reference plane. The scattering characteristics of such a multiport element are computed in the time domain by means of a three dimensional electromagnetic simulator based on the finite integration technique. The symmetry properties of the structure are exploited and its complete 104×104 scattering matrix is obtained in 3.5 hours of computation by a Pentium PC @ 450 MHz.

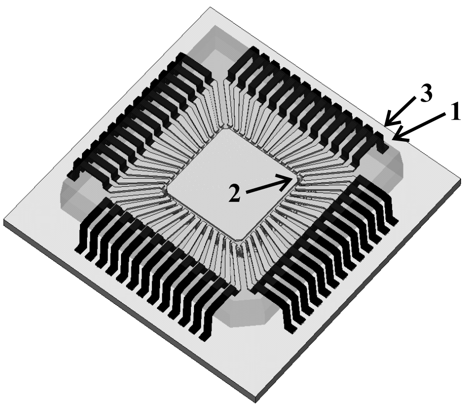

Figure 1: 52-pin package modeled by scattering data and BCFH

The input data for the modeling process are the input and output transient wave variables obtained from the time-domain fullwave simulation. Such responses are used to compute the moments of the elements of the scattering matrix and to estimate their poles via BCFH [4]. Time and frequency normalization of responses as well as moment scaling are used to generate well conditioned matrices of moments.

The search for poles of a scattering matrix element is computationally inexpensive, since it amounts to compute its first 15 – 20 moments for a few complex expansion points. Also, many poles detected from different scattering matrix elements are common and, therefore, only a subset of the scattering matrix elements must be searched. The complete set of estimated poles is then used to approximate every element of the scattering matrix by a partial fraction expansion, whose residues are obtained by least square fitting.

In this example, the BCFH search of the elements S_{ij}, $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ (ports numbered in counter-clockwise direction as shown in Fig. 1) leads to the same set of three couples of complex conjugate poles. Such poles allow a good modeling of all scattering matrix elements S_{ij}, $i = 1, 2, 3, 4$ $j = 1, 2, 3, 4$ over the bandwidth of available data, that is 7GHz. As an example, Fig. 2 shows the amplitude of $S_{11}(j\omega)$ and the phase of $S_{31}(j\omega)$ versus frequency compared with the corresponding responses of the model defined by the mentioned set of poles. The accuracy of the model can be clearly appreciated.

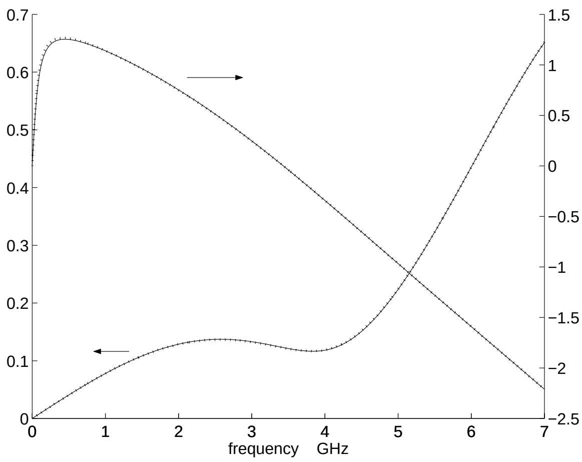

Figure 2: Reference (solid lines) and model (dotted lines) values for $|S_{11}|$ (left scale) and $\angle S_{31}$ (right scale, rad)

3 Subspace Methods

In the last years, many contributions on the estimation of time invariant, linear dynamic state-space models of multi-input multi-output systems have been published [6]. Such estimation approaches are collectively named *subspace methods* and are related to Matrix Pencil methods [7].

A discrete-time linear state-space model is defined by

$$\begin{cases} \mathbf{z}(k+1) &= \mathbf{A}\mathbf{z}(k) + \mathbf{B}\mathbf{u}(k) \\ y(k) &= \mathbf{C}\mathbf{z}(k) + \mathbf{D}\mathbf{u}(k) \end{cases} \tag{2}$$

where $\mathbf{z} \in \Re^r$ is the state vector and matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are the unknowns (parameters) of the model.

The key step of subspace methods to approximate systems with model (2) is the estimation of the column space of the extended observability matrix from suitable matrices containing the input/output perturbed data. The estimation is performed by the span of the column or row space of the matrices of input/output data. Then, the model unknowns, *i.e.,* the $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ matrices, are directly obtained from the estimation of the extended observability matrix. With the aid of subspace methods, the estimation of state-space models for circuit elements with a number of input/output of 10 and a number of states of the order of 100 is practicable and the numerical complexity is not critical [6].

3.1 Subspace estimation example

One of the ideal multiport elements used to test the modeling via subspace methods is the 2-port circuit shown in Fig. 3.

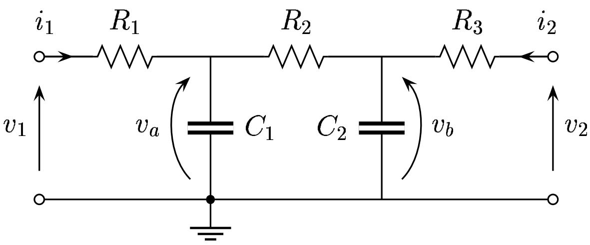

Figure 3: Example 2-port circuit element ($R_{1,3} = 150\,\Omega$, $R_2 = 200\,\Omega$, $C_1 = 10\,\mu F$, $C_2 = 1\,\mu F$).

The continuos-time state-space equations for such a circuit (*i.e.,* for the element under modeling) are

$$\begin{cases} \frac{d}{dt}\mathbf{z}(t) &= \mathbf{A}_c\mathbf{z}(t) + \mathbf{B}_c\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}_c\mathbf{z}(t) + \mathbf{D}_c\mathbf{u}(t) \end{cases} \tag{3}$$

where the state vector is $\mathbf{z} = [v_a, v_b]^T$, the output vector collects the outcoming voltage waves $\mathbf{y} = [b_1, b_2]^T$ ($b_j = \frac{1}{2}(v_j - R_0 i_j)$, $j = 1, 2$ and $R_0 = 50\ \Omega$), and the input vector collects the incoming voltage waves $\mathbf{u} = [a_1, a_2]^T$ ($a_j = \frac{1}{2}(v_j + R_0 i_j)$, $j = 1, 2$). With such definitions, the matrices of (3) write

$$\begin{aligned} \mathbf{A}_c &= \begin{bmatrix} -10^3 & 5\cdot 10^2 \\ 5\cdot 10^3 & -10^4 \end{bmatrix}, \ \mathbf{B}_c = \begin{bmatrix} 10^3 & 0 \\ 0 & 10^4 \end{bmatrix} \\ \mathbf{C}_c &= \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}, \ \mathbf{D}_c = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \end{aligned} \tag{4}$$

In order to apply subspace methods we need a discrete-time representation like (2). This is done by applying standard conversion routines such as the Matlab function `c2d` [9] to (3) and (4). In such a way, with a sampling time $T = 30\,\mu s$, the discrete-time state-space representation (2) for this specific example turns out to be defined by the following $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ matrices

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 0.97144550 & 0.01276185 \\ 0.12761851 & 0.74173217 \end{bmatrix} \\ \mathbf{B} &= \begin{bmatrix} 0.02956476 & 0.00202054 \\ 0.00202054 & 0.25927810 \end{bmatrix} \\ \mathbf{C} &= \mathbf{C}_c, \quad \mathbf{D} = \mathbf{D}_c \end{aligned} \tag{5}$$

Such state-space equations are then used to compute the response of the element under modeling $\mathbf{y}(k)$ (output identification sequence) to suitable input identification sequences $\mathbf{u}(k)$. Input and output identification sequences are processed by the subspace algorithm to estimate the unknown parameters of a state-space model. The algorithm [8] is applied to estimate matrices $\mathbf{A}$ and $\mathbf{C}$. This is done on the basis of the output identification sequence obtained by driving the element under modeling with white gaussian inputs $\mathbf{u}(k)$. Then, in order to get the best model, matrices $\mathbf{B}$ and $\mathbf{D}$ are obtained by minimizing the error between the model and the reference responses to a different input identification sequence (*e.g.,* a multilevel waveform). It can be shown that, once $\mathbf{A}$ and $\mathbf{C}$ are estimated, the response of a model defined by (2) can be turned into a linear combination of the elements of $\mathbf{B}$ and $\mathbf{D}$. In such a way, $\mathbf{B}$ and $\mathbf{D}$ can be obtained by solving a standard linear least square problem.

We checked the ability of the estimation algorithm to retrieve the parameters of the original system (*i.e.,* matrices (5)) when the identification output signals are noise free or corrupted with a superimposed gaussian noise (SNR=26 dB).

In the first case (noise free identification), the algorithm exactly estimates the matrices of the original system (5). In the second case (noisy identification), the estimated model approximates the original system very well, as shown by the validation response of Fig. 4.

The accuracy of the obtained model can be also appreciated by comparing the eigenvalues of the estimated matrix $\hat{\mathbf{A}}$ to those of the original matrix A, as shown below.

$$\begin{aligned} \mathrm{eig}(\mathbf{A}) &= \{0.97832914, 0.73484853\} \\ \mathrm{eig}(\hat{\mathbf{A}}) &= \{0.97777425, 0.73098978\} \end{aligned} \qquad (6)$$

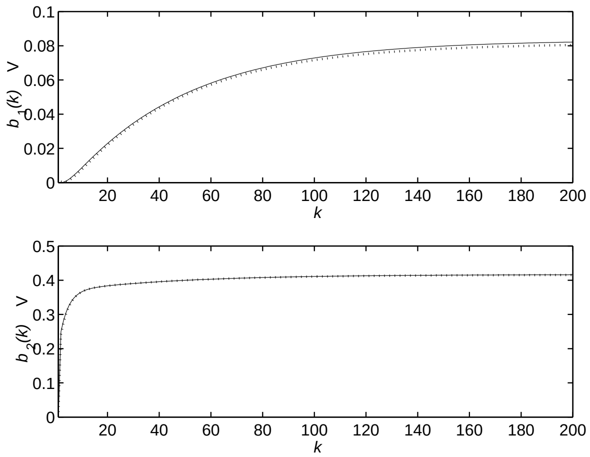

Figure 4: Discrete-time response $\mathbf{y}(k)$ of the original system and of the estimated model to the input sequence $\mathbf{u}(k) = [0, 0]^T$, $k < 0$; $\mathbf{u}(k) = [0, 0.5]^T$, $k > 0$. Solid line: reference response; dotted line: estimated model response.

4 Conclusions

We apply the BCFH and the subspace methods to the modeling of linear multiport elements from their sampled scattering characteristics. In most test cases considered, the accuracy of poles estimated by both methods turns out to be sufficient for a good fitting of the scattering characteristics.

From our tests, the poles estimated by BCFH are usually more accurate than those estimated via subspace methods. The main difficulty of BCFH is the computation of moments of the scattering characteristics and the large amount of time samples needed for such calculation. Such a difficulty, however could be overcome by different approaches to the evaluation of moments. On the other hand, subspace methods sometimes leads to models with spurious poles in the right half plane, whose elimination is an additional problem.

Though the present assessment is only preliminary, both methods seem well suited for the generation of models of linear junctions from measured or simulated scattering data to be used in real simulation problems.

Acknowledgement

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